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Automated Deduction in Equational Logic and Cubic Curves

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Preface

The aim of this monograph is to demonstrate that automated deduction is starting to become a practical tool for working mathematicians. It contains a set of problems and theorems that arose in correspondence between the authors starting early in 1993. One of us (Padmanabhan), a mathematician working in universal algebra and geometry, contacted the other (McCune), a computer scientist working in automated deduction, after reading a survey article by Larry Wos on applications of automated reasoning [80]. Padmanabhan sent McCune a few first-order theorems and asked whether Argonne's theorem prover Otter could prove them. Otter succeeded and, in a few cases, found slightly better results. The collaboration quickly took off.

We worked mostly in four areas: (1) equational proofs of theorems that had been proved previously with higher-order arguments, (2) a new equational inference rule for problems about cubic curves in algebraic geometry, (3) a conjecture about cancellative semigroups, proving many theorems that support the conjecture, and (4) equational bases with particular properties such as single axioms and independent self-dual bases. Some of the results presented here have also appeared (or will also appear) elsewhere in more detail.

The intended audience of this monograph is both mathematicians and computer scientists. We include many new results, and we hope that mathematicians working in equational logic, universal algebra, or algebraic geometry will gain some understanding of the current capabilities of automated deduction (and, of course, we hope that readers will find new practical uses for automated deduction). Computer scientists working in automated reasoning will find a large and varied source of theorems and problems that will be useful in designing and evaluating automated theorem-proving programs and strategies.

Otter (version 3.0.4) and MACE (version 1.2.0) are the two computer programs that made this work possible. Both programs are in the public domain and are available by anonymous FTP and through the World Wide Web (WWW). See either of

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ftp://info.mcs.anl.gov/pub/Otter/README
http://www.mcs.anl.gov/home/mccune/ar/
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for information on obtaining the programs. The primary documentation for the programs is [39] and [37, 38]; these are included with the programs when obtained from the above network locations. We also have the WWW document

`http://www.mcs.anl.gov/home/mccune/ar/monograph/`

associated with this work; it points to all of the Otter and MACE input files and proofs to which we refer, and it is particularly useful if the reader wishes to see precisely the search strategy we used or to experiment with related theorems.

Special thanks go to Larry Wos and to Ross Overbeek. Larry introduced the notions of strategy and simplification to automated deduction and invented the inference rule paramodulation for equality; all three of these concepts are at the center of this work. Also, Larry simplified several of our Otter proofs (in one case from 816 steps to 99!) so that they could be included in these pages. Ross is due a lot of the credit for Otter's basic design and high performance, because in building Otter, McCune borrowed so heavily from Ross's earlier theorem provers and ideas. Special thanks also go to Dr. David Kelly (of the University of Manitoba) and Dr. Stanley Burris (of the University of Waterloo) for encouraging Padmanabhan to get in touch with the Argonne group. Padmanabhan thanks Dr. Lynn Margarett Batten and Dr. Peter McClure, successive heads of the Department of Mathematics at the University of Manitoba, for creating a pleasant atmosphere conducive to creative research, without which this project could not have been completed so smoothly. We also thank Dr. Harry Lakser for compiling a Macintosh version of Otter, which enabled Padmanabhan to experiment with some of his conjectures on a Mac. And we are deeply indebted to Gail Pieper, who read several versions of the manuscript and made many improvements in the presentation.

The cubic curves in most of the figures were drawn with data generated by the Pisces software [77] developed at The Geometry Center of the University of Minnesota.

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