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With 222 Figures



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To Ming and Jingwen, Jessie and Lian

# Preface

Many techniques are available for designing linear multivariable analog controllers: pole placement using observer-based controllers, loopshaping, the inverse Nyquist array method, convex optimization in controller parameter space, and so on. One class of techniques is to specify a performance function and then optimize it, and one such performance function is the norm of the closed-loop transfer matrix, suitably weighted. The two most popular norms to optimize are the  $\mathcal{H}_2$  and  $\mathcal{H}_\infty$  norms. The fact that most new industrial controllers are digital provides strong motivation for adapting or extending these design techniques to digital control systems.

This book is intended as a graduate text in linear sampled-data (SD) control systems. The subject of SD control is a subdomain of digital control; it deals with sampled signals and their discrete-time processing, but not with quantization effects nor with issues of real-time software. SD control systems consist of continuous-time plants to be controlled, discrete-time controllers controlling them, and ideal continuous-to-discrete and discrete-to-continuous transformers.

As a prerequisite, the ideal reader would know multivariable analog control design, especially  $\mathcal{H}_2$  and  $\mathcal{H}_\infty$  theory—a user's guide to  $\mathcal{H}_2$  and  $\mathcal{H}_\infty$  theory is presented in Chapter 2. A prior course on digital control at the undergraduate level would also be an asset. Standard facts about state models in continuous and discrete time are collected in the appendix.

Part I (Chapters 2–8) is aimed at first-year graduate students, while Part II (Chapters 9–13) is more advanced. In particular, some of the development in the later chapters is framed in the language of operator theory.

In Part I we present two indirect methods of SD controller design:

- Discretize the plant and design the controller in discrete time.
- Design the controller in continuous time, then discretize it.

These two approaches both involve approximations to the real problem, which involves an analog plant, continuous-time performance specifications, and a SD controller. Part II proposes a direct attack in the continuous-time domain, where SD systems are time-varying (actually, periodic). The main problems addressed are  $\mathcal{H}_2$  and  $\mathcal{H}_\infty$  optimal SD control. The solutions are presented in forms that can readily be programmed in, for example, MATLAB. MATLAB

with the  $\mu$ -Tools toolbox was used for the examples.

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