# PERFORMANCE ANALYSIS OF RATE-BASED FLOW CONTROL UNDER A VARIABLE NUMBER OF SOURCES

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Abstract Rate-based flow control plays an important role for efficient traffic management of ABR service in ATM networks. In this paper, a performance analysis of a rate-based flow control mechanism is presented. In our analytical model, the number of active sources is variable. A new source arrives when a connection is established, and an existing source departs when it has transmitted its data. Hence our model not only reflect the real scenes, but also correctly estimate the effect of the rate-based flow control.

Due to this variation, the analysis of the steady state is not enough. Therefore the analysis of transient cycles is also developed. Using the results of both analyses, we derive the equations of cell loss probability and utilization.

Keywords: ATM Networks, ABR(Available Bit Rate), Rate-based Flow Control

# 1. INTRODUCTION

ATM (Asynchronous transfer mode) is the most promising transfer technology for implementing B-ISDN. It supports applications with distinct QoS requirements such as delay, jitter, and cell loss and with distinct demands such as bandwidth and throughput. To provide these services for a wide variety of applications, in addition to CBR (Constant bit rate), rt-VBR (Real-time variable bit rate), nrt-VBR (Non-Real-Time VBR), and UBR (Unspecified bit Rate) services, the ATM forum defined a new service class known as ABR (Available bit rate) service to support data applications economically. Also an end-toend adaptive control mechanism called closed-loop rate-based flow control is applied to this service. In this control scheme, the allowed cell transmission rate of each ABR connection is dynamically regulated by feedback information from the network [1-3]. If the network is congested, the source end decreases its cell transmission rate when it receives the congestion indication. Also, the source end increases its cell transmission rate when congestion is relieved. The rate-based control mechanism could efficiently control the connection flows and utilize the network bandwidth.

Recently several analyses and simulations have been conducted for ratebased control schemes. First Bolot and Shankar used differential equations to model the rate increase and decrease [4]. Yin and Hluchyj proposed analytical models for early versions of ABR control with a timer-based approach [5,6]. Ramamurthy and Ren developed a detailed analytical model to capture the behavior of a rate-based control scheme and obtain approximate solutions in closed forms [7]. Ohsaki et al. made an analysis and comparison between different switches in the steady state and initial transient state [8,9,10]. Ritter derived the closed form expression to quickly estimate the buffer requirements of different switches [11]. We derived the equations of cell loss probability and utilization for the rate-based control, and provided some rules to reduce cell loss probability and raise utilization [12]. These papers provide much insight into the effect of using the rate-based flow control. However, none of these papers consider the variable number of sources.

These researches under the assumption of a fixed number of sources are inaccurate. First, in real conditions, the number of sources is variable; a new connection may be established and an existing connection may be released. Second, using the rate-based control, the most obvious oscillations in buffer size happen at the time of a new source arrival or an existing source departure. Ignoring the fact of the variation about the number of sources does not correctly show the impact of rate-based control.

In this paper, we assume the number of sources to be variable to reflect the real conditions. An existing source may depart and a new source may arrive. We analyze the rate-based flow control under a variable number of sources, and deduce the equations of cell loss probability and utilization. In order to get the precise values of these parameters, besides the analysis for the steady state, the analysis for a transient state is developed. This is because the dynamic behavior is very different from the stable one when an existing source departs or a new source arrives.

### 2. ANALYTICAL MODEL

First we briefly introduce the basic operation of a close-loop rate-based control mechanism [3]. When a connection is established, the source end system (SES) sends the cells at the allowed cell rate (ACR) which is set as initial cell rate (ICR). In order to probe the congestion status of the network,

The SES sends a forward Resource Management (RM) cell every  $N_{RM}$  data cells. The destination end system (DES) returns the forward RM cell as a backward RM cell to the SES. Depending on the received backward RM cell, the SES adjusts its allowed cell rate, which is bounded between Peak cell rate (*PCR*) and Minimum cell rate (*MCR*).

The RM cell contains a 1-bit congestion indication (CI) which is set to zero, and an explicit rate (ER) field which is set to PCR initially by the SES. Depending on the different ways to indicate the congestion status, two types of switches are implemented. One is the Explicit Forward Congestion Indication (EFCI) switch, the other is the Explicit Rate (ER) switch. In the EFCI type, the switch in the congestion status sets the EFCI bit to one (EFCI=1) in the header of each passing data cells. The DES, if a cell with EFCI=1 has been received, marks the CI bit (CI=1) to indicate congestion in each backward RM cells. In the ER type, the switch sets the EFCI bit of the RM cells to indicate whether there is congestion or not, and sets the ER field to indicate the bandwidth the connection should use. The performance results and comparisons between the two types of switches are shown in [10] in detail.

When the SES receives a backward RM cell, it modifies its ACR using additive increase and multiplicative decrease. Depending on CI, the new ACR is computed as follows:

$$ACR = \max(\min(ACR + N_{RM} \cdot AIR, ER), MCR), \text{ if CI=0},$$
$$ACR = \max(\min(ACR \cdot (1 - \frac{N_{RM}}{RDF}), ER), MCR), \text{ if CI=1},$$

where AIR is the additive increase rate and RDF is the rate decrease factor. AIR and RDF are defined in the traffic management specification version 4.0 [3]. Although the new version of the specification has made some changes about AIR and RDF [13], the analysis for the new notations is easily translated from this paper.

In this paper, we focus on the EFCI switch and use a simple model as shown in Fig. 1. There are some ABR sources sharing a bottleneck link where the bandwidth is BW. We assume that these sources are homogeneous; that is, they all have the same parameters ICR, PCR, MCR, AIR and RDF. The number of sources is variable. Source arrival is according to a Poisson process and the duration of a source is according to an exponential process. Let  $\lambda$  be the source arrival rate and  $1/\mu$  be the mean source duration. Thus, the distribution of the number of sources is the same as the number in the system for an  $M/M/\infty$ queue. Also we assume that each source always has cells to send, i.e. it has infinite backlog. This assumption allows us to investigate the performance of an EFCI switch in the most stressful situations.

The buffer size at the switch is denoted by  $Q_B$ . The switch determines the congestion condition according to its queue length. There are two values, high



Figure 1 Analytic model for the rate-based flow control.

threshold  $Q_h$  and low threshold  $Q_l$ , which decide whether congestion occurs or not. When the queue length exceeds  $Q_h$ , the EFCI bit of passing data cells is set to one to indicate congestion. The congestion is relieved when the queue length drops below  $Q_l$ .

We define  $\tau_{sx}$  as the propagation delay between the SES and the switch, and  $\tau_{xd}$  as the propagation delay between the switch and the DES. Also, the feedback propagation delay from the switch to the SES is denoted by  $\tau_{xds}$  and the round trip propagation delay is denoted by  $\tau$ . Thus we get the relation  $\tau_{xds} = \tau_{sx} + 2\tau_{xd}$  and  $\tau = 2(\tau_{sx} + \tau_{xd})$ . The propagation delay is a critical parameter of system performance.

### **3. TRANSIENT CYCLES**

For lack of space, in this paper we skip the analysis of the case that the number of sources is fixed. However, those results are the base for the analysis of the case that the number of sources is variable. Hence the readers ought to reference the paper

[12] to know the meaning of the notations and the derivation of the analysis of steady state.

As we know, the cell loss probability and utilization during a transient cycle are determined by the time instant when the number of sources is changed. The worst case to cell loss probability happens as a new source arrives at the time instant when the high threshold is reached,  $t_{Q_h}^-$ . The reason is as follows. If a new source arrives after  $t_{Q_h}^-$ , this new source does not send its cells during the time interval between congestion detection and this new source arrival. Hence,

cell loss probability is smaller. On the other hand, the congestion is detected earlier if a new source arrives before  $t_{Q_h}^-$ . Therefore, the maximum ACR is smaller, which causes cell loss probability to be smaller [11]. Thus, the closer the new source arrival to  $t_{Q_h}^-$ , the higher cell loss probability we got. Similarly the worst case of utilization is that an existing source departs at the time instant when the low threshold is attained,  $t_{Q_l}^-$ . In this section, we only consider these worst cases. Therefore, we shall obtain the upper bound of cell loss probability and lower bound of utilization during transient cycles.

#### **3.1 SOURCE DEPARTURE**

Without loss of generality, we assume that the number of active sources is *i*. When an existing source departs at the time instant  $t_{Q_l}^-$ , the speed of adjusting ACR is not changed immediately. After the switch has sent all the cells in the buffer, then the SES speeds up its adjustment of the ACR. That is, the rate of SES receiving the backward RM cells is changed from  $BW/iN_{RM}$  to  $BW/(i-1)N_{RM}$  after the time  $Q_l/BW + \tau$ . Now we derive utilization at worst case.

The minimum rate,  $ACR_{min}$ , in a transient cycle is the same as in the steady state. Thus we get the time interval,  $\Delta \tilde{t}_{Q_{min}}^-$  [*i*-1], during a transient cycle where the number of active sources changes from *i* to *i*-1, by solving the equation,

$$(ACR_{min}[i] + \frac{BW \cdot AIR}{i} \cdot \frac{Q_l}{BW})(i-1) + BW \cdot AIR \cdot (\Delta \tilde{t}_{Q_{min}}[i-1] - \frac{Q_l}{BW}) = BW.$$

The minimum queue length,  $\hat{Q}_{min}[i-1]$ , during a transient cycle where the number of active sources changes from *i* to *i*-1, is given by

$$\tilde{Q}_{min}[i-1] = Q_l - \int_{\Delta t_{Q_l}}^{\Delta t_{Q_l} + \tau} (BW - (i-1) \cdot ACR_{t_0}(t)) dt$$

$$-\int_{0}^{rac{Q_{l}}{BW}}(BW-(i-1)(ACR_{min}[i]+rac{BW\cdot AIR}{i}t))dt$$

$$-\int_{0}^{(\Delta \tilde{t}_{Q_{min}}^{-}[i-1]-\frac{Q_{l}}{BW})} (BW-(i-1)(ACR_{min}[i]+\frac{Q_{l}AIR}{i})-BW\cdot AIR\cdot t)dt.$$

Then we get

$$\tilde{Q}_{min}[i-1] = Q_l - \tau \cdot BW + (i-1) \cdot RDF \cdot e^{-\frac{BW}{i \cdot RDF} \cdot \Delta t Q_l} \cdot (1 - e^{-\frac{BW}{i \cdot RDF} \cdot \tau})$$

$$-\frac{(2BW - (i-1)(2ACR_{min}[i] + \frac{Q_lAIR}{i}))\frac{Q_l}{BW}}{2} - \frac{(BW - (i-1)(ACR_{min}[i] + \frac{Q_lAIR}{i}))^2}{2 \cdot BW \cdot AIR}$$
(1)

After the queue length reaches the minimum, the system enters the steady state where  $N_{VC}$  is equal to *i*-1. Hence cell loss probability and utilization during this transient cycle are calculated as

$$\tilde{\rho}_{1}[i-1] = 1 - \frac{\tilde{N}_{waste1}[i-1]}{\tilde{T}_{cycle1}[i-1] \cdot BW}, \text{ and}$$

$$\tilde{P}_{loss1}[i-1] = \frac{N_{loss}[i-1]}{\tilde{T}_{cycle1}[i-1] \cdot \tilde{\rho}_{1}[i-1] \cdot BW + N_{loss}[i-1]}.$$
(2)

where

$$\tilde{N}_{waste1}[i-1] = \max(0, 0 - \tilde{Q}_{min}[i-1])$$

 $\tilde{T}_{cycle1}[i-1] = 2\tau + \Delta t_{Q_l}[i-1] + \Delta \tilde{t}_{Q_{min}}^-[i-1] + \Delta t_{Q_h}[i-1] + \Delta t_{Q_{max}}^-[i-1].$ 

# **3.2 SOURCE ARRIVAL**

In the steady state or the transient cycle of a source departure, because the ACR of each source is the same, the speed of adjusting the ACR is the same among all sources. Therefore, we do not need to take care the scenarios that different sources bring. In contrast, in the transient phase of a new source arrival, the ACR of the new source is different from the existing sources. Then, for this new source, the speed of adjusting the ACR is also different from the existing sources. It depends on a '*rate ratio*' a queueing delay before, i.e., right before the RM cell joins the switch queue. The rate ratio, R, is the ratio between the ACR of the new source and the ACR of the existing sources.

Since the rate ratio and queueing delay are changed continuously, analyzing the speed of adjusting the ACR during this transient cycle is very difficult. We introduce the concept of 'average interval' to alleviate this difficulty. The forepart of this transient cycle is divided into some average intervals, whose lengths are not fixed, as shown in Fig 2. In the *n*th average interval  $\Delta I_n$ , the switch sends all the cells which have been sent by the sources in the previous average interval  $\Delta I_{n-1}$ . Also the variable behavior of rate ratio is approximated by a constant, which is the ratio between the average ACR of the new source and the average ACR of the existing sources during this average interval. Hence the speed of adjusting ACR in the  $\Delta I_n$  completely depends on the rate ratio,  $R_{n-1}$ , in the  $\Delta I_{n-1}$ . The approximation works well when the length of average intervals is not large.

The evolution of the number of active sources.

When a new source arrives at the time instant  $\Delta t_{Q_h}^-$ , the SES sends its cells at the rate *ICR*. The new source keeps this rate until the first RM cell returns. Let the first average interval begin at the time instant  $t_{Q_h}^-$ , and end at the time instant when the first RM cell of this new source returns. Hence the length of



Figure 2 Transient cycle of a new source arrival.

the first average interval is

$$\Delta I_1 = \frac{Q_h}{BW} + \tau.$$

For simplicity of presentation, let  $ACR''_j(t)$  and  $ACR'_j(t)$  be the dynamic behavior of the new source and of those existing sources in the *j*th average interval, respectively. Note that *t* is the escaped time from the beginning time of the average interval. Now we get the dynamic behavior of the new source and the existing sources in the first average interval as

$$\begin{aligned} ACR_1''(t) &= ICR, \quad 0 \le t < \Delta I_1, \\ ACR_1'(t) &= \frac{BW}{i} (1 + AIR(\Delta t_{Q_h} + t)), \quad 0 \le t < \tau, \\ ACR_1'(t) &= \frac{BW}{i} (1 + AIR(\Delta t_{Q_h} + \tau))e^{-\frac{BW}{iRDF}(t-\tau)}, \quad \tau \le t < \Delta I_1. \end{aligned}$$

The rate ratio  $R_1$  is given by

$$R_{1} = \frac{\int_{0}^{\Delta I_{1}} ACR_{1}''(t)}{\int_{0}^{\Delta I_{1}} ACR_{1}'(t)}$$

The system is in phase II or IV after the first average interval, so the dynamic behavior is as

$$ACR''_{n}(t) = ACR''_{n}(0)e^{-\frac{BW \cdot R_{n-1}}{(i+R_{n-1})RDF}t}, \quad 0 \le t < \Delta I_{n},$$
$$ACR'_{n}(t) = ACR'_{n}(0)e^{-\frac{BW}{(i+R_{n-1})RDF}t}, \quad 0 \le t < \Delta I_{n}.$$

where  $ACR''_{n}(0)$  and  $ACR'_{n}(0)$  equal to  $ACR''_{n-1}(\Delta I_{n-1})$  and  $ACR'_{n-1}(\Delta I_{n-1})$ , respectively. The length of the second and *n*th average intervals is

$$\Delta I_2 = \frac{\int_0^{\Delta I_1} (iACR'_1(t) + ACR''_1(t))dt}{BW} - \tau,$$

$$\Delta I_n = \frac{\int_0^{\Delta I_{n-1}} (iACR'_{n-1}(t) + ACR''_{n-1}(t))dt}{BW}, \quad n > 2.$$

The rate ratio of the *n*th average interval is

$$R_n = \frac{\int_0^{\Delta I_n} ACR''_n(t)dt}{\int_0^{\Delta I_n} ACR'_n(t)dt}$$

Without loss of generality, we assume that the queue length reaches maximum at the mth average interval.

$$iACR'_m(\tilde{t}[i+1]) + ACR''_m(\tilde{t}[i+1]) = BW.$$

Thus we get

$$ilde{Q}_{max}[i+1] = (\Delta I_m - ilde{t}[i+1])BW + \int_0^{ ilde{t}[i+1]} (iACR'_m(t) + ACR''_m(t))dt$$

We assume that the system enters the steady state after the time instant when the queue length reaches the maximum. Hence cell loss probability and utilization during this transient cycle is calculated as

$$\tilde{\rho}_2[i+1] = 1 - \frac{N_{waste}[i+1]}{\tilde{T}_{cycle2}[i+1] \cdot BW},$$

$$\tilde{P}_{loss2}[i+1] = \frac{\tilde{N}_{loss2}[i+1]}{\tilde{T}_{cycle2}[i+1] \cdot \tilde{\rho}_{2}[i+1] \cdot BW + \tilde{N}_{loss2}[i+1]},$$
(3)

where

$$\tilde{N}_{loss2}[i+1] = \max(0, \tilde{Q}_{max}[i+1] - Q_B),$$

 $\tilde{T}_{cycle2}[i+1] = 2\tau + \Delta t_{Q_h}[i+1] + \sum_{j=1}^{m-1} \Delta I_j[i+1] + \tilde{t}[i+1] + \Delta t_{Q_l}[i+1] + \Delta t_{Q_{min}}^-[i+1].$ 

# 4. COMBINATION OF STEADY STATE AND TRANSIENT CYCLES

Now we derive the equations of cell loss probability and utilization. Remind that source arrival is according to a Poisson process with parameter  $\lambda$ , and the source existence duration is according to an exponential process with parameter  $1/\mu$ . The distribution of the number of sources is the same as the number in the system for an M/M/ $\infty$  queue. Hence we get

$$P_{i} = \frac{\left(\frac{\lambda}{\mu}\right)^{i}}{i!} e^{-\frac{\lambda}{\mu}},$$
$$T_{i} = \frac{1}{\lambda + i\mu},$$



Figure 3 The evolution of the number of active sources.

where  $P_i$  is the probability that *i* sources are active in the system, and  $T_i$  is the mean duration when *i* sources are active.

Because the event that a new source arrives or an existing source departs happen not very often,  $\lambda$  and  $\mu$  are small. Therefore many cycles may pass by between two events. There are few opportunities that an event happens during a transient cycle. So the behavior of system looks like Fig. 3. When there are i + 1 sources in the system and an existing source departs, the system passes the time interval of a transient cycle, and then enters the steady state. Hence the time of the system staying in the transient cycle is  $\tilde{T}_{cycle1}[i]$ , and the time of the system staying in the steady state is  $T_i - \tilde{T}_{cycle1}[i]$ . We use the probability method to obtain cell loss probability and utilization as

$$P_{loss1}[i] = \tilde{P}_{loss1}[i] \frac{\tilde{T}_{cycle1}[i]}{T_i} + \bar{P}_{loss}[i] \frac{T_i - \tilde{T}_{cycle1}[i]}{T_i},$$

$$\rho_1[i] = \tilde{\rho}_1[i] \frac{\tilde{T}_{cycle1}[i]}{T_i} + \bar{\rho}[i] \frac{T_i - \tilde{T}_{cycle1}[i]}{T_i}.$$
(4)

When there are i - 1 sources in the system and a new source arrives, similarly we get  $P_{loss_2}[i]$  and  $\rho_2[i]$ .

As described above, there are two ways that the system enters the condition that *i* sources are active. The departure rate of an existing source when there are i + 1 sources is  $P_{i+1}(i+1)\mu$ . On the other hand, the arrival rate of a new source when there are i - 1 sources is  $P_{i-1}\lambda$ . Hence the probability of first way is  $\frac{P_{i+1}(i+1)\mu}{P_{i+1}(i+1)\mu+P_{i-1}\lambda}$ , and the probability of second way is  $\frac{P_{i-1}\lambda}{P_{i+1}(i+1)\mu+P_{i-1}\lambda}$ . Also according to the local-balance equation of the queueing theory, the rate  $P_{i+1}(i+1)\mu$  is equal to  $P_i\lambda$ , and the rate  $P_{i-1}\lambda$  is equal to  $P_ii\mu$ . Therefore the probabilities of first and second ways are  $\frac{\lambda}{\lambda+i\mu}$  and  $\frac{i\mu}{\lambda+i\mu}$ , respectively.

Finally we obtain cell loss probability and utilization as

$$P_{loss} = \sum_{i=0}^{\infty} P_i \left(\frac{\lambda}{\lambda + i\mu} P_{loss1}[i] + \frac{i\mu}{\lambda + i\mu} P_{loss2}[i]\right),$$

$$\rho = \sum_{i=0}^{\infty} P_i \left(\frac{\lambda}{\lambda + i\mu} \rho_1[i] + \frac{i\mu}{\lambda + i\mu} \rho_2[i]\right). \tag{5}$$

Although the summation is infinite, we have to limit the number of sources. We can set a reasonable bound for  $N_{VC}$  so that boundary states have probability very close to 0.

# 5. CONCLUSION

When the variation of the number of sources is neglected, the derived cell loss probability and utilization do not correctly reflect the effect of the ratebased flow control. Meanwhile, although the variation in the number of sources is considered, the results of ignoring the transient cycles are still unsatisfactory. In this paper, an accurate analysis for rate-based flow control under a variable number of sources is provided.

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