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# **ANALYSIS AND DESIGN OF DISCRETE PART PRODUCTION LINES**

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# **Analysis and Design of Discrete Part Production Lines**

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*Dedicated to those who are so much more  
important to us than mere production lines,  
but who could have reasons to inquire  
‘why then spend so much time with  
production lines?’*

*Maridora, Katerina, Elisavet — Jane, John,  
Eamon, Ita, Paul, David, Jenny — Katerina,  
Ioannis, Athanasia, Daphni — Eliza,  
Dionysis, Eleana*

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## Preface

Initially, during discussions among the four colleagues about this writing project, we used “on the optimal design of production lines” as the working title of the book. However, it must be understood that all models involve assumptions and unless these assumptions are valid, the results could not be described as optimal. So basically, what this text is offering is a set of best solutions to the models as described in the various chapters. The models and the algorithms presented are generally accepted by internationally respected scholars to give very good solutions following extensive simulation and comparison with actual systems. We, therefore, see the process of the optimal design of production lines as a complementary activity between the scholars and the practitioners. The scholars provide models and associated algorithms and the practitioners, in their turn, ensure the appropriateness of the assumptions of the models used together with the validity of the data used, and hence, in effect there is a joint responsibility to achieve the optimal or near optimal design of production lines. It is our experience that practitioners in industry and consultancy companies often have considerable difficulties with the academic and research papers which appear in international journals due to the complexity of the mathematical analysis involved and the lack of readily available efficient algorithms for the solution of the models presented. The literature consists of a large number of excellent papers and it is extremely difficult for the practitioner to have an opportunity to examine the appropriateness of each paper to the design problem on hand. We thought that this project could assist the practitioner in this regard by providing a set of models which have been found useful in specific situations. Of course, it is not claimed that these models cover every conceivable situation, but the authors believe that they provide an extremely useful starting point for the understanding of production lines. Furthermore, we thought that it would be very useful to have in one place a collection of relevant analysis and design material of production lines. For this reason, we decided to put the algorithms on a web site: <http://purl.oclc.org/NET/prodline>. Here, we would like to sincerely acknowledge the generosity of many colleagues across the world who gave us access to the relevant algorithms. Without such generosity and cooperation our project would have been a total failure.

Production lines in the context of this work are a subset of general manufacturing systems. There are various types of manufacturing systems such as job shops, flexible manufacturing systems (FMS), flexible assembly systems (FAS), production or flow lines and automatic transfer lines. The usual features of a production or flow line are dedicated work-stations, manual or automatic, usually producing a single product with a fixed routing and an asynchronous movement of material between the work-stations and a high mean production rate (throughput). Production lines are complex systems. Full understanding of such systems requires skilled analysis in order to facilitate the development of a competent design. Some important design problems associated with production lines consist of decisions in relation to three main issues, viz., work-load allocation, buffer allocation and server allocation. The objective of this book is to provide the reader with a set of models and solutions to these problems (work-load allocation problem (WAP), buffer allocation problem (BAP) and server allocation problem (SAP)) which are accepted by experienced researchers and practitioners to be of value in the design of these systems. To assist in the solution of these design problems, it is necessary to make use of both evaluative and generative (optimization) algorithms.

During the course of a project like this, a number of changes of perspective and vision, as time progresses, are inevitable. Accordingly, we decided to change the working title of the book to “Discrete Part Production Lines.” It is the authors’ view that the models presented may be used in either of two modes, viz., analysis and design. For actual existing lines, the models may be used to predict performance under existing conditions or if certain changes are made, for example, to the number of buffer slots before a particular station. If a new design is contemplated, then, of course, a range of models may be used having in mind the objectives of the design including cost considerations. We hope that the Analysis and Design Decision Network, given in the book’s web site, will assist the readers in choosing appropriate models for their investigations. Researchers and practitioners alike have sometimes questioned the usefulness as well as the benefits derived from very detailed and somewhat complex analysis of production lines. It is, of course, not always feasible to adopt in practice what may be the theoretical optimal or near optimal solution to a design problem in production lines, developed from system modeling. However, if one knows the optimal or near optimal solution, the theoretical prime cost of adopting a more ‘practical’ solution would be of interest. Clearly, the software associated with this text would be of assistance in discussions of these matters.

In Chapter 1, “*Manufacturing Systems: Types and Modeling*,” an overview of the evolution and classification of manufacturing systems is given as well as an introduction to models and modeling.

Chapter 2, “*Evaluative Models of Discrete Part Production Lines*,” describes four predictive models of performance evaluation of production or flow lines: the Markovian model, the expansion method, the aggregation method and the decomposition approach applied both to single-machine station and parallel-machine station production lines. A short section on simulation modeling is given at the end of this chapter.

Chapter 3, “*The Design of Production Lines*,” introduces the reader to the design problems of production lines and the concept of improvability.

Chapter 4, “*Work-Load and Server Allocation Problems*,” describes two separate problems, viz., the work-load allocation problem and the server allocation problem.

Chapter 5, “*The Buffer Allocation Problem*,” describes this important problem within the context of production lines.

Chapter 6, “*Double and Triple Optimization*,” considers the combinations of the three pure work-load allocation, server allocation and buffer allocation problems, taken two at a time or all three together.

Chapter 7, “*Cost Considerations*,” examines cost considerations in the design of production lines using profit maximization and cost minimization objective functions.

In Appendix A, a review of some mathematical fundamentals is given, mainly from linear algebra, probability theory, discrete Markov processes (Markov chains) and queueing theory.

Appendix B contains details concerning the code available on the book’s web site. For each algorithm we provide its author, its coder, a short description, the corresponding output, and key bibliographic references.

Appendix C gives the glossary.

The authors are conscious of the debt of gratitude they owe to a very large number of researchers and practitioners, much too numerous to list, in the area of the design of production lines and manufacturing systems in general. We believe that we must make a special mention of those colleagues who participated either as presenters or attendees at the five Hellenic International Conferences on Analysis, Design and Optimization of Manufacturing Systems which were held in Greece (four at the Islands of the Aegean Archipelagos at Samos, Tinos, Tinos and Samos, respectively, and one at Zakynthos Island of the Ionian Sea) and at the 30th Computers & Industrial Engineering International Conference which was held on Tinos Island and who assisted us so much in crystallizing our understanding of the research work in this area. As we are reluctant to list any specific colleagues for special acknowledgment, we give in Appendix D a list of all colleagues who participated as presenters or attendees at the five Hellenic International Conferences on Analysis, Design and Optimization of Manufacturing Systems.

Appendix E presents an Arena simulation model of a reliable production line.

In conclusion, the authors hope that the background theory, details of the relevant algorithms, tabulations of actual computer runs and the provision of the algorithms at the website associated with this text will together form a reservoir of knowledge to assist the designers of practical production lines. In particular, the authors hope that the guides to the use of these algorithms given throughout the text and in Appendix B will assist the busy designers and practitioners in choosing appropriate computational tools for their analyses. The individual contributions of the authors are given in the book, but, of course, the composite contributions of many other researchers which are included and acknowledged in the text far outweigh what any one of the authors could hope to contribute.



Although Dr. Alexandros Diamantidis's name appears in both Appendix B and Appendix D, all the authors wish to make a special acknowledgment of his contribution to our work particularly in relation to the development of the effective evaluative decomposition algorithm for solving multi-station multi-server production lines and for running various problems sets at our request.

Needless to say, as any academic will attest, we are individually very much in debt to our students who over the years have assisted us in advancing our understanding of the fascinating subject of production lines.

We wish to acknowledge very sincerely the patience of the publisher with the delay in producing this text caused inter alia by one of the authors being indisposed for a relatively long period of time.

Finally, the authors would be very pleased to hear from researchers or practitioners who wish to have an algorithm/procedure, developed by them, to be considered for inclusion at the website. No claim is made, at this point in time, that the algorithms presented can handle all possible realistic design problems for either short or long production lines and it is in that spirit that the authors invite other researchers to make available their algorithms so that the issues related to the design of production lines are finally closed. Hopefully, in time, a very comprehensive set of algorithms/procedures for the analysis/design of production lines would become available for all to use. This could well be the first step to having on a website a set of algorithms/procedures which have been found to be of value in the design and analysis of general manufacturing systems.

Thessaloniki, 2009  
Waterford, 2009  
Chios, 2009  
Athens, 2009

*Chrissoleon T. Papadopoulos*  
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