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Distributed-Order Dynamic Systems

Stability, Simulation, Applications and Perspectives

Springer

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ISSN 2191-8112
ISBN 978-1-4471-2851-9
DOI 10.1007/978-1-4471-2852-6
Springer London Heidelberg New York Dordrecht

e-ISSN 2191-8120
e-ISBN 978-1-4471-2852-6

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

Library of Congress Control Number: 2012931946

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To our colleagues, friends, mentors and families

Preface

Fractional calculus is now being more widely accepted. The (constant) order of differentiation and/or integration can be an arbitrary real number including integers as special cases. For example, a low pass filter (LPF) with a fractional order pole can be written as $H(s) = 1/(\tau s^{\alpha_0} + 1)$, where $\alpha_0 > 0$ is a constant. Its corresponding governing differential equation is

$$\tau \frac{\mathrm{d}^{\alpha_0}}{\mathrm{d}t^{\alpha_0}} y(t) + y(t) = u(t),$$

where $u(t)$ and $y(t)$ are input and output signals, respectively; $\frac{\mathrm{d}^{\alpha_0}}{\mathrm{d}t^{\alpha_0}} y(t)$ or $y^{(\alpha_0)}(t)$ is the notation of fractional order derivative of $y(t)$. It is mathematically immediate to generalize this constant-order LPF in distributed-order sense as

$$H_{do}(s) = \frac{1}{b-a} \int_a^b \frac{1}{\tau s^{\alpha} + 1} \mathrm{d}\alpha,$$

where a and b are given constants and the term $\frac{1}{b-a}$ is for scaling the DC gain to be 0 dB. The distributed-order dynamics can be characterized by the following distributed-order differential equation

$$\int_a^b w(\alpha) \frac{\mathrm{d}^{\alpha}}{\mathrm{d}t^{\alpha}} y(t) \mathrm{d}\alpha + y(t) = u(t),$$

where $w(\alpha)$ can be regarded as order-dependent time constant or "order weight/distribution function."

Note that, the above constant-order model is in the same form of the famous classic Cole–Cole relaxation model, which can be recovered from the distributed-order model by setting the order distribution function $w(\alpha) = \delta(\alpha - \alpha_0)$, where $\delta(\cdot)$ is the well known Dirac Delta function. So, it is natural to believe that distributed-order Cole–Cole model $H_{do}(s)$ may be in a better position to characterize the *complex* material properties when the distribution function $w(\alpha)$ is properly chosen. The wisdom in modeling "*All models are wrong but some are useful*" and

"*All models are wrong but some are dangerous*", in fact, encourages us to explore the distributed-order generalization since we believe this notion is helpful, at least partially, as demonstrated in this Brief, with no harm.

With the above in mind, this Brief presents a general approach of distributed-order operator which can and will find its use for real world applications, as being observed from recent literature in many fields of science and engineering. It is devoted to provide an introduction of the latest research results about distributed-order dynamic system and control as well as distributed-order signal processing, which are based on the distributed-order differential/integral equations, to serve the control and signal processing community as a guide to understanding and using distributed-order differential/integral equations in order to enlarge the application domains of its disciplines, and to improve and generalize well established (constant-order) fractional-order control methods and strategies.

A major goal of this Brief is to present a concise and insightful view of the relevant knowledge by emphasizing fundamental methods and tools to understand why distributed-order concept is useful in control and signal processing, to understand its terminology, and to illuminate the key points of its applicability. The Brief is suitable for science and engineering community for broadening their toolbox in modeling, analysis, control, filtering tasks, with a hope that, transformative progress can be made in their respective research projects.

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Technical University of Kosice Igor Podlubny
September 2011

Acknowledgements

As an emerging research topic, distributed-order systems started to draw attention from the research community. With this SpringerBriefs, we wish to provide an almost exhaustive, comprehensive literature review and a summary of our research efforts during the past few years on distributed-order systems. This Brief thus contains materials from papers and articles that were previously published. We are thankful and would like to acknowledge the copyright permissions from the following publishers who have released our work on that topic:

Acknowledgement is given to the Institute of Electrical and Electronic Engineers (IEEE) to reproduce material from the following papers:

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Acknowledgement is given to Elsevier B.V. to reproduce material from the following papers:

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The research described in this Brief would not have been possible without the inspiration and help from the work of individuals in the research community, and we would like to acknowledge their help.

There are many people to whom the authors are obliged for their help and support.

Zhuang Jiao would like to express his sincere thanks to his tutor, Prof. Yisheng Zhong, the Tsinghua-Santander Postgraduate Research Scholarship for financial support to his research study in the USA, former and current CSOIS members: Dr. Yan Li, Dr. Hu Sheng for their support during his Ph.D. studies in CSOIS at Utah State University as an exchange Ph.D. visiting scholar. In particular, he appreciates the members of AFC (Applied Fractional Calculus) at Utah State University: Calvin Coopmans, Hadi Malek, Prof. Deshun Yin, Dr. Dali Chen, Dr. Haiyang Chao, Jinlu Han, Long Di, Yaojin Xu, Dr. Xuefeng Zhang, Dr. Kecai Cao, Peng Guo, Bo Li, Shuai Hu, Zhuo Li, Ms. Pooja Kavathekar, Ms. Sara Dadras.

YangQuan Chen would like to thank his wife Dr. Huifang Dou and his sons Duyun, David, and Daniel, for their patience, understanding, and complete support throughout this work. He is also thankful to Caibin Zeng for assistance in final round of proofreading.

Igor Podlubny is thankful to Aleksandr N. Vityuk and Viktor V. Verbitsky from the Odessa National University (Odessa, Ukraine) for fruitful discussions on distributed-order operators, and to Anatoly A. Alikhanov from the Kabardino-Balkarian State University (Nalchik, Russia), for providing help with obtaining some sources. Support from the BERG Faculty of the Technical University of Kosice (dean: Professor Gabriel Weiss) and from the grant agencies VEGA (grant 1/0497/11) and APVV (grants APVV-0040-07, APVV-0482-11, SK-UA-0042-09) is gratefully acknowledged.

Finally, we would like to thank Oliver Jackson of Springer for his interest in this Brief project and to Charlotte Cross, Editorial Assistant (Engineering), Springer London, for many good suggestions.

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Acronyms

BIBO	Bounded-Input Bounded-Output
CPE	Constant Phase Element
DOD	Distributed-order Differentiator
DODE	Distributed-order Differential Equation
DODS	Distributed-order Dynamic System
DOI	Distributed-order Integrator
DOIE	Distributed-order Integral Equation
DOLTIS	Distributed-order Linear Time-Invariant System
DOPDE	Distributed-order Partial Differential Equation
FC	Fractional Calculus
FODE	Fractional-order Differential Equation
FOIE	Fractional-order Integral Equation
FO-LTI	Fractional-order Linear Time-Invariant
FOPDE	Fractional-order Partial Differential Equation
IAE	Integral of Absolute Error
ISE	Integral of Squared Error
ISTE	Integral of Squared Time multiplied Error
ITAE	Integrated of Time multiplied Absolute Error
ITSE	Integrated of Time multiplied Squared Error
LPF	Low Pass Filter
NILT	Numerical Inverse Laplace Transform
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation