

FORMALIZING MEDIEVAL LOGICAL THEORIES

LOGIC, EPISTEMOLOGY, AND THE UNITY OF SCIENCE

VOLUME 7

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Formalizing Medieval Logical Theories

*Suppositio, Consequentiae and
Obligationes*

By

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Springer

A C.I.P. Catalogue record for this book is available from the Library of Congress.

ISBN 978-1-4020-5852-3 (HB)
ISBN 978-1-4020-5853-0 (e-book)

Published by Springer,
P.O. Box 17, 3300 AA Dordrecht, The Netherlands.

www.springer.com

Printed on acid-free paper

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For Reinout, Marie and Little-one-on-its-way

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INTRODUCTION

Perhaps one of the most striking characteristics of later medieval philosophy and science is the remarkable unity with which the different fields of investigation were articulated to each other, in particular with respect to the methodology used. While it is fair to say that current science is characterized by a plurality of methodologies and by a high degree of specialization in each discipline, in the later medieval period there was one fundamental methodology being used across disciplines, namely logic. One can say without hesitation that logic provided unity to knowledge and science in the later medieval times. Logic (which was then understood more broadly than it is now, including semantics and formal epistemology) was one of the first subject-matters in the medieval curriculum; it was thought that the knowledge of logic was a necessary, methodological requirement for a student to move on to the other disciplines. And indeed, the widespread use of this logical and semantic methodology can be perceived in disciplines as diverse as natural philosophy (physics), theology, ethics and even medicine.

Besides the fact that medieval logic provided unity to science then, while modern logic does not play the same role now (if anything at all, it is mathematics that might be considered as the fundamental methodology for current investigations), it is also widely acknowledged that the medieval and modern traditions in logic are very dissimilar in many other respects. Of course, this holds of most domains of knowledge: Copernican astronomy also has little resemblance to current astrophysics; current chemistry came a long way from long-forgotten alchemy. Nevertheless, even if the main assumptions and methods are radically different, most present-time disciplines share at least a common subject matter with their predecessors; indeed, Copernican astronomy and astrophysics both have stars, planets and the universe as their subject matter.¹ But the same cannot be said of logic: at first sight, the subject matters of current logic seem to have no counterpart in, for example, Aristotelian or medieval logic, to name but two of its ‘predecessors’. In fact, we may doubt whether

¹ Even though our conceptions of what planets, stars and the universe are have changed considerably, as these are essentially theory-laden concepts.

these past traditions should be viewed as predecessors of what we now call logic, or, alternatively, whether what is now known as logic deserves this name at all, in light of its history. In other words, can we really speak of a unified discipline – logic – or is each of these traditions a discipline in its own right? This seems a hard pill to swallow, but at the same time it is not evident what, if anything, would constitute the very nature of logic, that is, the traits common to all these different traditions.

This apparent lack of uniformity in logic lies at the origin of the main question driving the present investigation: in which senses (if any) can medieval logic be viewed as logic (in particular from the viewpoint of modern logic)? It is not so much that medieval logic is of interest to us only insofar as it satisfies modern criteria of what is to count as logic; rather, it is the quest for the common grounds of these two traditions that motivates the search for the senses in which medieval logic is to be seen as logic also by us, 21st century philosophers and logicians. In other words, this investigation seeks to outline unity in two main respects: the unity of medieval science and knowledge provided by medieval logic, and the diachronic unity of logic as a discipline, in spite of the apparent profound dissimilarity between the traditions of medieval logic and modern logic.

Of course, there is a fundamental disparity in their respective general approaches: while, for medieval logicians, their investigations were very closely related to the general study of language, logic is nowadays a part of mathematics. This, among other reasons, is held to justify the skepticism with which medieval logic and other past logical traditions are often viewed by modern logicians (not to mention the widespread positivistic credo to the effect that everything that is ‘old’ is necessarily obsolete). Notwithstanding (or because of?) these dissimilarities, the degree of sophistication attained by medieval logicians is impressive, just as much as what are, to my mind, significant resemblances (albeit not easily perceived at first sight) between the medieval investigations and current developments in logic and philosophy.

At the same time, it appears that many lessons can be learned from the medieval logicians, as they were aware of some of the intricacies of logic and language whose importance we seem to have forgotten. That is, while the quest for the common grounds of the two traditions is essentially motivated by an inquiry on the nature of logic, the aspects in which medieval logic differs from modern logic are just as significant, as they are a potential source of inspiration for new developments within the current tradition. At any rate, it is clear that to establish a dialogue between the two traditions can only be beneficial.

How can this be done? From a modern perspective, the medieval writings in logic are incomprehensible. Not only is the language (Latin) a barrier; medieval logic was embedded in a complex conceptual framework, with constant use of highly technical jargon. But the most serious obstacle may be the modern tendency to express logical theories in especially devised notations, and with a certain axiomatic structure, which are not to be found in the medieval writings. Either way, it is clear that one way of establishing such a dialogue between these two traditions is to *formalize*

fragments of medieval logic. And this is precisely what I set out to do. In particular, the objects of formalization in the present study are three topics from medieval logic, namely supposition, *consequentia* and *obligationes*; each can be seen as a case study demonstrating the fruitfulness of formalizing medieval logic.

By the term ‘formalization’, one usually understands the ‘translation’ of something expressed in ordinary language into a symbolic counterpart. In fact, as I carried out the formalizations presented here, it became increasingly evident that, for an adequate formalization, more important than just the choice of symbols is a suitable conceptual analysis of the theory to be formalized. For this reason, the project presupposed an in-depth conceptual understanding of the topics and theories being formalized. In this sense, the present work is just as much a conceptual-historical examination of these topics as it is an attempt at formalization.

Moreover, the term ‘formalization’ obviously refers to the notion of the formal. This is a rather telling element; currently, formality is often thought to be what is distinctive about logic, so that, for a theory to deserve the attribute ‘logical’, it must be formal.² Therefore, to formalize a theory, that is, to render it (more) formal, is also to show that it is (or the extent to which it is) logical and/or essentially grounded on logical concepts.³

However, that formality is what is characteristic of logic is indeed a strong assumption, which must not be plainly taken for granted; in effect, one of the important upshots of examining other logical traditions is to put this assumption to test. Four views are possible: (i) the theoretical constructs of a given logical tradition do conform to the formality criterion; (ii) these theories do not conform to the formality criterion, and thus are not logical theories properly speaking; (iii) these obviously logical theories do not conform to the formality criterion, so the criterion may have to be modified; (iv) formality is irrelevant as a criterion demarcating what is to count as logic.

Obviously, the very notion of the formal demands careful consideration, as it is clear that distinct concepts of the formal are in play. I will argue that, according to some suitable notions of the formal, some of the medieval logical theories are (at least to some extent) formal – and this is made patent by means of the formalizations offered here – in particular if this notion is understood more broadly than it usually is in current developments (especially with respect to permutation invariance – cf. MacFarlane 2000). In other words, I defend view (iii) as defined above: I maintain that the notion of the formal is relevant at least as a necessary condition for what is to count as logic, but that it must go beyond the rather restricted concept of the formal as permutation invariance.

² It is disputable whether formality is a sufficient condition for what is to count as logic, but it seems to me that it is in any case a necessary condition.

³ One may argue that this does not hold, as a formalization of a mathematical theory does not turn it into a logical theory. But it is not a coincidence that the usual practitioners of formalization in mathematics are advocates of mathematical logicism; the underlying idea seems to be that a formalization of a mathematical theory corroborates the view that mathematics ultimately rests on logical concepts.

Overall, the aims of the present investigation can be summarized as follows:

1. Historical aim: an investigation of some aspects of medieval logic and semantics, so as to obtain a better understanding of them. In particular, I investigate the extent to which these theories are *formal*, in such a way that they could play the methodological role ascribed to them in medieval science.
2. Pedagogical aim: the attempt to make these medieval theories more easily understandable from a modern vantage point.
3. Philosophical aim: the search for the common grounds underlying different logical traditions (medieval vs. modern), in order to explore the nature and unity of logic as such. The underlying assumption is that logic is formal, but that of itself does not say much as long as it is not clear what is meant by ‘formal’.

Given these aims, the use of formalization as the main tool seemed to impose itself. Now, this decision is of itself not of much help, as one can hardly speak of well-defined guidelines as to how a formalization must be carried out. In fact, this is rather murky terrain; several different loose ideas seem to be associated with the concept of formalization, so it became clear that a philosophical reflection on this very concept was not only a necessary addendum to this project; it might also be a welcome contribution to the philosophy of logic in general. As a consequence, in addition to the three case studies on medieval logic, this work contains a fourth chapter on the philosophy of formalization. In this chapter, I argue that formalization corresponds to three distinct but related tasks, that is, axiomatization, symbolization and conceptual translation of a non-formalized theory into an already existing formal theory. A formalization may consist of one of these three procedures, or, more typically, of a combination of them.

HISTORICAL PRELUDE

A systematic overview of the history of later medieval logic is not to be found in the present work. For this, the reader is referred elsewhere.⁴ Here, the main goal is that of conceptual analysis, presupposing familiarity with the medieval logical framework. But a few preliminary words on the history behind the authors that figure prominently in my investigation can certainly do no harm.

The later medieval period in (Christian) philosophy starts in the 12th century, with Abelard. This 12th century tradition is a world of its own, extremely complex and interesting, which requires separate attention. Therefore, in the present work, I have deliberately chosen not to deal with the 12th century tradition. It should be mentioned, though, that, while philosophy and theology were still essentially part of the same broad domain of investigation, it is in the 12th century that laymen such as Abelard

⁴The *Cambridge History of Later Medieval Philosophy* (Kretzmann, Kenny and Pinborg 1982) is particularly useful for this purpose, as is (Spade 1996).

(who became a cleric only later in life⁵) became important figures in the Christian intellectual environment.⁶

The 13th century witnessed the emergence of terminist logic, that is, the tradition marked by the study of the so-called properties of terms, such as signification, supposition etc. (cf. Read 2002, De Rijk 1962/67). Two authors from this period will often be referred to in the present work, namely William of Sherwood and Peter of Spain. Both wrote what we could call ‘textbooks’ in logic, which were then widely used for the study of logic.

But most of the authors considered here belong to the 14th century. At that time, there were two major traditions, namely the English tradition revolving around the University of Oxford, and the continental tradition, whose center was the University of Paris (cf. De Libera 1982). Burley, Ockham, Swyneshed and Strode all stem from the English tradition, while Buridan, Albert of Saxony and Marsilius of Inghen, among many others, belong to the continental tradition. For sure, there are points of contact and exchanges between these two traditions, but each has its own distinctive spirit.

That is, this work is mainly based on 14th century authors, predominantly from the English tradition. Earlier authors are considered only insofar as their writings offer elements for the conceptual understanding of the 14th century theories that are my object of analysis.

SUBJECT-MATTER

I have chosen three topics from medieval logic as objects of formalization: supposition, *consequentia* and *obligationes*. Why these topics, and not others? There is no principled answer to this question. Various contingent reasons led me to focus on these three topics.

The concept of supposition was already the topic of my master thesis, where I dealt with Ockham’s truth conditions for the main propositional forms, leaving aside the different kinds of supposition that are my concern here. Besides, supposition is a crucial concept in the medieval semantic framework, so it seemed appropriate to treat of it in the present context – even more so since supposition remains an unfinished topic within medieval scholarship. My main tenet is that, contrary to the accepted view, theories of supposition should not be compared to modern theories of reference. Within the modern framework, they are best seen as theories of meaning, more specifically as theories for the algorithmic generation of the meanings that a certain body of propositions may carry. This insight came to me from a switch of perspective: theories of supposition should not be seen as static, but rather as procedural, in a sense that has recently become influential in logic.

⁵ Cf. (King 2004).

⁶ Notice that the (very rich) Arab and Jewish traditions of the time also fall out of the scope of this work.

As for *consequentia*, it was not obvious to me at first in which way the medieval discussions on the topic had something to add to the current state of affairs (notwithstanding the central position occupied by consequence and related notions in logic, then as well as now). But I quickly realized that these medieval discussions touched upon various important topics. In particular, Buridan's commitment to tokens as truth-value bearers leads him to inquiries that are strikingly similar to current investigations in two-dimensional semantics. Moreover, the distinction material vs. formal semantics as found in Buridan turns out to have important connections with the modern debate on logical consequence. That is, while, on the one hand, some of the modern apparatus of two-dimensional semantics is crucial for spelling out the details of Buridan's views, on the other hand, his notion of formal consequence offers an interesting vantage point for current discussions of the notion of logical consequence; that is to say, the dialogue seems to benefit both sides, as I show in part 2.

Lastly, *obligationes*. It is a doubly fashionable topic: at present, *obligationes* is a popular subject matter among medievalists, and the modern counterpart that I found for it, namely the application of the game-theoretical framework to logic, is equally popular among logicians. Of course, this is not the (only) reason why I chose *obligationes* to be one of my objects of formalization; in fact, it is a remarkable case of conceptual similarity between a medieval and a modern theoretical framework and, accordingly, one of the best examples of the fruitfulness of this kind of investigation. Most of all, recent research on *obligationes* has made important progress, but we are still a long way from totally understanding this genre. There is certainly room for further research on the topic, and with the formalization presented in part 3, I hope to offer further insight using the framework of logical games as point of vantage.

Moreover, these three topics are related to one another in many important ways. As already said, in the later medieval period, logic was a tool to be used for a wide variety of intellectual investigations; in particular, a given logical theory or topic was often used for the analysis of other logical theories or topics (that is, logic as a discipline was not articulated in a strict, foundational way). The notion of supposition was at the core of the medievals' machinery of semantic analysis, and thus was used virtually everywhere; the notion of *consequentia*, or entailment, was of course at the center of all investigations, since it permeates the all-crucial notion of inference of new knowledge from known premises; the obligational framework, which may seem to us a rather artificial and regimented construction, amply underlined the analysis of a variety of topics. The specific connections between each of these topics shall be pointed out in due course, but for now it is important that the organic character of the articulation of the different topics and theories in later medieval logic be borne in mind.

In sum, the present text is composed of four main parts: part 1 is dedicated to supposition theory, part 2 to the notion of *consequentia*, part 3 to *obligationes* and part 4 to the philosophy of formalization. Finally, in the conclusion, I draw some general remarks on the nature of logic, inspired by the foregoing analyses and formalizations.