GAMES: UNIFYING LOGIC, LANGUAGE, AND PHILOSOPHY

LOGIC, EPISTEMOLOGY, AND THE UNITY OF SCIENCE

VOLUME 15

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Introduction

Ondrej Majer, Ahti-Veikko Pietarinen, and Tero Tulenheimo

1 Games and logic in philosophy

Recent years have witnessed a growing interest in the unifying methodologies over what have been perceived as pretty disparate logical 'systems', or else merely an assortment of formal and mathematical 'approaches' to philosophical inquiry. This development has largely been fueled by an increasing dissatisfaction to what has earlier been taken to be a straightforward outcome of 'logical pluralism' or 'methodological diversity'. These phrases appear to reflect the everyday chaos of our academic pursuits rather than any genuine attempt to clarify the general principles underlying the miscellaneous ways in which logic appears to us.

But the situation is changing. Unity among plurality is emerging in contemporary studies in logical philosophy and neighbouring disciplines. This is a necessary follow-up to the intensive research into the intricacies of logical systems and methodologies performed over the recent years.

The present book suggests one such peculiar but very unrestrained methodological perspective over the field of logic and its applications in mathematics, language or computation: games. An allegory for opposition, cooperation and coordination, games are also concrete objects of formal study.

As a metaphor for argumentation Aristotle's *Topics* and its reincarnations such as the scholastic *Ars Obligatoria* are set up as dialogical duels (Pietarinen, 2003a). Logics exploiting this idea resurface in the twentieth century attempts to clarify the concepts of argument and proof. The game metaphor has retained its strength in contemporary theories of computation (Pietarinen, 2003b, Japaridze, this volume), in which computation is recast in terms of the symbiosis between the Computing System ('Myself') and its Environment ('Nature'). In mathematics, the benefits of doing so were noted decades ago by Stanislaw Ulam (1960), who wrote how amusing it is "to consider how one can 'gamize' various mathematical situations (or perhaps the verb should be 'paizise' from the Greek word $\pi\alpha\iota\varsigma\iota\eta$, to play)."

Games as explications of the core philosophical questions concerning the scientific methodologies were on the brink of being born in the writings of the early unificators, including Rudolf Carnap, Otto Neurath, Charles Morris and Carl Gustav Hempel. But they never operationalised the key notions. The term 'operationalisation' is apt, since what was attempted was to give meaning to 'operationalisation'. According to operationalism, a concept is synonymous with the set of operations correlated with it. Influenced by Percy Bridgman's and Alfred Einstein's thoughts, the early workers on what was later to become the Unity of Science Movement inherited the better parts of the Viennese verificationism in the methodology of science which, in turn, was allied to, though also significantly different from, the pragmaticism of Charles Peirce. Moreover, Pietarinen and Snellman (2006) show that the kernel of pragmaticism is, in turn, essentially game-theoretical in nature.

Accordingly, a sustained attempt has existed in the history and philosophy of science to articulate the interactive, the strategic and the pragmatic in logic. The chief reason for the failure of the early philosophers working on uniting the foundations of scientific methodology was their stout belief in the explanatory capacities of singular behaviour. In game theory, in contrast, the success lies in the possibility of there being general, or strategic, habits of acting in a certain way whenever certain kinds of situations are confronted.

How coincidental it must have been that many of the logicians working on the operative definitions of logical concepts, including Hugo Dingler and Paul Lorenzen, were not only champions of the Husserlian notion of *Spielbedeutungen* (Pietarinen, 2008), but also immersed in the continental branch of operationalism, which in various forms had already been in vogue around the exiting new projects emerging in the philosophy of science since the 1920s. Meanwhile, game theory proper was in the making, first in the urban atmospheres of the continental triangle of Berlin, Vienna and Göttingen, and later on in the singular intellectual concentrate of the ludic post-war Princeton Campus.

But these historical events constitute just the beginnings of the story, the impact of which is only beginning to unravel. The present book itself constitutes only a modest fragment of that narrative. The book consists of 12 chapters divided into four parts: *Philosophical Issues* (Part I), *Game-Theoretic Semantics* (Part II), *Dialogues* (Part III), and *Computation and Mathematics* (Part IV). The individual topics covered include, in Part I, the philosophy of logical games (Chapter 1, Mathieu Marion), the epistemic characterisation results in game theory, scientific explanation and the philosophy of the social sciences (Chapter 2, Boudewijn de Bruin), rationality, strategic interaction, focal points, radical interpretation and the selection of multiple Nash-equilibria (Chapter 3, Hykel Hosni) and the notion of cognitive agency, cognitive economy and fallacies (Chapter 4, John Woods and Dov M. Gabbay). In Part II, the central methodology is that of game-theoretic semantics, where the germane topics are independence-friendly (IF) logic, imperfect-information games and weak dominance (Chapter 5, Merlijn Sevenster), fuzzy logic (Chapter 6, Petr Cintula and Ondrej Majer) and generalised quantifiers and natural-language semantics (Chapter 7, Robin Clark). Part III is devoted to the method of dialogues, and it deals with the relationships between the game-theoretic and dialogic notions of truth and validity (Chapter 8, Shahid Rahman and Tero Tulenheimo), fuzzy logic, vagueness, supervaluation and betting (Chapter 9, Christian G. Fermüller) and epistemic and intuitionistic logic (Chapter 10, Manuel Rebuschi). Part IV is on the application and use of games in computation and mathematics. Topics covered have to do with computability logic, game semantics and affine linear logic (Chapter 11, Giorgi Japaridze) and determinacy, infinite games and intuitionism in mathematics (Chapter 12, Wim Veldman).

As is evident from this impressive list of topics, the method of games is so widespread across studies in logic and the neighbouring disciplines—including applications to linguistic semantics and pragmatics, the social sciences, philosophy of science, epistemology, economics, mathematics and computation—that it prompts us to take seriously the possibility that there is some "greater conceptual rationale of what it is to be a *bona fide* science" (Margolis, 1987, p. xv). Games, as applied to logic, philosophy, epistemology, linguistics, cognition, computation or mathematics, provide at the same time a notably modern, rigorous and creative formal toolkit that lays bare the structures of logical and cognitive processes—be they proofs, dialogues, inferences, models, arguments, negotiations, bargaining, or computations—while being the product of an age-old enquiring mind and human rational action.

To what extent such methods and tools are able ultimately to reconcile the human and natural sciences (Margolis, 1987) remains to be seen. After all, the first steps in any expansion over multiple disciplines must begin from the beginning; in logic, it would begin from charting what the foundational perspectives are that logic provides to those fields of intellectual pursuit amenable to fruitful formalisations. But we believe that the existence of methods inescapably linked with the ways in which human rational thought processes and actions function supports the wider scenario.

Whether the unity holds in those nooks and corners of scientific and intellectual pursuits covered in the present essays we leave for the readers to judge—it is a question of not only method of logic but also ontology, history of ideas, scientific practices, and, ultimately, of the fruits that the applications of games to the multiplicity of intellectual tasks are capable of bearing.

In the remainder of this introduction, we outline the essentials of two major approaches to how games have been used to explicate logical notions: game-theoretical semantics and dialogical logic.

2 Game-Theoretical Semantics

Hintikka (1968) introduced Game-Theoretical Semantics (GTS) for first-order logic. From the very beginning, the idea was driven by philosophical considerations. Hintikka's goal was not merely to provide an alternative characterisation of truth for first-order logic, but to lay down a theory of meaning making use of—and sharpening—Wittgenstein's idea of 'language game', relating these considerations to Kantian thought and to the idea that logic has to do with synthetic activity (Hintikka, 1973).

Hintikka extended the game-theoretic interpretation that Henkin (1961) had in effect provided to quantified sentences in prenex normal form; this interpretation will be discussed further below. He explained how a semantic game is played with an arbitrary first-order sentence as input.[1] He observed that conjunctions and disjunctions can be treated on a par with universal and existential quantifier, respectively. After all, $(\phi \wedge \chi)$ holds if and only if all of the sentences ϕ, χ hold, and $(\phi \vee \chi)$ holds if and only if at least one of the sentences ϕ, χ holds. Accordingly, a game for $(\phi \wedge \chi)$ proceeds by the "universal" player picking out one of the conjuncts $\theta \in \{\phi, \chi\}$, after which the play is continued with respect to the sentence θ. Similarly, in connection with a game for $(\phi \vee \chi)$, it is the "existential player" who makes a choice of a disjunct $\theta \in \{\phi, \chi\}$. (The objects chosen are syntactic items in connection with conjunction and disjunction, whereas the moves for quantifiers involve choosing objects out there in the domain.)

What about negation, then? Hintikka observed that negation has the effect of changing the roles of the players. After any sequence of moves that the players have made while playing a game, one of the players has the role of 'Verifier' and the other that of 'Falsifier'. Now a game corresponding to $\neg\phi$ continues with respect to ϕ, with the players' roles reversed: the player having the role of 'Verifier' relative to $\neg\phi$ assumes relative to ϕ the role of 'Falsifier', and vice versa.

GTS provides a game-theoretic counterpart to the model-theoretic notion of truth. In this way, the notions of truth for a great variety of logics can be provided. Cases in point are propositional logic, first-order logic, modal and temporal logics, independence-friendly logics (Hintikka, 1995, 1996; Sandu, 1993; Hintikka and Sandu, 1989, 1997), logics with Henkin quantifiers (Henkin, 1961; Krynicki and Mostowski, 1995), infinitely deep languages (Hintikka and Rantala, 1976; Karttunen, 1984; Hyttinen, 1990) and the logic of Vaught sentences (Vaught, 1973; Makkai, 1977).

Semantic games are two-player games; we may call the two players Eloise or the 'initial Verifier' and Abelard or the 'initial Falsifier'. The truth of a sentence φ in a model $\mathcal{M}$ corresponds to the existence of a winning strategy for

[1] The game interpretation goes back to Charles Peirce's investigation in the algebra of logic and graphical logic (Hilpinen, 1982; Pietarinen, 2006b).

Eloise in the semantic game $G(\varphi, \mathcal{M})$ correlated with φ and played on $\mathcal{M}$. The falsity of φ corresponds to the existence of a winning strategy for Abelard. Intuitively, Eloise can be thought of as defending the claim "φ is true in $\mathcal{M}$" against any attempts of Abelard to refute this claim. Similarly, Abelard defends the claim "φ is false in $\mathcal{M}$" against any attempted refutations of this claim by Eloise. The games $G(\varphi, \mathcal{M})$ are so defined that φ is indeed true (false) in $\mathcal{M}$ iff there exists a method for Eloise (Abelard) to win against all sequences of moves by Abelard (Eloise).

The mathematical reality behind semantic games may be less picturesque than the above description in terms of defences against refutations suggests. Given a semantic game $G(\varphi, \mathcal{M})$, the existence or non-existence of a winning strategy for either player is an objective fact about the model $\mathcal{M}$. Whether the players' actions bear relevance to the truth or falsity of the sentence is thus arguable.[2]

The roots of semantic games go back to the Tarskian definition of truth. According to Tarski, to test whether a sentence such as $\forall x \exists y P(x, y)$ is true in a model $\mathcal{M}$, reference to objects a and b of the domain M of $\mathcal{M}$ is needed. The sentence is true iff it is the case that for any a there is an object b such that $P(a, b)$ holds. Thus understood, the truth of the sentence $\forall x \exists y P(x, y)$ does not require the existence of a function $f\colon M \to M$ such that $b = f(a)$ for any $a \in M$. It only requires the existence of a relation $R \subseteq M \times M$ such that for every a there is at least one b with $R(a, b)$ such that $P(a, b)$ holds in $\mathcal{M}$. To get from the statement involving relations to the statement concerning functions, the Axiom of Choice is, in general, needed (Hodges, 1997a). On the other hand, assuming the Axiom of Choice, the truth-condition of $\forall x \exists y P(x, y)$ can indeed be stated as the requirement that there be a function f such that for any value a interpreting $\forall x$, the function produces a witness $b = f(a)$ for $\exists y$. Such functions, introduced by Skolem (1920), are known as Skolem functions.

Henkin (1961) considered logical systems in which infinitely long formulas with infinitely many quantifier alternations are allowed; one of the examples he mentions is the formula

$$\exists x_1 \forall x_2 \exists x_3 \forall x_4 \ldots P(x_1, x_2, \ldots). \tag{1}$$

In connection with such formulas, Henkin suggested that the procedure of picking up objects corresponds to moves in a game between two players, which we might for simplicity call the universal player (Abelard) and the existential

[2] Hodges (2006a, b; Hodges and Krabbe, 2001) has levelled critique on the idea that logical games shed new light on the semantics of quantifiers, or that logical games could actually have conceptually important roles to play in justifying certain logical procedures or in defining meanings. But see the rejoinders in Pietarinen (2006b, Chapter 9) and Hodges and Krabbe (2001) and Marion, this volume, as well as earlier discussion in Hand (1989).

player (Eloise). The former is responsible for choosing objects corresponding to universally quantified variables while the latter similarly interprets existentially quantified variables.

Admittedly, Henkin used the notion of game quite metaphorically. But he pointed out that logical games are related to Skolem functions and observed that winning strategies for the existential player are sequences of Skolem functions. For instance, when evaluating the above formula (1) relative to a model $\mathcal{M}$, any sequence $\langle f_1, f_3, f_5, \ldots \rangle$ of Skolem functions, one for each existential quantifier $\exists x_{2n+1}$ in (1), gives a winning strategy for the existential player in the game correlated with the formula (1) in the model $\mathcal{M}$. In other words, the formula (1) is true in $\mathcal{M}$ if and only if the following second-order formula is true in $\mathcal{M}$[3]:

$$\exists f_1 \exists f_3 \exists f_5 \ldots \forall x_2 \forall x_4 \forall x_6 \ldots P(f_1, x_2, f_3(x_2), x_4, f_5(x_2, x_4), x_6, \ldots). \quad (2)$$

Let us give a precise definition of semantic games for first-order logic. First we agree on some terminology. If τ is a vocabulary, ψ is a first-order τ-formula and c is an individual constant (not necessarily from the vocabulary τ), then $\psi[x/c]$ will stand for the $(\tau \cup \{c\})$-formula that results from substituting c for all free occurrences of the variable x in ψ. Whenever $\mathcal{M}$ is a τ-structure (model), by convention M will stand for the domain of $\mathcal{M}$. If $\mathcal{M}$ is a τ-structure, $\mathcal{M}'$ is a τ'-structure, and $\tau \subset \tau'$, then $\mathcal{M}'$ is an expansion of $\mathcal{M}$, provided that $M = M'$ and $\mathcal{M}'$ agrees with $\mathcal{M}$ on the interpretations of the symbols from τ.

With every vocabulary τ, τ-structure $\mathcal{M}$ and first-order τ-sentence φ, a two-player, zero-sum game $G(\varphi, \mathcal{M})$ of perfect information is associated. The games are played with the following rules.

- If $\varphi = R(a_1, \ldots, a_n)$, the play has come to an end. If $(a_1^{\mathcal{M}}, \ldots, a_n^{\mathcal{M}}) \in R^{\mathcal{M}}$, the player whose role is 'Verifier' wins, and the one whose role is 'Falsifier' loses. On the other hand, if $(a_1^{\mathcal{M}}, \ldots, a_n^{\mathcal{M}}) \notin R^{\mathcal{M}}$, then 'Falsifier' wins and 'Verifier' loses.

- If $\varphi = (\psi \vee \chi)$, then 'Verifier' chooses a disjunct $\theta \in \{\psi, \chi\}$, and the play continues as $G(\theta, \mathcal{M})$.

- $\varphi = (\psi \wedge \chi)$, then 'Falsifier' chooses a conjunct $\theta \in \{\psi, \chi\}$, and the play continues as $G(\theta, \mathcal{M})$.

[3] In order for the second-order sentence (2) to be equivalent to the sentence (1), the standard interpretation of second-order logic in the sense of Henkin (1950) must be applied (the other requisite assumption being the Axiom of Choice). In particular, n-ary function variables are taken to range over arbitrary n-ary functions on the domain. Note that in (2) a Skolem function f_{2n+1} for the quantifier $\exists x_{2n+1}$ is a function of type $M^n \to M$. Hence a Skolem function for $\exists x_1$ is a zero-place function, that is, a constant.

- If $\varphi = \exists x\psi$, then 'Verifier' chooses an element $b \in M$, gives it a name, say n_b, and the play goes on as $G(\psi[x/n_b], \mathcal{N})$, where $\mathcal{N}$ is the $(\tau \cup \{n_b\})$-structure expanding $\mathcal{M}$ and satisfying $n_b^{\mathcal{N}} = b$.
- If $\varphi = \forall x\psi$, then 'Falsifier' chooses an element $b \in M$, gives it a name, say n_b, and the play goes on as $G(\psi[x/n_b], \mathcal{N})$, where $\mathcal{N}$ is the $(\tau \cup \{n_b\})$-structure expanding $\mathcal{M}$ and satisfying $n_b^{\mathcal{N}} = b$.
- If $\varphi = \neg\psi$, then the play continues as $G(\psi, \mathcal{M})$, with the players' roles switched: the 'Verifier' of game $G(\neg\psi, \mathcal{M})$ is the 'Falsifier' of game $G(\psi, \mathcal{M})$, and vice versa.

In applying the above game rules, any play of $G(\varphi, \mathcal{M})$ reaches an atomic sentence and hence comes to an end after finitely many moves. These rules follow Hintikka's original definition (Hintikka, 1968); in particular, whenever $G(\varphi, \mathcal{M})$ is a game, φ is a sentence—formula with no free occurrences of variables. However, no conceptual difficulties are involved in generalising the definition so as to apply to first-order formulas with any number of free variables. This is accomplished by providing variable assignments γ as an extra input when specifying games. Accordingly, for every τ-formula φ, τ-structure $\mathcal{M}$, and assignment γ mapping free variables of φ to the domain M, a game $G(\varphi, \mathcal{M}, \gamma)$ can be introduced. The game rules for quantifiers become simpler when phrased in terms of variable assignments. If for instance $\varphi = \exists x\psi$, then game $G(\varphi, \mathcal{M}, \gamma)$ proceeds by 'Verifier' choosing an element $b \in M$, whereafter the play continues as $G(\psi, \mathcal{M}, \gamma')$, where γ' is otherwise like γ but maps x to b. Unlike in the games defined for sentences, now the vocabulary considered is not extended by a name for the element b, and the model $\mathcal{M}$ is not expanded.

To make proper use of games for semantic purposes, having laid down a set of game rules is not enough. We also need the notion of strategy. To this end, some auxiliary notions must be defined. A history (or, partial play) of game $G(\varphi, \mathcal{M})$ is any sequence of moves, made in accordance with the game rules. A terminal history (or, play) is a history at which it is neither player's turn to move. The set of non-terminal histories can be partitioned into two classes $\mathcal{P}_\exists$ and $\mathcal{P}_\forall$: those at which it is Eloise's turn to move and those at which it is Abelard's turn to move.

Write $O_\exists$ for the set of those tokens of logical operators in φ for which it is Eloise's turn to move in $G(\varphi, \mathcal{M})$, namely for all existential quantifiers and disjunction signs with positive polarity, and for all universal quantifiers and conjunction signs with negative polarity.[4] Likewise, write $O_\forall$ for the set of the tokens of operators for which it is Abelard's turn to move. Then the histories in

[4]A logical operator has a positive polarity in a formula φ, if it appears in φ subordinate to n negation signs with $n \in \{2m : m \in \mathbb{N}\}$; otherwise it has a negative polarity.

the set $\mathcal{P}_\exists$ can be further partitioned according to the logical operator to which they correspond: for each $O \in \mathcal{O}_\exists$ there is a subset $\mathcal{P}_\exists^O$ of $\mathcal{P}_\exists$ of those histories at which Eloise must make a move to interpret O. The set $\mathcal{P}_\forall$ is similarly partitioned by $\mathcal{P}_\forall^O$ with $O \in \mathcal{O}_\forall$.

For each $O \in \mathcal{O}_\exists$, Eloise's strategy function is a function that provides a move for her at each history belonging to $\mathcal{P}_\exists^O$. It is commonplace to stipulate that at a history $h \in \mathcal{P}_\exists^O$, Eloise's strategy function for O takes as its arguments Abelard's moves made in h.[5] A strategy for Eloise is a set of her strategy functions, one function for each operator in $\mathcal{O}_\exists$. A strategy for Eloise is winning, if it leads to a play won by Eloise against any sequence of moves by Abelard. The notions of strategy function, strategy, and winning strategy are similarly defined for Abelard.

Assuming the Axiom of Choice, it can then be shown that a first-order sentence φ is true (false) in a model $\mathcal{M}$ in the usual Tarskian sense if and only if there exists a winning strategy for Eloise (Abelard) in game $G(\varphi, \mathcal{M})$, (see Hodges, 1983; Hintikka and Kulas, 1985).[6]

The fact that any formula φ is either true or false in any given model $\mathcal{M}$ manifests on the level of games in that all semantic games for first-order logic are determined: in any game $G(\varphi, \mathcal{M})$, either Eloise or Abelard has a winning strategy. Semantic games are zero-sum, two-player games of perfect information with finite horizon. The fact that they are determined follows from the Gale-Stewart theorem (Gale and Stewart, 1953).

The framework of semantic games makes it possible to pursue research at the interface of game theory and logic. Once a parallel between logical and game-theoretic notions has been successfully drawn—as it has, for instance, in connection with the notion of truth-in-a-model for first-order logic and the game-theoretic notion of the existence of a winning strategy for Eloise in a semantic game—one can meaningfully bring in further game-theoretic notions and go on studying the resulting logical systems.

One such avenue is opened up by subjecting games to imperfect information. The goal is then to study the 'information flow' in logical formulas, or the various relations of dependence and independence between logical constants. This type of research has led to the investigation of a family of independence-friendly logics (IF logics), studied in various publications by Jaakko Hintikka, Gabriel Sandu and many others (Hintikka, 1995, 1996; Hintikka and Sandu, 1989, 1997; Hodges 1997a, b; Pietarinen, 2001b, 2006a; Sandu, 1993; Väänänen, 2007).

[5] Normally, allowing Eloise's own moves as arguments of her strategy functions would not make it any easier for Eloise to have a winning strategy.

[6] The Axiom of Choice could be avoided when formulating the relation of the game-theoretic truth-definition to the Tarskian truth-definition, if strategies in the above sense, namely deterministic strategies, were replaced by nondeterministic strategies (Hodges, 2006b; Väänänen, 2006).

The framework of semantic games with imperfect information has been applied to a host of variants of IF logic, including IF propositional logic (Pietarinen, 2001a; Sandu and Pietarinen, 2001, 2003; Sevenster, 2006a), IF modal logic (Bradfield, 2006; Bradfield and Fröschle, 2002; Hyttinen and Tulenheimo, 2005; Pietarinen, 2001c, 2003c, 2004; Tulenheimo, 2003; Tulenheimo and Sevenster, 2006; Sevenster, 2006b), IF fixpoint logic (Bradfield, 2004) and IF fuzzy logics (Cintula and Majer, this volume).

Another example of game-theoretic conceptualisations in connection with logic is furnished by systematically investigating how far the common ground between logic and game theory can be pushed (van Benthem, 2001). The paper of Sevenster (this volume) belongs to that tradition.

3 Dialogical logic

Dialogical logic (a.k.a. dialogic) offers a game-theoretic approach to the logical notions of validity and satisfiability. In so doing, it contributes to two of the four objectives mentioned by Erik C. W. Krabbe in his apology of the dialogical standpoint, "Dialogue Logic Restituted" (Hodges and Krabbe, 2001): the foundations of mathematics and the addition of a third approach to logic next to model theory and proof theory. The two further objectives are related to argumentation theory and systematic reconstruction of the language of science and politics. Let us concentrate here on dialogical logic seen from the logic-internal viewpoint.

Given a formula φ of, say, propositional logic, it is associated with a game $\mathcal{D}(\varphi)$ referred to as dialogue about φ. Such games are between two players, called the Proponent and the Opponent. Games are so defined that a formula φ of classical propositional logic is valid under the usual criteria (that is, true under all valuations) iff there is a winning strategy for the Proponent in the dialogue about φ. The framework is flexible—a game-theoretic characterisation is obtained similarly, for instance, for validity in first-order logic and in various modal logics. It has also been applied to paraconsistent, connexive and free logics (Rahman et al., 1997; Rahman and Rückert, 2001; Rahman and Keiff, 2005). What is more, the contrast between classical and intuitionistic logic has a clear-cut characterisation in terms of dialogues. Indeed, Paul Lorenzen's characterisation of validity in intuitionistic propositional logic in his 1959 talk "Ein dialogisches Konstruktivitätskriterium" (Lorenzen, 1961) in terms of dialogues was of crucial importance to the very birth of dialogical logic. With hindsight, we may observe that, given rules that define dialogues corresponding to intuitionistic propositional logic, there is a systematic liberalisation that can be effected with respect to these rules so as to yield classical propositional logic (Lorenz, 1968).

The rules of dialogues are divided into two groups—particle rules and structural rules. The former rules specify, for each logical operator (or 'logical particle'), how a formula having this operator as its outmost form can be criticised, and how such a critique can be answered. Structural rules, by contrast, lay down the ways in which the dialogues can be carried out—they specify, for instance, how the dialogue is begun, what types of attacks and defenses are allowed, and what counts, for a given player, as a win of a play of a dialogue. As it happens, dialogues for intuitionistic logic are obtained from those of classical logic by changing a single structural rule, while keeping the particle rules intact. (In classical dialogues, a player may defend himself or herself against *any* previously effected challenge, including those that the player has already defended at least once; while in intuitionistic dialogues, the player may only defend himself or herself against the most recent of those challenges that have not yet been defended.)

Dialogical logicians tend to see dialogues as a sui generis approach to logic, a third realm in addition to proof theory and model theory. Be that as it may, there is a clear sense in which dialogical logic is naturally coupled with proof theory, whereas game-theoretical semantics, in contrast, is coupled with the study of model-theoretic properties. Think of a logic $\mathcal{L}$ that admits, as a matter of fact, a sound and complete proof system, say classical propositional logic or classical first-order logic. Dialogues provide such a proof system for $\mathcal{L}$. A winning strategy of the Proponent in a dialogue about φ counts as a proof of φ. Crucially, dialogues for the logic $\mathcal{L}$ serve to recursively enumerate the set of valid formulas of $\mathcal{L}$. (Given a valid formula of $\mathcal{L}$, the Opponent's choices can only give rise to finitely many moves before a play is reached which is won by the Proponent and which cannot be further extended.) It is natural to consider systems of semantic tableaux (Hintikka, 1955; Beth, 1959; Smullyan, 1968; Fitting, 1969) as mediating the connection between proof theory and dialogues; there is an important, yet straightforward connection between tableaux on the one hand, and the totality of plays of dialogues on the other (Rahman and Keiff, 2005). In particular, for a given refutable formula φ of, say, propositional logic, there is a one-one correspondence between open maximal branches of a tableau for the signed formula $F\varphi$ and winning strategies of the Opponent in the dialogue about φ. And for a given valid propositional formula φ, there is a way of mechanically transforming the totality of closed branches of a tableau for $F\varphi$ to a winning strategy of the Proponent, and vice versa.

The moves in dialogues are formal, they do not involve objects out there (elements of the domains of models). All that is involved is manipulation of linguistic items, such as individual constants substituted for variables. Hintikka (1973) has called his semantic games 'games of seeking and finding', or 'games of exploring the world'. Semantic games are 'outdoor' games, they are related to the activities of verifying or falsifying (interpreted) formulas, while dialogues are 'indoor' games, related to proving—by suitably manipulating sequences of symbols—that certain (uninterpreted) formulas are valid (Hintikka, 1973, pp. 80–81). From Hintikka's vantage point, only 'outdoor' games can build a bridge between logical concepts and the meaningful use of language.

Naturally, the realism-antirealism dispute looms large here.[7] As is typical in connection with logics driven by proof theory, philosophically dialogical logic tends to be associated with antirealism or justificationism, namely the idea that semantic properties such as truth or validity can only be ascribed to sentences which can be recognised as having this property. In the transition from premises to conclusion, inference rules preserve assertibility rather than truth in abstracto. Therefore, a dialogician would typically not accept Hintikka's arguments for the 'semantic irrelevance' of dialogues. Rather, he or she would argue in favour of a justificationist theory of meaning, whereby an informal notion of proof would become a central semantic notion. A dialogician might further hold that dialogues capture such a notion of informal proof. It would be possible, but not necessary, to combine this view with the conception that dialogues actually introduce a third realm for logical theorising, adding to what proof theory and model theory have on offer.

Without entering philosophical discussions on the fundamental nature of dialogues, it can be observed that the notion of proof or inference to which dialogues give rise is distinct from the fully formal notion of proof operative in sound and complete proof systems. One may, at least so it seems, formulate reasonable dialogues—and reasonable tableau systems—even for pathologically incomplete logics, namely logics which simply do not admit of any sound and complete proof system. If so, the type of inference with which dialogues are concerned is semantic inference—with no a priori claim to always yield a recursive enumeration of the (uninterpreted) formulas of the language considered. If dialogues were all about formal proofs, it would be a contradiction in terms to speak of formal dialogues for incomplete logics.[8]

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[7]On antirealism (see, e.g., Dummett, 1978, 2004, 2006) and Marion, this volume.
[8]For discussion, see the contribution of Rahman and Tulenheimo in this volume (Subsection 7.2).

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