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Vittorio Maniezzo, Thomas Stützle and Stefan Voß

# Matheuristics

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## Hybridizing Metaheuristics and Mathematical Programming

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# Preface

The field of metaheuristics has traditionally been very receptive to proposals about how to structure algorithms in order to effectively solve optimization problems. Innovation of solution approaches has always been one of the traits of the field, and design paradigms have succeeded as inspiration for algorithm designers: inspiration from nature, improvements of local search, logics and probability, etc. The paradigm put forth in this book represents a “back to the roots” for computational optimization: use mathematics!

Albeit people working on metaheuristics have always been full citizens of the mathematical programming and operations research community and the main results have always been well-known, interactions (cross-fertilizations as it used to be fashionable to say) have always been limited. Core mathematical programming (MP) approaches have little to share with metaheuristics and mainstream metaheuristics include little or no mathematics.

This book shows how both metaheuristics and MP can leverage on one another. It shows how it is possible both to include MP techniques into metaheuristic frameworks and metaheuristic concepts inside MP systems. This follows a trend in hybridization, which appeared in several forms in the last years: metaheuristics are being hybridized with artificial intelligence, with constraint programming, with statistics, not to mention among themselves. However, the combination of metaheuristics and MP has a set-apart condition. Including MP techniques for a metaheuristic designer does not mean looking for contributions, which could possibly derive from another research area, it means looking inside one’s own cultural baggage, it means using in a different way something one has already had experience with.

The contributions included in this collection comprise invited chapters that give an overview on specific topics in matheuristics and articles that were selected among the presentations at the *Matheuristics 2008* workshop. This was the second edition of a workshop series centered on the above ideas, having the stated objective of giving group identity to all researchers who share the interest in the synergies existing between the two related research lines metaheuristics and MP. The success of the first edition, which was upon

invitation only, suggested to open to submissions for the second. We scored an higher than 30% rejection rate at the conference, and acceptance barriers were high also for the present volume. We thus believe to have collected a set of good quality contributions, which will help in increasing the awareness of the possibilities offered by this new research direction.

The book includes 11 contributions, which span over a variety of topics.

Caserta and Voß open with an up-to-date overview of the field of metaheuristics, which helps to frame the matheuristics contributions in the more general context of metaheuristics advances.

Fischetti, Lodi and Salvagnin provide a survey on the use of mixed integer programming (MIP) solvers as subroutines for solving NP-hard subproblems, which arise while solving a more complex problem. Different success cases are reported, which follow this general idea.

Puchinger, Raidl and Pirkwieser review possibilities of how to use greedy heuristics, iterative improvement algorithms, and metaheuristics to improve the performance of MIP solvers. The possibilities for doing so are varied, and range from providing good quality starting solutions to using metaheuristics for cut separation or column generation.

Dumitrescu and Stützle review approaches that combine local search and metaheuristics with MP techniques. In particular, they focus on algorithms where the metaheuristic is the master solver and MP techniques are used to solve subproblems arising in the search process.

Boschetti, Maniezzo and Roffilli show how it is possible to use decomposition techniques, which were originally conceived as tools for exact optimization, as metaheuristic frameworks. The general structure of each of the best known decomposition approaches (Lagrangian, Benders and Dantzig-Wolfe) can, in fact, be considered as a general structure for a corresponding metaheuristic.

Gutjahr proposes a more theoretical contribution. His chapter gives an overview on techniques for proving convergence of metaheuristics to optimal or sufficiently good solutions. Two particularly interesting topics that are covered are convergence of metaheuristic algorithms for stochastic combinatorial optimization, and estimation of expected runtime, i.e., of the time needed by a metaheuristic to hit the first time a solution of a required quality.

The remaining papers address specific problems by means of matheuristic algorithms, rather than presenting general overviews as the ones above.

Dolgui, Ereimeev and Guschinskaya address the problem of balancing transfer lines with multi-spindle machines. They present two algorithms that integrate traditional metaheuristics and a MIP solver. Specifically, they design a GRASP and a genetic algorithm, which both use a MIP solver as a subroutine for solving subproblems arising in the search process of the metaheuristics.

Gruber and Raidl work on exact algorithms for the bounded diameter minimum spanning tree problem. They use simple heuristics and a tabu search algorithm for solving the separation problem in their branch-and-cut algo-

rithm. Moreover, they include in their approach a variable neighborhood descent for finding good primal solutions.

Liberti, Nannicini and Mladenović present a work in mixed integer non-linear programming, having the objective of identifying good quality, or at least feasible solutions for difficult instances. They propose a method, called RECIPE (for Relaxed-Exact Continuous-Integer Problem Exploration), combining variable neighborhood search, local branching, sequential quadratic programming and branch-and-bound.

Mitrović-Minić and Punnen present their method consisting in a local search where large neighborhoods are explored by means of ancillary MIP subproblems. They present results on the basic generalized assignment problem (GAP) and on the multi-resource GAP showing the potential of the approach.

Finally, Ulrich-Ngueveu, Prins, and Wolfler-Calvo study the  $m$ -peripatetic vehicle routing problem, which is a special vehicle routing problem, asking that each arc is used in the solution at most once for each set of  $m$  periods considered in the plan. The approach uses a perfect  $b$ -matching to define the candidate sets used in a granular tabu search algorithm.

We conclude expressing our gratitude to all authors who submitted their works to Matheuristics 2008 and to this post-conference collection, to the members of the international program committee and to all external reviewers. We are confident that this book, as a result of their joint effort, will provide a basis to support the increasing interest on the covered topics. We hope and believe that the result constitutes a significant achievement in the direction of establishing matheuristics as a credible tool for obtaining fast and reliable solutions to real-world problems.

Bologna, Brussels, Hamburg,  
April 2009

*Vittorio Maniezzo  
Thomas Stützle  
Stefan Voß*

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