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Max Bramer

# Principles of Data Mining

Fourth Edition

 Springer

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# *About This Book*

This book is designed to be suitable for an introductory course at either undergraduate or masters level. It can be used as a textbook for a taught unit in a degree programme on potentially any of a wide range of subjects including Computer Science, Business Studies, Marketing, Artificial Intelligence, Bioinformatics and Forensic Science. It is also suitable for use as a self-study book for those in technical or management positions who wish to gain an understanding of the subject that goes beyond the superficial. It goes well beyond the generalities of many introductory books on Data Mining but — unlike many other books — you will not need a degree and/or considerable fluency in Mathematics to understand it.

Mathematics is a language in which it is possible to express very complex and sophisticated ideas. Unfortunately it is a language in which 99% of the human race is not fluent, although many people have some basic knowledge of it from early experiences (not always pleasant ones) at school. The author is a former Mathematician who now prefers to communicate in plain English wherever possible and believes that a good example is worth a hundred mathematical symbols.

One of the author's aims in writing this book has been to eliminate mathematical formalism in the interests of clarity wherever possible. Unfortunately it has not been possible to bury mathematical notation entirely. A 'refresher' of everything you need to know to begin studying the book is given in Appendix A. It should be quite familiar to anyone who has studied Mathematics at school level. Everything else will be explained as we come to it. If you have difficulty following the notation in some places, you can usually safely ignore it, just concentrating on the results and the detailed examples given. For those who would like to pursue the mathematical underpinnings of Data Mining in greater depth, a number of additional texts are listed in Appendix C.

No introductory book on Data Mining can take you to research level in the subject — the days for that have long passed. This book will give you a good grounding in the principal techniques without attempting to show you this year’s latest fashions, which in most cases will have been superseded by the time the book gets into your hands. Once you know the basic methods, there are many sources you can use to find the latest developments in the field. Some of these are listed in Appendix C. The other appendices include information about the main datasets used in the examples in the book, many of which are of interest in their own right and are readily available for use in your own projects if you wish, and a glossary of the technical terms used in the book.

Self-assessment Exercises are included for each chapter to enable you to check your understanding. Specimen solutions are given in Appendix E.

## Note on the Fourth Edition

Since the first edition there has been a vast and ever-accelerating increase in the volume of data available for data mining. According to IBM (in 2016) 2.5 billion billion bytes of data is produced every day from sensors, mobile devices, online transactions and social networks, with 90 percent of the data in the world having been created in the last two years alone. Today the amount of healthcare data available in the world is estimated as over 2 trillion gigabytes. To reflect the growing popularity of ‘deep learning’ a new final chapter has been added which gives a detailed introduction to one of the most important types of neural net and shows how it can be applied to classification tasks.

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I would like to thank my daughter Bryony for drawing many of the more complex diagrams and for general advice on design, Dr. Frederic Stahl for advice on Chapters 21 and 22 and my wife Dawn for her very valuable comments on draft chapters and for preparing the index. The responsibility for any errors that may have crept into the final version remains with me.

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