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Jochen Renz

Qualitative Spatial Reasoning with Topological Information

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Preface

Spatial knowledge representation and reasoning with spatial knowledge are important in many application areas such as robotics, geographical information systems, and computer vision. While most research addresses the problems connected with spatial knowledge from a purely quantitative point of view, more recently methods for representing and reasoning with spatial knowledge in qualitative ways have been developed. The motivation behind such qualitative approaches is the desire to represent and reason on a more abstract level that is closer to the way humans think and speak.

Jochen started research in this area when he did his Master's thesis in 1995 under my supervision. In 1996, the German national science foundation DFG initiated a priority research program on spatial cognition, which had as one of its topics research on the computational properties of qualitative, spatial representation approaches. Of course, Jochen became one of the researchers in this priority program. And as this book proves, this was not a mistake! His work in the priority program led to the solution of a number of open research problems and to the contribution of new research methods.

Jochen started out with the qualitative, topological constraint calculus RCC8 proposed by Randell, Cui, and Cohn. This calculus has eight different basic relations and allows one to describe spatial configurations of regions in a very abstract, qualitative way. As had been shown by Bennett – using results by Tarski – reasoning in the calculus is decidable. However, it was by no means clear, whether there are efficient ways to reason in the calculus or fragments of it. It was not clear how to represent the notion of "region" inside a computer, and it was not obvious which role the dimension of space would play for the reasoning process. Even when having answered all these questions, one might wonder what the "practical" performance of reasoning algorithms for RCC8 is? Can we deal with 10, 100, or 1000 regions? Finally, one might wonder whether the RCC8 relations are "cognitively plausible."

Jochen found answers to all of these questions. This implies that he had to use methods from logic and theoretical computer science in order to address questions concerning computational properties. The questions concerning practical performance had to be answered by large-scale empirical computational experiments. Finally, the cognitive questions were answered using the experimental, psychological paradigm. The most impressive result is pro-

bably the complete classification of computational properties for all fragments of RCC8. This result was achieved using a new method for determining the computational complexity of constraint calculi with infinite domains, a method that can be used for any qualitative constraint calculus.

All in all, I believe that this book will become one of the standard references in the area of qualitative spatial reasoning. Although it could not solve all problems in this area – which is probably impossible – it is a very large step ahead.

January 2002 Bernhard Nebel

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Remarks

This book is a slightly revised version of my PhD thesis submitted to the Faculty of Applied Science at the Albert-Ludwigs-University of Freiburg, Germany in May 2000. Parts of it have already been published in journals, in conference proceedings, and in collections. Parts of Chapter 5 appear in the *Proceedings of the 3rd International Conference on Spatial Information Theory (COSIT'97)* [102], other parts have been published in the collection *Spatial Cognition II* [145]. The psychological background (Section 5.1.) is based on a first version written by Reinhold Rauh. Parts of Chapter 6 have been published in the *Artificial Intelligence* journal [143], in the *Proceedings of the 15th International Joint Conference on Artificial Intelligence (IJCAI'97)* [141], and in the *Proceedings of the 4th International Conference on Principles and Practice of Constraint Programming (CP'98)* [68]. Parts of Chapter 7 appear in the *Proceedings of the 16th International Joint Conference on Artificial Intelligence (IJCAI'99)* [140], parts of Chapter 8 in the *Journal of Artificial Intelligence Research* [144] and in the *Proceedings of the 13th European Conference on Artificial Intelligence (ECAI'98)* [142], and parts of Chapter 9 in the *Proceedings of the 6th International Conference on Principles of Knowledge Representation and Reasoning (KR'98)* [139].

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