PERFORMANCE ANALYSIS OF CELLULAR SYSTEMS WITH DYNAMIC CHANNEL ASSIGNMENT

D.Tissainayagam and D.Everitt
Department of Electrical & Electronic Engineering
The University of Melbourne, Australia
{d.tissainayagam, d.everitt}@ee.mu.oz.au

Abstract We present a quasi-analytical method to compute the call blocking probabilities at a particular traffic density in a cellular system that employs dynamic channel assignment. The method is based on enumerating all the states in the entire state-space, a task that is beyond the capabilities of even the fastest computers when the cellular system has many cells and channels. We show how this task can be done by only focusing on the same system having just one channel. The results demonstrate that our combinatorial approach is very accurate.

Keywords: Dynamic Channel Assignment, State Space.

1. INTRODUCTION

By the end of the year 1998, the number of subscribers to a wireless communication system anywhere in the world was estimated to have been around 500 million. Presently, the average growth rate around the world is steady at just over 40%, which means that the wireless world would double in size in about 2 years' time. It has therefore become important to find means of increasing the capacity of cellular systems. This increase in capacity should not only absorb the expanding customer-base, but also maintain the present levels of quality of services. Many techniques have been employed to do just that: cell splitting [4], cell sectoring [7], frequency hopping [2], reuse partitioning [12], channel coding [9] are some of those that have already been implemented in cellular systems. A further option is to have in place a Dynamic Channel Assignment (DCA) algorithm.

Under DCA, channels are not distributed to cells in advance; all channels are deemed common to all the cells. A channel can be assigned to two callers in any two cells as long as they are separated by a 'buffer zone' of a cell's diameter. This is equivalent to saying that if a channel is used in one particular

D. H. K. Tsang et al. (eds.), *Broadband Communications*

cell, then it cannot be used in any of the surrounding ring of six cells. This constraint is known as one-cell buffering. A call request in a cell is assigned a channel if and only if it is not in use anywhere in that cell's buffer zone.

Finding analytical estimations for the increase in capacity of cellular systems under DCA has proven to be a difficult task. Good approximations have been made in [1][3] [5][6][8][11]. However, an exact analysis has not been forthcoming. This paper attempts to do that in part.

We will name the cell in which a call request arrives to be the **call-cell**. In DCA, a channel that is in use within a cell is said to be **busy**. All other channels are considered **idle** in that cell. Not all idle channels in a cell, however, can be assigned to call attempts originating there. Some of them are **unavailable** in that cell for assignment because those channels are already busy in other cells that fall within its buffer zone. **Available** channels to a cell are those that are not busy anywhere within that cell and its buffer zone. Note that only available channels of a cell are assignable to call attempts there.

We denote the state of a channel in a cell by either a '1' (if it is busy) or a '0' (if it is idle). Throughout this paper, we let n be the number of channels at the disposal of the system. Then the state of a cell can be expressed in n bits. The state of the system is simply the collective states of all the cells. The set of all possible states of a system is its **state space**. In this paper, we first calculate the number of states in the state space S of a given cellular system. Then we calculate the number of those states in S that have at least one available channel in the call-cell - when the call-cell is specified. These are the states that permit a successful call attempt in the call-cell. The above computations are entirely mathematical. Furthermore, the enumeration of such states is independent of the traffic across the system. Finally, the probability of a state in S having at least one available channel in the call-cell is estimated via simulations. These traffic density dependent estimates are then made use of in computing the probability of a call attempt being blocked in a cell.

The **State Space Method**, as we call the above approach, is explained through an example and tested on two cellular systems. Later, it is generalised to a system containing any number of cells and channels.

2. EXAMPLE : THE 7-CELL SYSTEM

We take the cellular system shown in Figure 1 to explain our State Space method initially. The system has 7 cells, n channels and uses one-cell buffering. We shall treat cell 1 as the call-cell for this example. Note that a busy channel in the call-cell cannot be assigned to another user anywhere in the system.

First let us find the number of states in this cellular system's state space S. Call that number S_n. Before doing that, a simpler problem is tackled: let S_1 be the number of states for the same system when it has just one channel - that

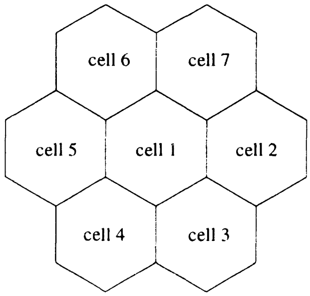

Figure 1 The 7-cell system

is, $n = 1$. S_1 can be found by direct enumeration. It turns out that there is 1 state with no calls, 7 states with one call, 9 states with two calls and 2 states with three calls. Hence,

$$S_1 = 19. \qquad (1)$$

We also find in Figure 1 that for the first state, the number of calls in progress across the system is 0. For the next seven states, it is 1 and for the nine states thereafter it is 2. The number of calls in progress across the system is 3 for each of the last two states. There can be no state where 4 or more calls are in progress. We can express the above observations neatly by a **generator polynomial** which we define thus for the 7-cell, 1-channel system:

$$G(x) = 1 + 7x + 9x^2 + 2x^3. \qquad (2)$$

The number of states with i calls in progress is simply the coefficient of x^i in the generator polynomial. Furthermore, $S_1 = G(1)$.

What if there are more than one channel in the system, though? The state of the 7-cell, n-channel system can always be broken down into a sum of the states of n identical 7-cell, 1-channel systems. An example is shown in Figure 2 for $n = 3$.

Figure 2(a) depicts the state of the 7-cell, 3-channel system. Channel #1 is busy in cell 1; channels #2 and #3 are busy in cell 2, and so on. This state can be decomposed into the states of three 7-cell, 1-channel systems as in Figures 2(b), 2(c) and 2(d). The channel that the 7-cell, 1-channel system possesses in Figure 2(b) is #1. In Figures 2(c) and 2(d), they are channels #2 and #3, respectively. We will call each of the the last three figures a *configuration*. The xth configuration only shows in which cells channel #x is busy (by a '1' in them) and idle (by a '0' in them). It will be realised that this decomposition into configurations is possible regardless of the value of n and regardless of the cellular topology of the system. Each configuration is totally independent of the rest. And each of them can exist in 19 different states - the number of states

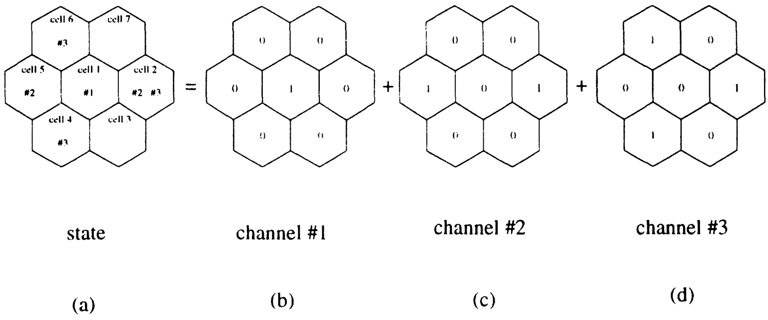

Figure 2 The break-down for a state in a 7-cell, 3-channel system

for the 7-cell, 1-channel system. Hence, the total number of possible states for the 7-cell, n-channel system is $19 \times 19 \times \ldots \times 19$ (n times). *i.e*

$$S_n = 19^n. \tag{3}$$

It will also be realised that the generator polynomial for the 7-cell, n-channel system is $G^n(x)$, where $G(x)$ has been defined in equation 2. It follows that the maximum number of calls that can be in progress, at a time, is $3n$, with each channel being busy in three cells. It also follows that $S_n = G^n(1)$.

Now, we calculate the number of states that have at least one available channel in cell 1 for the 7-cell, n-channel system. Let us call this number T_n. Focus back on the 7-cell, 1-channel system and its state space in Table 1. Out of the 19 possible states in it, only one state has the channel being available in cell 1 - namely column 1. If the system happens to be in this state when a call attempt is received in cell 1, then it will be successful. The rest of the 18 states have the channel being either busy in the call-cell (state 2) or unavailable to the call-cell (states 3 - 19). If the system were in any one of these 18 states, then a call attempt in cell 1 would be blocked. We note in passing that $T_1 = 1$, then.

Generalising to the n channel case, each of the n configurations in the decomposition of the state of the 7-cell, n-channel system can exist in 19 states, as was seen earlier. Out of which, only one state in nineteen in each configuration carries an available channel in the call-cell. Therefore, the number of states in the 7-cell, n-channel system which have *no* available channels in the call-cell is $18 \times 18 \times \ldots \times 18$ (n times). Hence,

$$T_n = S_n - 18^n = 19^n - 18^n. \tag{4}$$

Comparing equations 4 and 3, it is seen that the proportion of states in the state-space that have at least one available channel in the call-cell increases with n. For instance, just over half the states fall into this category when $n = 13$. Note that T_n can be derived from the generator polynomial $G(x)$: $T_n = G^n(1) - [G(1) - 1]^n$.

Now we move on to finding the probability that a cellular system is in a state where it has at least one available channel in the call-cell. This is an important quantity because it is also the probability that a call attempt in the call-cell will be successful. To begin with, turn again to the 7-cell, 1-channel system and its state space in Table 1. States 18 and 19 are equiprobable. This is so because under uniform traffic across the system, it does not matter in which three cells the channel is busy. Similarly, all of the states having two calls in progress are also equiprobable (states 9 to 17). Whether a state having two calls in progress is more probable than a state having three calls in progress depends on the traffic density. Under heavy traffic, the former state may be less probable than the latter state. Note also that only states 3 to 8 are equiprobable because cell 1 has six neighbours while the rest have only three; hence, the chances of a call attempt in cell 1 being successful (and being in state 2) are less than the chances of a call attempt elsewhere being successful.

Let the probability of this system having no calls in progress be π_0. Let the probability of cell 1 having a busy channel be π_a and the probability of any other cell alone having a busy channel be π_b. Also let the probabilities that the system has two and three calls in progress be π_2 and π_3, respectively. It must be noted that theses probabilities depend on the traffic density in the system. As the system has to be in one of the 19 states possible, we have that

$$\pi_0 + \pi_a + 6\pi_b + 9\pi_2 + 6\pi_3 = 1. \tag{5}$$

Now,

Prob (call attempt in cell 1 is successful) $=$ Prob (system in state 1) $= \pi_0$.

Therefore, the blocking probability for the call-cell in the 7-cell, 1-channel system is given by

$$B_1 = 1 - \pi_0. \tag{6}$$

With increased traffic density, π_0 decreases and B_1 increases.

Now, these probabilities are estimated when the system has n channels. Firstly, an assumption is made: the DCA algorithm used by the system is such that it does not consistently choose one particular available channel over another in the call-cell. This is necessary to maintain the independence assumption between configurations. Therefore, the responsibility of assigning an available channel to the offered traffic stream in the call-cell falls evenly on all n channels. Hence, it is seen that the probability that a channel is busy in 0, 1, 2 or 3 cells in the 7-cell, n-channel system under a uniform traffic density of ρ Erlangs per cell is the same as the probability that the channel is busy in 0, 1, 2 or 3 cells in the 7-cell, 1-channel system under a uniform traffic density of ρ/n Erlangs per cell, respectively. Having found $\pi_0, \pi_a, \pi_b, \pi_2$ and π_3 via this technique,

$$\begin{aligned} & \text{Prob (call attempt in cell 1 is successful)} \\ = \; & \text{Prob (at least one available channel in cell 1)} \\ = \; & 1 - \text{Prob (no available channels in cell 1)} = 1 - (1 - \pi_0)^n. \end{aligned}$$

Therefore, the blocking probability for the call-cell in the 7-cell, n-channel system is given by

$$B_n = (1 - \pi_0)^n. \tag{7}$$

3. APPLICATION TO THE 25-CELL SYSTEM

The State Space method is now applied to a larger cellular system shown in Figure 3. The number of channels at its disposal is n. One-cell buffering is used. We refrain from specifying a particular cell as the call-cell. The reason is that we want the estimated blocking probabilities to be applicable to any cell in the system - not just for cell 1 as in our example. To do that, first all cells must be made identical. Hence, we only analyse the system under uniform traffic density. Furthermore, all the cells are made to have exactly six neighbouring cells. This is achieved by juxtaposing replicas of the system to the top and bottom, and left and right of it. The resulting toroidal topology of the system is edgeless. Figure 3 depicts how cells 1 and 5, or cells 3 and 23 are really adjacent to each other.

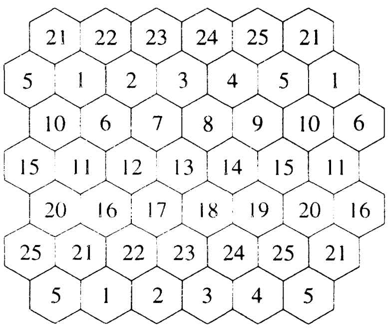

Figure 3 The 25 cell system

The example showed that all enumeration can be done from the generator polynomial of a cellular system with $n = 1$. Therefore, the generator polynomial for the 25-cell, 1-channel system is found first. It turns out to be

$$G(x) = 1 + 25x + 225x^2 + 900x^3 + 1600x^4 + 1100x^5 + 225x^6. \tag{8}$$

A short program was written to directly enumerate all possible states on a computer to determine the coefficients above. Therefore, the total number of states in the state space of the 25-cell, 1-channel system is given by

$$S_1 = G(1) = 4076. \tag{9}$$

Out of all those, how many states have the channel as available in one particular cell? It is not simply the first of the 4076 states; the one where no call is in progress in the system. This happened to be the case in our example because the call-cell and its buffer zone comprised the entire system. Now, there can be calls in progress on a channel outside a particular cell's buffer zone and still that channel would be available in that cell. Hence, we need to find the number of states in the state space where no calls are in progress within a particular cell and its buffer zone. The result is the following generator polynomial:

$$G_0(x) = 1 + 18x + 108x^2 + 256x^3 + 220x^4 + 54x^5. \tag{10}$$

This means that if, say, cell 13 happens to the call-cell, then there are 54 states where the call-cell and its buffer zone have no calls in progress and the rest of the cells collectively have five calls in progress - in cells 1, 4, 16, 19 and 23, for example. The point is that every state that contributes to $G_0(x)$, and there are $G_0(1) = 657$ of them, allows the call-cell to have the channel as available. And every state that does not contribute to $G_0(x)$ makes the channel unavailable to the call-cell. Generalising to the 25-cell, n-channel system, we have the following: the total number of state in the state space is

$$S_n = G^n(1) = 4076^n. \tag{11}$$

and number of states that have at least one available channel in a particular cell is

$$T_n = G^n(1) - [G(1) - G_0(1)]^n = 4076^n - 3419^n. \tag{12}$$

It takes just four channels to have a state space where half the elements in it would have at least one available channel to a given cell.

What is the probability that the 25-cell, 1-channel system is in a state where a call attempt in a cell will be successful? Note that there are a_i states, with i calls in progress in each, that have the channel as available in a cell. Here, a_i's are the coefficients of the generator polynomial $G_0(x)$. The total probability that the system is in one of these states is then given by $\sum_{i=0}^{5} a_i \pi_i$, where π_i is the probability that the channel is busy in i cells at some particular uniform traffic density ρ Erlangs per cell. The π_i's also have to satisfy the following equation (*c.f.* Equation 5):

$$\pi_0 + 25\pi_1 + 225\pi_2 + 900\pi_3 + 1600\pi_4 + 1100\pi_5 + 225\pi_6 = 1. \tag{13}$$

The probability of blocking for a call attempt in any cell is thus given by

$$B_1 = 1 - (\pi_0 + 18\pi_1 + 108\pi_2 + 256\pi_3 + 220\pi_4 + 54\pi_5). \tag{14}$$

Just as in the example, the probability of blocking in any cell in the 25-cell, n-channel system, at a traffic density of ρ Erlangs per cell, can be expressed as

$$B_n = [1 - (\pi_0 + 18\pi_1 + 108\pi_2 + 256\pi_3 + 220\pi_4 + 54\pi_5)]^n. \qquad (15)$$

The π_i's have to be obtained from simulating the 25-cell, 1-channel system at a uniform traffic density of ρ/n Erlangs per cell.

4. RESULTS

The State Space method for calculating blocking probabilities is put to the test in the 25-cell system with $n = 45$. The traffic density ρ was varied from 8 to 20 Erlangs per cell. A simulation is run for the 25-cell, 1-channel system to obtain the probabilities $\pi_i, i = 0, 1, 2, 3, 4, 5$ and 6. Note that in the simulations the offered traffic would have to be in the range $\frac{8}{45} - \frac{20}{45}$ Erlangs per cell. Note also that no consideration need be given to the choice of DCA algorithm that has to be employed in the system - as there is only one channel. Once the π_i's are found, the blocking probabilities are estimated from equation 15. These estimates are compared against the blocking probabilities obtained by running the full simulation for the 25-cell, 45-channel system. Here, a DCA algorithm is necessary and the one chosen was called Random Channel Search algorithm [10]. This algorithm assigns any available channel, picked randomly, in the call-cell to a call attempt there and blocks it if none is available.

The comparison of blocking probabilities is shown in Figure 4. The State Space method approximates the actual call blocking probabilities very well across all traffic densities.

5. CONCLUSIONS

We proposed a novel method for estimating the call blocking probabilities for a cellular system with arbitrarily many cells and arbitrarily many channels. For a given system, by assuming that only one channel is available for users anywhere in the system, two generator polynomials $G(x)$ and $G_0(x)$ are found. This allows us to compute the total number of states in the state space when there are n channels in the system that can be assigned dynamically. The number of states that have at least one available channel in a particular cell can also be found. Both of the above computations are exact. The probabilities of call blocking at some traffic density can be estimated if the probabilities of i calls in progress in the system is known in advance. We approximate them from simulations. The simulations are confined to the simple case when the system has just one channel. The results demonstrate that the State Space method is accurate.

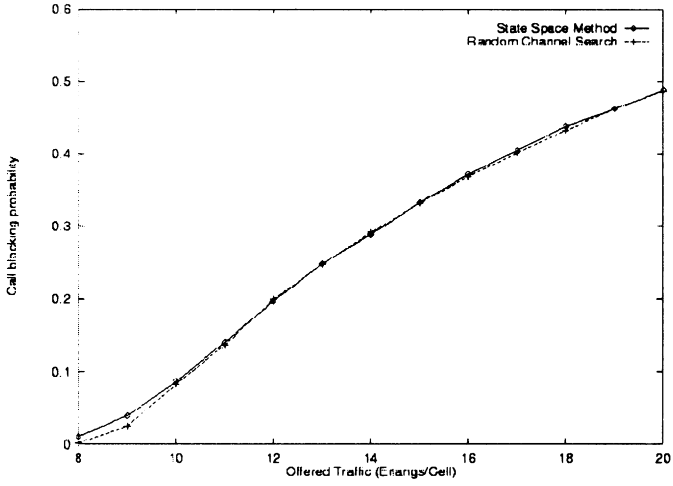

Figure 4 Call blocking probabilities for the 25-cell, 45-channel system

References

[1] D.Everitt and D.Mansfield, "Performance Analysis of Cellular Mobile Communication Systems with Dynamic Channel Assignment", *IEEE Journal on Selected Areas in Communications*, Vol. 7, pp. 1172-1180, 1989.

[2] M.Madfors, K.Wallstedt, S.Magnusson, H. Olofsson, P-O.Backman and S.Engstrom, "High Capacity with Limited Spectrum in Cellular Systems", *IEEE Communications Magazine*, Vol. 35, pp. 38-45, 1997.

[3] D.L.Pallant, "A Reduced Load Approximation for Cellular Mobile Networks Including Handovers", *Aust. Tel. Res.*, Vol. 26, pp. 21-29, 1992.

[4] T.Rappaport, "Wireless communications : principles and practice", IEEE Press, 1996.

[5] P.Raymond, "Performance Analysis of Cellular Networks", *IEEE Transactions on Communications*, Vol. 39, pp. 1787-1793, 1991.

[6] K.N.Sivarajan, R.J.McEliece and J.W.Ketchum, "Dynamic Channel Assignment in Cellular Radio", *Proc. of the 40th IEEE Vehicular Technology Conference*, pp. 631-637, 1990.

[7] G.Stuber, "Principles of Mobile Communication", Kluwer Academic, Boston, 1996.

[8] D.L.Pallant and P.G.Taylor, "Approximation of Performance Measures in Cellular Mobile Networks with Dynamic Channel Allocation", *Telecommunication Systems*, Vol. 3, pp. 129-163, 1994.

[9] J.Tisal, "GSM cellular radio telephony", Wiley, 1997.

[10] D.Tissainayagam, D.Everitt and M.Palaniswami, "Self Organising Feature Map Implementation of the Random Channel Search Algorithm", *Proc. of Asia-Pacific Conference on Communications*, Vol. 2, pp. 921-925, Dec. 1997.

[11] P.Whiting and S.Borst, "Performance Bounds for Dynamic Channel Assignment Schemes Operating under Varying Reuse Constraints", in *Proc. of INFOCOM '98*, Atlanta, USA, April 1998.

[12] J.Zander and M.Frodigh, "Capacity Allocation and Channel Assignment in Cellular Radio Systems Using Reuse partitioning", *Electronic Letters*, Vol. 28, No. 5, pp. 19-25, Feb. 1992.