UNITY, TRUTH AND THE LIAR

LOGIC, EPISTEMOLOGY, AND THE UNITY OF SCIENCE

VOLUME 8

Logic, Epistemology, and the Unity of Science aims to reconsider the question of the unity of science in light of recent developments in logic. At present, no single logical, semantical or methodological framework dominates the philosophy of science. However, the editors of this series believe that formal techniques like, for example, independence friendly logic, dialogical logics, multimodal logics, game theoretic semantics and linear logics, have the potential to cast new light no basic issues in the discussion of the unity of science.

This series provides a venue where philosophers and logicians can apply specific technical insights to fundamental philosophical problems. While the series is open to a wide variety of perspectives, including the study and analysis of argumentation and the critical discussion of the relationship between logic and the philosophy of science, the aim is to provide an integrated picture of the scientific enterprise in all its diversity.

Unity, Truth and the Liar

The Modern Relevance of Medieval Solutions to the Liar Paradox

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Preface

And in my haste, I said: "All men are Liars"
—Psalms 116:11[1]

The Original Lie

Philosophical analysis often reveals and seldom solves paradoxes. To quote Stephen Read:

> A paradox arises when an unacceptable conclusion is supported by a plausible argument from apparently acceptable premises. [...] So three different reactions to the paradoxes are possible: to show that the reasoning is fallacious; or that the premises are not true after all; or that the conclusion can in fact be accepted.[2]

There are sometimes elaborate ways to endorse a paradoxical conclusion. One might be prepared to concede that indeed there are a number of grains that make a heap, but no possibility to know this number. However, some paradoxes are more threatening than others; showing the conclusion to be acceptable is not a serious option, if the acceptance leads to triviality. Among semantic paradoxes, the Liar (in any of its versions) offers as its conclusion a bullet no one would be willing to bite.[3]

One of the most famous versions of the Liar Paradox was proposed by Epimenides, though its attribution to the Cretan poet and philosopher has only a relatively recent history. It seems indeed that Epimenides was mentioned neither in ancient nor in medieval treatments of the Liar

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[1] Jewish Publication Society translation.
[2] Read [1].
[3] Save, perhaps, for the dialetheist, who would be willing to accept that the Liar sentence is both true and false.

Paradox.[4] One source of this attribution is most probably Paul of Tarsus, who in his *Epistle to Titus* (1:12) writes:

> One of themselves, a prophet of their own, said, Cretans are always liars, evil beasts, idle gluttons.[5]

This saying, which Paul does not explicitly mention as paradoxical, but only as 'true'[6] has been later identified with a fragment of Epimenides' writings (though we do not dare to enter the scholarly disputes around the identification of the proper fragment).[7] It seems that Epimenides was condemning the widespread Cretan religious belief of Zeus' mortality (for whom the Cretans had allegedly erected a tomb), though once again this is a disputed conjecture.[8] That this utterance was not considered paradoxical, though widely discussed and commented on by the early Church Fathers, may at first be puzzling. But we may suppose (applying some version of the Charity Principle) that if this was so, it was not because they failed to notice its paradoxical nature, but because they considered that Epimenides' saying was not meant to apply to itself. Such a proposal might indeed be seen as a way out of the paradoxes arising from self-reference. Excluding Epimenides from the set of *intended* Cretans would offer a kind of solution, since if this was how Epimenides' claim should be interpreted, there would be no paradox left. The same result would be obtained by proposing that one should simply exclude Epimenides' utterance of the sentence 'The Cretans are always liars' from the set of utterances made by the Cretans, all the members of which Epimenides wishes to claim are false.

Another famous paradox, namely the Barber Paradox, has a simple-minded solution in the very same spirit. Assume that the barber shaves all and only those who do not shave themselves; assume also (or, on a suitable view of presupposition, presuppose) that we are only talking about those who *actually* need to shave, thus excluding women from the range of the quantifier 'all and only those who . . . '. Now, one way to avoid paradoxical conclusions regarding who shaves the barber, while maintaining

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[4]It is a matter of scholarly dispute as to who was the first to attribute to Epimenides the paternity of the Liar as a paradox. Since Russell and Prior this attribution is a matter of common knowledge among logicians and philosophers (see [2, 3]).

[5]American Standard Version translation.

[6]He indeed adds: 'This testimony is true' (1:13).

[7]The identification of the anonymous Cretan with Epimenides is, however, ancient, and dates back to Clement of Alexandria (*Stromata*, 14) who himself does not consider the saying as paradoxical, but rather finds the attribution of a true statement to a pagan surprising.

[8]It was Origen (*Against Celsus*, 3:53) and Athenagoras (*A Plea for the Christians*, 30) who connected the Liar Paradox to this religious belief, though the quote 'Cretans, always Liars' stems from a hymn to Zeus attributed to Callimacchus of Cyrene who, living in a Greek Libyan colony, was hence not uttering a paradox.

that the description of the barber is correct, would be to suggest that the barber lies outside the range of the relevant quantifier: the barber is a woman. Such a dissolution strategy simply excludes those individuals from the range of the relevant quantifier whose inclusion therein would give rise to a paradox.

It may be a disappointment to the reader that Epimenides' original statement was not, in its most likely rendering, a paradox after all. In any event, there is a lesson to be learned from Epimenides' hasty statement: in some cases at least, one may reasonably attempt to rid oneself of a paradox by suitably restricting the range of the relevant quantifiers. As history teaches us, this is the way Russell wished to avoid the paradox of the set of all sets. Tarski acknowledges Russell's solution as a source of his own systematic distinction between language and metalanguage. But such a solution may have unintended consequences when one wishes to model the semantics of a natural language employing a formal one.

Why Paradoxes Matter

Semantics deals with meaning – even though one of the tasks of semantics, as an object of study of the philosopher, is to explain away 'meanings' conceived as abstract entities attached to our words by some sort of metaphysical glue. Since the work of Frege, Wittgenstein and Tarski, the notion of truth has been seen by many as a relatively well understood notion, in terms of which the notion of meaning could be explicated. Adopting this perspective, one might suggest that to grasp the meaning of a sentence can be equated with grasping its truth conditions; and to propose that the meaning of a word consists of the contribution it makes to determine the truth condition of a sentence in which it occurs. To give but two examples, Carnap's replacement of the Fregean distinction sense/denotation by the distinction intension/extension, or Davidson's 'bold conjecture' that a theory of meaning for a language is a theory of truth for that language, are both proposed moves from meaning to truth conditions.

Truth-conditional semantics is often linked with the idea that questions about meaning are best interpreted as questions about understanding. To understand the meaning of a sentence, it is said, is to know its truth conditions. However, it is by no means philosophically unproblematic to attempt reducing considerations about meaning to considerations of truth conditions. Michael Dummett, who has delved into the notion of understanding, has launched the so-called 'anti-realist' (or 'justificationist') theory of meaning, according to which the understanding of a sentence consists of an ability to recognize, in appropriate circumstances,

whether it is true or false – grasping the sense of a statement consists of an ability to recognize evidence for the statement when presented with it.[9] Under this view, if a putative truth condition of a sentence S is of such a kind that it could not even in principle be recognized whether the condition prevails or not, then S will not qualify as true or false in any circumstances. Jaakko Hintikka's game-theoretical semantics can be seen – as Hintikka himself argues – as a semantic theory that is at the same time verificationist (hence arguably doing justice to the anti-realists' concerns) *and* truth-conditional: truth conditions are themselves defined in terms of activities of verification and falsification.[10]

The notions of meaning and truth are intimately related, be one's overall position that of an anti-realist or that of a proponent of (some variant of) truth-conditional semantics. In a semantic theory of truth, the truth conditions of sentences of the formal language considered are specified inductively. This may happen in various ways. In Tarski's truth definition, one can compute the satisfaction conditions of a complex formula on a model from the satisfaction conditions of its simpler constituents and the way in which these are put together, while, for instance, in Hintikka's game-theoretical semantics, two-player games are inductively associated with pairs of sentences and models, and truth conditions are specified by reference to the existence or non-existence of a winning strategy for a certain player. Generally, one faces the question of rationally choosing a semantic theory of truth to serve as a basis of a theory of meaning. Tarski's theory is fitting when attention is restricted to a formal language (such as first-order logic), but cannot unproblematically and without further ado be applied in connection with natural languages. Once again, to mention but two phenomena ubiquitous in ordinary language, self-reference and indexicality cannot be dealt with on the object language level in Tarski's theory.[11]

Tarski saw in the Liar sentence a possibility of arguing generally for the undefinability of truth. No sufficiently strong language (one capable of speaking of its own semantics) can define its own truth. For if it was possible, the sentence 'This sentence is false' could be formulated within such a language. Now this sentence would either be true, or else fail to be true. It would follow that the sentence is neither true nor false, contrary to the law of excluded middle, which Tarski took for granted. Tarski's

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[9]Cf. [4]. For more recent expositions of the justificationist standpoint, see [5, 6]. (For an approach to the anti-realist theory of meaning, based on Martin-Löf's intuitionistic type theory, see [7].)

[10]Hintikka argues that Wittgenstein implicitly held such a view on meaning. See, e.g., [8, 9].

[11]S. Read's target paper (Chap. 1) reminds us of some of the well-known failures of Tarski's T-scheme in similar cases (but see also Jan Woleński's defense, Chap. 12).

famous theorem that there can be no explicit first-order truth definition
for a first-order language of arithmetic is a precise mathematical result
whose proof makes use of a formalized version of the Liar sentence. If
there was an arithmetical predicate $\tau(\cdot)$ true of all and only the Gödel-
numbers $\ulcorner\psi\urcorner$ of sentences ψ true in the standard model $\mathcal{N}$ of arithmetic,
it could be shown that there is a sentence ϕ true in $\mathcal{N}$ if and only if the
sentence $\neg\tau(\ulcorner\phi\urcorner)$ is true in $\mathcal{N}$. But this is impossible, for if $\tau(\cdot)$ is a truth
predicate, $\neg\tau(\ulcorner\phi\urcorner)$ in effect asserts of ϕ that it is not true.[12]

Given Tarski's assumptions, no explicit truth definition of a language
in that language itself is, then, possible. On the other hand, there is
no obstacle to giving a truth definition for the object language in a
metalanguage that is essentially stronger than the object language. This
is why the systematic distinction between an object language and a
metalanguage is crucial for Tarski's semantic theory of truth.[13] If not
all Tarski's assumptions are accepted, it even becomes possible to find
a language explicitly defining its own truth predicate. Examples are fur-
nished by Kripke's three-valued interpretation (to which we will return
soon) and Hintikka's independence-friendly logic.[14] In both cases a truth-
definition is made possible by the fact that not all instances of the law
of excluded middle hold. For Kripke, this happens because the semantics
of first-order logic is suitably redefined at the outset (the union of the
extension and antiextension of a unary predicate need not exhaust the
domain), while for Hintikka this is due to the way he extends first-order
syntax and interprets the resulting language utilizing game-theoretical
semantics.

Let us spend some more words on Tarski's theory, as it is what most
contributions in this volume are concerned with. In Tarski's theory, it
is possible to define truth while avoiding the Liar paradox, precisely by
making a rigid distinction between an object language and a metalan-
guage. Since there is no such distinction in connection with bare natural
languages, no such remedy to the paradox seems open to them. Given
Tarski's assumptions (notably the law of excluded middle, and compo-
sitionality of semantics), the option that remains is of course to accept

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[12]It is useful to bear in mind that Tarski proved — in addition to the result that first-order
arithmetical truth is not explicitly definable in first-order logic — that first-order arithmeti-
cal truth *is* implicitly definable in first-order logic! That is, if the arithmetical vocabulary
considered is L, there is a first-order formula $\Psi(x)$ of a larger vocabulary $L \cup \{SAT\}$ such
that for any first-order L-sentence ψ, we have $\mathcal{N} \models \psi$ if and only if there is an interpretation
$S \subseteq \mathbb{N}^2$ of the binary predicate SAT such that $(\mathcal{N}, S) \models \Psi(\ulcorner\psi\urcorner)$. Intuitively, $SATxy$ says
that x is the Gödel-number of an L-sentence ψ, and y encodes a variable assignment γ such
that $\mathcal{N}, \gamma \models \psi$.

[13]See [10].

[14]See [11, 8].

non-definability of truth in natural languages. As truth is a key semantic notion, Tarski's result has been widely taken as supporting the idea that natural language semantics cannot be expressed in these languages.[15]

On the other hand, even if no explicit truth predicate can be formulated in natural languages (supposing Tarski's assumptions hold for them), still one may of course put into use, in studying semantics, various aspects of the Tarskian framework. Hence for instance compositionality has become, for better or worse, virtually a standard in much semantic theorizing. Formal semantics plays an important role in the study of natural language: Montague semantics, Hintikka's game-theoretical semantics, Kamp's Discourse Representation Theory, as well as Groenendijk and Stokhof's Dynamic Predicate Logic – all make essential use of formal model-theoretic tools to analyze natural language phenomena which were beyond the scope of formal semantics as originally formulated by Tarski. Against this background, semantic paradoxes appear to be of general interest: as in the case of the Liar paradox, they may reveal important limitations with which a semantic theory of truth is bound to comply.

Medieval Solutions. . .

Medieval logicians, for reasons of their own, developed several lines of response against the Liar Paradox. Some advocated *restrictionism*, or the idea that what a significant term stands for in the context of a proposition (its *suppositio*) is determined via an appropriate contextual restriction imposed on the precontextually given extension of the term. From this view, self-reference was ruled out by suitable postulates concerning restrictions. Others attempted to provide a modified account of meaning, based on the idea that meaning is closed under implication and entailment (*closure postulate of meaning*). In fact, in the target paper of this volume, Stephen Read develops a solution to the Liar that was first formulated by Thomas Bradwardine (1295–1349) who, like William Heytesbury (before 1313 – ca. 1373), opposed the solution proposed by the restrictionists and who explicitly stated the closure postulate of meaning.[16]

It is important to note that both restrictionists and their opponents assumed a rather precise theory of truth conditions for categorical

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[15]Tarski himself seems to have endorsed the view that natural language semantics was ineffable (see [12]).
[16]S. Read's reconstruction of Bradwardine's theory is not uncontroversial, however, for it rests on a 'charitable' reading of Bradwardine's closure principle (discussed and defended against other interpretations, including those of many contributors to this volume, at the end of his response, Chap. 13, Sect. 1).

sentences, and that this theory was based on the notion of *suppositio*. Thus the medieval discussions on the Liar were centered around the link between truth and meaning – just as the discussion in contemporary (analytic) philosophy. Because of this fact it is hardly surprising that medieval solutions, once recast in modern parlance and formalism, still offer valuable insights. They even point at an alternative conception (or definition) of truth, which can be made use of in attempting to escape the Liar tangle. It may be particularly useful when moving beyond formal languages.

One issue, addressed repeatedly in the present volume – though not often explicitly – is a strong assumption made by Tarski, namely the *uniqueness of meaning*.[17] If sentences have exactly one meaning, or (to use S. Read's expression) 'say' exactly one thing, then the Liar sentence and its variants are paradoxical because they seem to have an indeterminate meaning. What a sentence[18] says is at the very heart of the Aristotelian conception: as Ross translates *Metaphysics* IV, 7:

> To say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, and of what is not that it is not, is true; so that he who says of anything that it is, or that it is not, will say either what is true or what is false.[19]

In interpreted formal languages, as normally conceived, the uniqueness assumption comes of course for free, since extensions are assigned to non-logical symbols by *functions*, therefore, applying usual definitions of logical operators, a given sentence says exactly one thing (there is, up to logical equivalence, one and only one metalanguage truth condition for the sentence). If one wishes to entertain a broader conception of meaning, perhaps with the motivation of doing justice to the complexity of everyday speech, the question can be raised whether (an utterance of) a sentence can say more than one thing. A positive answer to this last question immediately raises another one: how will quantification over those things that (an utterance of) a sentence says enter into an account of its truth conditions?

According to S. Read, the heart of Bradwardine's solution is this: once it is admitted that a sentence possibly says a great many things,[20] and

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[17]Or, as C. Dutilh Novaes names it (Chap. 3) the *Principle of Uniformity*.

[18]Or a given utterance of a type sentence, or a given speaker by uttering a given occurrence of a given type of sentence.

[19]Tarski mentions explicitly this passage (quoted without reference in Tarski (1944), *op. cit.*, Sect. 3), and introduces it writing that he 'should like [his] definition to do justice to the intuitions which adhere to the *classical Aristotelian conception of truth* – intuitions which find their expression in the well-known words of Aristotle's *Metaphysics*'.

[20]But not generally of itself that it is true, on pain of trivializing the account of truth conditions (see S. Read's target paper, Chap. 1, Sect. 6).

given a reasonably accurate account of 'saying that', the Liar sentence
turns out to be simply false: among what the Liar sentence in effect 'says',
given suitable closure postulates (e.g. that a sentence says whatever is
implied by what it says), there is a contradiction.[21] This volume is an
attempt at evaluating the consequences of this solution, once it is recast
in 'modern' terms, and given a precise (formal) expression.

...And Modern Approaches

This modern *aggiornamento* of Bradwadine's solution may be more
telling if contrasted with the (nearly) dominant approach to paradoxes
– one we have already mentioned – namely the Kripke (or Kripke–
Feferman) theory. The Kripke theory is very famous, having dominated
most discussions of the Liar for the past 30 years, but has still not
been universally accepted as a solution. Kripke's basic idea was to drop
Tarski's assumption of bivalence (or, as we said, assumption of the ex-
cluded middle), and give a partial interpretation to the truth predicate.
Strictly speaking, there are still two values, true and false, but not every
proposition need take either value. Kripke's theory proceeds defining the
truth predicate recursively from the set of well-formed sentences not con-
taining it. Unlike Tarski's hierarchy of self-contained truth predicates,
Kripke allows for the truth being defined as the union of all the defin-
ition stages. Yet paradoxes are avoided since, after denumerably many
stages, the recursive procedure reaches a fixed point. Some sentences may
not be assigned any value at those fixed points, sentences that are not
"grounded" in the (original) set of sentences (not containing the truth
predicate). Neither the Liar ("This sentence is not true") nor the Truth-
teller ("This sentence is false") can be assigned any value, true or false,
in any fixed point, since they are *ungrounded*.

That way, Kripke can retain semantic closure, so that each language
contains its own truth predicate, without contradiction nor (usual) para-
doxes. Kripke is also concerned to retain (the spirit of) Tarski's T-scheme,
in that a sentence s and the sentence 's is true' have always the same
value, provided they have a value at all.[22]

This is at the expense of bivalence, for ungrounded sentences do not re-
ceive any truth-value. Though widely popular nowadays, Kripke's theory

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[21]Indeed, as conceded by S. Read in his response (Chap. 13), that at least *some* sentences
(namely, those saying of themselves that they are false) *necessarily* say more than one thing
is a necessary condition for Bradwardine's (and hence, Read's) solution to the Liar to work.
[22]However, instances of the T-scheme are not expressible in the original Kripkean theory, since
the three-valued Kleene semantics it is based on does not contain an adequate conditional
connective.

has not imposed itself as a solution, any more than Tarski's. Appealing as it is, it still has the same consequences as Tarski's theory: some *semantic* concepts cannot be explicitly defined in the object language. Just as Tarski would have denied the question "Is this sentence true?" any assignable meaning,[23] so the question "Is this sentence grounded?" cannot be answered, and for analogous reasons: its presupposition fails. Without some story, yet to be told, about which sentences are grounded and which are not, it is unclear whether some conceptual gain has been obtained moving from Tarski's theory to Kripke's.

Yet the motivation to maintain semantic closure clearly relates Bradwardine's solution to Kripke's, though it falls short of uniting them under the same banner. Bradwardine's approach is very different, for he rejects the kind of equivalence Tarski frames in the T-scheme. Rather, Bradwardine defines truth in terms of signification: s is true iff things are only as s signifies, and this is a universally quantified formula, not a schema as in Tarski. And signification is (allegedly) a simpler concept to grasp than 'groundedness', or at least, it has some intuitive pedigree. Yet Bradwardine's (and Read's) move is likely to close the door to an extensional theory of meaning, since it is usually understood in terms of a grasp of truth conditions, and this would be blatantly circular.

A recent offspring of the family of solutions spawned by Kripke's initial theory is due to Tim Maudlin.[24] In one sense, it is more relevant to the present discussion than Kripke's original theory, once rephrased in terms of one of its commentators, Hartry Field [14]. Maudlin presents his theory as one where so called *T-inferences* – the 'upward' inference from s to 's is true', and the 'downward' inference from 's is true' to s – are both valid, but where the Excluded Middle fails. Field argues that this theory is better viewed as one where upward T-inference fails *inferentially*, though not semantically, because, according to Field, Maudlin's notion of validity is primarily inferential. It is not our intention to evaluate the relative merits of Maudlin's and Field's formulations, but the latter has the advantage of allowing a clearer comparison with Bradwardine's solution. Indeed summarized by Field,

> When Maudlin's theory is put in terms of inferential validity, it turns out to be (an extension of) what's often called the Kripke–Feferman theory (KF): a classical logic theory that allows Downward T-Inference but not Upward T-Inference (but contains the weaker rule that T<A> ['A' is true] implies T<T<A>> ['‘A' is true' is true]).

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[23] We assume that the meaning of a yes-or-no question is given through the truth conditions of its potential answers, and that it is meaningful in a given situation (model, fixed point, etc.) if the disjunction of its potential answers is true in this situation (in the model, at the fixed point).

[24] See [13].

According to Field's account, distinguishing what he calls 'semantic validity' and 'inferential validity', while Upward T-Inference is semantically valid, it is not inferentially valid. Indeed, Maudlin's own account is that the Liar sentence, L, is false, and that it is false that L is false. From someone's belief that L is false, one cannot then *infer* the belief that it is true that L is false (even though it is valid, i.e., semantically valid, but from this it does not follow that it also is inferentially valid). Since however belief is norm-dependent, in that the property of 'being correct to believe' is governed by norms none of which is uniquely best, it is (according to Maudlin, and in his own terms) *permissible* to believe that it is true that L is false, though Upward T-Inference is not what warrants this belief. There are other fascinating features of Maudlin's theory (such as his account of groundedness), but for the purpose of drawing a comparison to Bradwardine's theory, the overall picture is already sufficiently clear.

Describing Maudlin's theory in Field's terms makes conspicuous the possible analogies and disanalogies with Bradwardine: both reject Upward T-Inference (though Maudlin does not describe it that way), but while Maudlin is in the strange (Moorean) position of believing that L is false and that it is false that L is false (though it is permissible to believe that it is true that L is false), Bradwardine distinguishes the statement, L, that L is false, which is false, from his own statement that L is false. Bradwardine's explanation is that the first is self-referential while the second is not, and this is in essence his way out of the so-called revenge problem – that is, for any account of the Liar according to which L is false, to distinguish between the judgment made from the theory's standpoint, and the Liar sentence itself.[25]

So the two theories, Bradwardine's and Maudlin's, have something in common, but differ in their explanation of why Upward T-Inference is to be rejected: for Maudlin it is (as described by Field) semantically valid but not inferentially so. For Bradwardine it is simply not valid, being a fallacy *secundum quid et simpliciter*, moving from a (true) statement which says only that L is false to (a false) one that says both that L is false and that L is true. Once again, the heart of the matter is that L says too many things, and if we follow Bradwardine and Read, what it says (of itself) is contradictory. Once we are in the Bradwardinean position to talk about L without self-reference, because we say that what L says is contradictory, hence false, without uttering the (self-referential) L, we need not be in the Maudlin–Moorean position which severs the link between doxastic permissibility and inferential validity. According

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[25]On this, see B. Armour-Garb's contribution, and S. Read's response.

to Bradwardine, one can infer that it is true that L is false from L being false, but not thanks to Upward T-Inference, but because what L says is a contradiction. Subsequently one can believe that it is true that L is false, not because of some norm-relative reason, but because of logic being a standard of doxastic health.

Content of this Volume

This volume consists of two parts. The first part is somewhat reminiscent of medieval regimented controversies, and named after them *Disputatio*, with Stephen Read as *Proponent*. His reconstruction of Bradwardine's solution (Chap. 1), as attempting both to revive a *historical* proposal, and to give it a *formal* reconstruction, can be discussed on several grounds: the adequacy of the interpretation of Bradwardine's theory it offers, the nature of the formal apparatus used to express it, and its alleged merits (or flaws) with respect to other – historical or formal – proposals. The other contributors of this part, acting as *Opponents*, may be grouped into (somewhat unevenly between) those who: (1) discuss the theory itself; (2) discuss the nature of its formal reconstruction; (3) do both. The second part provides a comprehensive background for apprehending the conceptual contribution of medieval theories of truth conditions. The 'restrictionist' theory is especially discussed; this theory is fully compatible with uniqueness of meaning, as opposed to the 'manifold' theory of meaning.

Bradley Armour-Garb (Chap. 2: "Read and Indirect Revenge") calls into question whether Read's account is able to cope with the problem of 'indirect revenge': finding a way to express that the Liar sentence is false without uttering the Liar sentence itself. Along the way he discusses Read's position against the background of contemporary theories of truth (including deflationism and dialetheism). Central to this discussion is the notion of assertion, and the fact that a truth predicate is critical in expressing assertive commitments (such as 'I believe that what X just said is true').

Catarina Dutilh Novaes (Chap. 3: "Tarski's Hidden Theory of Meaning") discusses one of the core features of S. Read's account: the eventual plurality of meaning of a sentence. Giving independent grounds for denying the 'Principle of Uniformity' (of meaning), she explores the consequences of adopting a *quantificational* conception of truth. Important issues that arise in this connection are the need for a proper way to define *falsity* (a recurring concern in several other contributions of this volume), and the doubt that *meaning*, as a semantic notion, could be explained in non-semantic terms.

Laurence Goldstein (Chap. 4: "Doubting Thomas: From Bradwardine back to Anon") – while challenging S. Read's solution to the Liar Paradox on the grounds that it may not be able to answer other semantic paradoxes – also attempts to articulate a solution of his own. He does so following a line rejected by Bradwardine (and hence by Read himself), the *cassationist* view, according to which some utterances of a given type of sentence can 'say' nothing, while others do indeed say something. Goldstein claims that his solution fares better as regards other semantic paradoxes (especially 'iterative' paradoxes) than Read's. In return, Read argues against this claim in his response.[26] But the success of Goldstein's argument against Read's position is independent of the relative merits of the two solutions. Namely, Read assumes explicitly that each sentence (to which his schema **(A)** applies, thus including any form of the Liar) says something,[27] and it is this general contention that Goldstein challenges.

Gyula Klima's contribution (Chap. 5: "Logic Without Truth: Buridan on the Liar") manifests the same type of interest for medieval solutions as Goldstein's, in attempting to reconstruct Buridan's solution. Klima argues, *contra* Read, that Buridan's solution is independent of a specific theory of truth, since Buridan's account of logical consequence is not defined in terms of truth. According to this reconstruction, Buridan aims at dealing with the problem of consistent use of semantic predicates in the presence of semantic closure – not at providing a solution to a problem pertaining to the theory of truth. Klima's paper shows that Buridan's theory nevertheless fails for other reasons, though he eventually suggests that a somewhat charitable interpretation of the theory might be defensible.

In the body of papers of the present volume, Eugene Mills' article (Chap. 6: "Scheming and Lying: Truth-Schemas, Propositions, and the Liar") presents the peculiarity of attempting to argue *against* Read from the standpoint of a theory having so much in common with Read's that one might expect any argument presented against the former to 'bounce back' to the latter. But the crucial point relates, according to Mills, to linguistic − and ultimately, logical − *ontology* (though Mills does not himself use these terms). Attacking Read's 'unacknowledged commitment to semantic essentialism' (p. 122), Mills argues that his own solution is unhindered by this commitment. Read eventually 'plead[s] guilty' of this essentialism in his response (Sect. 2, p. 210). Mills' contribution forces

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[26] Another feature of Goldstein's position is that it considers *utterances* as bearers of truth-values, just as the rival (to both the Bradwardine–Read and Anon–Goldstein) 'weak restrictionism' considered (and defended) by Panaccio (Chap. 14) does.

[27] See Chap. 1, p. 8.

anyone evaluating the eventual merits of the proposed solutions to pay due attention to their 'ontological commitments'.

Terence Parsons (Chap. 7: "Comments on Stephen Read's 'The Truth-schema and the Liar'") mounts a short but sharp attack against Read's solution. He notes that Read does not accept the principle according to which *every sentence says that it itself is true*; and that Read's argument for this conclusion is that if this principle indeed held, a theory incorporating this principle could determine no truth-value of any sentence. Parsons argues, however, that Read's argument is not sound; according to Parsons, it merely follows from the acceptance of the principle *every sentence says that it itself is true* that a theory complying with it could determine no sentence to be *true* (while nothing would prevent determining some sentences to be *false*). Parsons goes on to argue that the failure of the stronger entailment leads Read's original view into difficulties: he attempts to show that by the same token Read's very theory cannot be used in determining any sentence to be true.

Greg Restall's contribution (Chap. 8: "Models for Liars in Bradwardine's Theory of Truth") proposes a thorough analysis of the kind of conditional by means of which the "Bradwardine Axiom" (that every proposition signifies everything which follows from it) should be formalized. After having shown the difficulties associated with the material conditional reading, Restall explores two ways: a modal reading of the conditional, and a relevant reading (within the Anderson, Belnap and Dunn tradition of relevant logic). Restall argues that Bradwardine's argument fails in some models of both families (modal and relevant): the Liar sentence fails to assert its own truth, and then to signify a contradiction. Restall proposes several options to develop a "rich theory of signification" along the lines advocated by Bradwardine, none of which leaves intact the original argument.

José M. Sagüillo's contribution (Chap. 9: "On a New Account of the Liar") attempts to compare 'logics' underlying, respectively, the orthodox Tarskian T-schema and the principle by which Read wishes to replace it. Thereby Sagüillo is led to discuss the nature of the 'saying that'-relation as involving a 'semantic closure' postulate. Read addresses these issues in his response (Sects. 2 and 3 of Chap. 1, respectively).[28]

György Serény's (Chap. 10: "The Liar Cannot be Solved") detailed argument receives an extensive response from Read, and leads Read to

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[28]It is to be noted that Sagüillo's contribution – even if more sympathetic to Read's approach than Jan Woleński's (see below) – partially overlaps with the latter's criticisms. Namely, both authors want to locate the difference between **(T)** and **(A)** (respectively, Tarski's T-schema and Read's analysis of truth) – and subsequently, the difficulties in applying **(A)** – in the fact that the former is extensional, while the latter is intensional.

clarify his way of reformulating Bradwardine's theory (nearly the whole Sect. 3 of Chap. 13, and a substantial part of Sect. 4, are devoted to meet Serény's arguments). Nowhere in the present volume does the controversy resemble more closely a medieval dispute than in Serény's attempt to refute Read's solution, and Read's subsequent attempt at defending it.

Hartley Slater, in his paper (Chap. 11: "Out of the Liar Tangle"), tries to restate Read's solution within a more parsimonious logical framework: he diagnoses the difficulties in logically representing propositional referring phrases as being responsible for the paradoxical appearance of the Liar sentence, proposes how to properly carry out such representation, and criticizes Read for appealing to Church's type theory in his proposed solution.[29]

Jan Woleński is the only contributor to the present volume who (Chap. 12: "Read about T-scheme") attempts to defend Tarski's semantic definition of truth. In doing so, he reminds us of its scope as well as of its limits. He also raises important questions concerning the relationship between the notion of 'saying that' and the notion of *interpretation*. Woleński wishes to stress that Read's schema is *intensional*, while Tarski's semantic definition of truth is *extensional* and operates with respect to a fixed interpretation. A further important point to which Woleński calls attention is the behavior of Read's truth predicate in the presence of negation.

In the second part of the volume, consisting of the contributions of Claude Panaccio and Fabienne Pironet, the reader will find a comprehensive account of the medieval conceptual landscape with respect to the question of *insolubilia* – against the background opposition between uniqueness vs. plurality of meaning.

Panaccio's contribution (Chap. 14: "Restrictionism: A Medieval Approach Revisited") is an attempt to carefully expound and reconstruct the medieval *restrictionist* approach. Panaccio argues that not only may a specific (weak) form of restrictionism, when properly reconstructed, be defensible even from a modern viewpoint, but that it actually enjoys appealing features, when compared with other approaches to truth – such as the Tarskian and Kripkean analyses.

Focusing on the debate on the fourteenth century, Pironet (Chap. 15: "William Heytesbury and the Treatment of *Insolubilia* in Fourteenth-Century England") offers a more general contribution than the title of her article might suggest: Heytesbury's solution is indeed 'Bradwardinian'

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[29]It is also worth noticing that Slater states briefly the very same point as C. Dutilh Novaes, when mentioning that " 'singleness of saying' [is required] before there can be truth assessments of sentences in the traditional T-schema form" (p. 194).

in nearly all respects, and was historically of considerable influence, as Pironet's thorough paper demonstrates. Furthermore, Pironet presents a critical edition of three medieval treatises inspired by Heytesbury (and hence, indirectly, by Bradwardine), which testify to the long-standing popularity and conceptual robustness of such a type of solution.

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