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Alexandre Rademaker

A Proof Theory for Description Logics

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ISSN 2191-5768 ISSN 2191-5776 (electronic)
ISBN 978-1-4471-4001-6 ISBN 978-1-4471-4002-3 (eBook)
DOI 10.1007/978-1-4471-4002-3
Springer London Heidelberg New York Dordrecht

Library of Congress Control Number: 2012937297

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Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

To my family, Carla, Gabriela and Sofia. To my parents, André and Silvia whom I thank for their constant encouragement. I am specially thankful to my wife, Carla, for her love and friendship.

Preface

This book is the reprint of my thesis defended in 2010 at PUC-Rio. The content is mostly identical and contains only few adjustments and complements to the original thesis.

For instance, I took advance of some experiences gathered from the Semantic Web community during the 10th Semantic Web Conference at Bonn. A presentation during that event reminded me that I should write about how our systems can be used in the problem called “justification” by the Semantic Web community. I also add some brief comments about the extension of our deduction systems to deal with intuitionistic version of $\mathcal{ALC}$. This research was started right after I got my PhD and it is now referenced in the conclusion.

I specially thank Edward Hermann Haeusler, my advisor and friend, who never refuses to give me some advice and share his ideas. I also thank Leonardo Magalhes de Araujo Monteiro for his careful revision of the English language.

Rio de Janeiro
January

Alexandre Rademaker

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