Topological Methods in Complementarity Theory

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by

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Springer-Science+Business Media, B.V.

A C.I.P. Catalogue record for this book is available from the Library of Congress.

ISBN 978-1-4419-4828-1 ISBN 978-1-4757-3141-5 (eBook)
DOI 10.1007/978-1-4757-3141-5

Printed on acid-free paper

Originally published by Kluwer Academic Publishers in 2000.
Softcover reprint of the hardcover 1st edition 2000

To my parents Dumitru and Elena Isac

"Ne vous occupez pas des fautes
d'autrui, ni de leurs actes,
ni de leurs négligences. Soyez plutôt conscients de vos
propres actes et de vos propres négligences."

(Bouddha)

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PREFACE

This book is intended for mathematicians, engineers, economists, and specialists working in operations research or in optimization and for anybody interested in applied mathematics or in mathematical modelling.

The Complementarity Theory, is a new domain in applied mathematics and its subject is the study of complementarity problems. Complementarity problems represent a wide class of mathematical models related to optimization,game theory, economics, engineering, mechanics, elasticity, fluid mechanics, stochastic optimal control et cetera.

The complementarity condition is a kind of general equilibrium containing the equilibrium in the physical sense and in the economical sense.

The concept of equilibrium is central to the understanding of many problems in physics, engineering, economics, and other fields. Equilibrium is frequently used in the study of competitive systems arising in different disciplines.

Particularly, in economics, examples of equilibrium problems include: markets in which firms compete to determine their profit-maximizing production outputs, general economic equilibrium problems in which all the commodity prices are to be determined, congested urban transportation systems in which users seek to determine their cost-minimizing routes of travel et cetera.

An interesting characteristic of Complementarity Theory is the fact that it has multiple connections with other domains including: Linear Algebra, Functional Analysis, Topology, the Fixed Point Theory, the Theory of Variational Inequalities, the Topological Degree and Numerical Analysis. Because the diversity of its application, the Complementarity Theory is a good stimulant for research in fundamental mathematics.

This book is especially dedicated to the study of nonlinear complementarity problems in infinite dimensional spaces.

The literature on this subject is very large. Concerning the subjects presented in this book, we selected only results related to some topological methods and susceptible to new developments. The numerical methods are not considered, as this subject may be developed in a future book.

The structure of this book is as follows:

In the first *Chapter* we present the necessary background on topological vector spaces and especially on convex cones in topological vector spaces. Several classes of cones used currently in Complementarity Theory are studied.

The origins of Complementarity Theory and the definitions of the most important complementarity problems are presented in *Chapter 2.*

Chapter 3 is devoted to the description of a long list of mathematical models based on complementarity problems.

The study of many complementarity problems is based on the fact that some particular complementarity problems are equivalent to some special nonlinear functional equations. These equivalencies are presented in *Chapter 4.*

Chapter 5 is large, as it is, dedicated to the study of several solvability theorems. In this chapter we present several classical and some recent existence results.
Applications of the topological degree to the study of complementarity problems are presented in *Chapter 6.*

The concept of zero-epi mapping is a new concept, similar to the concept of topological degree, but much simpler and more refined. Zero-epi mappings and their application to the Complementarity Theory are presented in *Chapter 7.*

A new topological method, recently introduced in Complementarity Theory, is based on the concept of Exceptional Family of Elements for a continuous mapping. This concept is related to the Leray-Schauder alternative. *Chapter 8* is dedicated to this subject.

Condition $(S)_+$ and $(S)_+^1$ were introduced as a good substitute of compactness when this is missing. Several applications of condition $(S)_+$ and $(S)_+^1$ to the Complementarity Theory are presented in *Chapter 9.*

It is well known that the Complementarity Theory has interesting and deep relations with the Fixed Point Theory. *Chapter 10* is dedicated to this subject.

Finally, in *Chapter 11* we present some recent topological results based on a special topological index on cones, or on the Mountain Pass Theorem or on connectedness. An application of the concept of Exceptional Family of Elements to the study of the Implicit Complementarity Problem is also given in this last chapter.

Each chapter is followed by *References* and the book is concluded with a *Bibliography* on complementarity problems.

This book is an interesting volume for graduate courses and it covers in particular our book "**Complementarity Problems** *Lecture Notes in Mathematics, Nr. 1528, Springer-Verlag (1992)* ".

I would like to express my sincere thanks to my friends, Prof. M. M. Kostreva (Clemson University), Prof. A. Ebiefung (The University of Tennessee at

Chattanooga) and Prof. Byung-Soo Lee (Kyungsung University) for their fruitful remarks and improvements of the manuscript of this book. The author is very much obliged to each of them. Many thanks also to Prof. P. M. Pardalos (University of Florida, Gainesville) for his encouragement in publishing this book.

My wife Viorica Isac has carefully prepared the manuscript. She supported with unlimited enthusiasm and kindness this long and very hard work. Many, many thanks for all her support.

Last, but not least it is a pleasure to acknowledge the excellent assistance that the staff of Kluwer Academic Publishers has provided in the publication of this book.

George Isac
December 21, 1999