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Daniel Liberzon

# Switching in Systems and Control

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# Preface

Many systems encountered in practice involve a coupling between continuous dynamics and discrete events. Systems in which these two kinds of dynamics coexist and interact are usually called *hybrid*. For example, the following phenomena give rise to hybrid behavior: a valve or a power switch opening and closing; a thermostat turning the heat on and off; biological cells growing and dividing; a server switching between buffers in a queueing network; aircraft entering, crossing, and leaving an air traffic control region; dynamics of a car changing abruptly due to wheels locking and unlocking on ice. Hybrid systems constitute a relatively new and very active area of current research. They present interesting theoretical challenges and are important in many real-world problems. Due to its inherently interdisciplinary nature, the field has attracted the attention of people with diverse backgrounds, primarily computer scientists, applied mathematicians, and engineers.

Researchers with a background and interest in continuous-time systems and control theory are concerned primarily with properties of the continuous dynamics, such as Lyapunov stability. A detailed investigation of the discrete behavior, on the other hand, is usually not a goal in itself. In fact, rather than dealing with specifics of the discrete dynamics, it is often useful to describe and analyze a more general category of systems which is known to contain a particular model of interest. This is accomplished by considering continuous-time systems with discrete switching events from a certain class. Such systems are called *switched systems* and can be viewed as higher-level abstractions of hybrid systems, although they are of interest in their own right.



The present book is not really a book on hybrid systems, but rather a book on switched systems written from a control-theoretic perspective. In particular, the reader will not find a formal definition of a hybrid system here. Such a definition is not necessary for the purposes of this book, the emphasis of which is on formulating and solving stability analysis and control design problems and not on studying general models of hybrid systems. The main goal of the book is to bridge the gap between classical mathematical control theory and the interdisciplinary field of hybrid systems, the former being the point of departure. More specifically, system-theoretic tools are used to analyze and synthesize systems that display quite nontrivial switching behavior and thus fall outside the scope of traditional control theory.

This book is based on lecture notes for an advanced graduate course on hybrid systems and control, which I taught at the University of Illinois at Urbana-Champaign in 2001–2002. The level at which the book is written is somewhere between a graduate textbook and a research monograph. All of the material can be covered in a semester course, although the instructor will probably need to skip some details in the treatment of more advanced topics and assign them as supplementary reading. The book can also serve as an introduction to the main research issues and results on switched systems and switching control for researchers working in various areas of control theory, as well as a reference source for experts in the field of hybrid systems and control.

It is assumed that the reader is familiar with basic linear systems theory. Some results on existence and uniqueness of solutions to differential equations, Lyapunov stability of nonlinear systems, nonlinear stabilization, and mathematical background are reviewed in suitable chapters and in the appendices. This material is covered in a somewhat informal style, to allow the reader to get to the main developments quickly. The level of rigor builds up as the reader reaches more advanced topics. My goal was to make the presentation accessible yet mathematically precise.

The main body of the book consists of three parts. The first part introduces the reader to the class of systems studied in the book. The second part is devoted to stability theory for switched systems; it deals with single and multiple Lyapunov function analysis methods, Lie-algebraic stability criteria, stability under limited-rate switching, and switched systems with various types of useful special structure. The third part is devoted to switching control design; it describes several wide classes of continuous-time control systems for which the logic-based switching paradigm emerges naturally as a control design tool, and presents switching control algorithms for several specific problems such as stabilization of nonholonomic systems, control with limited information, and switching adaptive control of uncertain systems. At the moment there is no general theory of switching control or a standard set of topics to discuss, and the choice of material in this part is based largely on my personal preferences. It is hoped, however, that the

book will contribute to creating a commonly accepted body of material to be covered in courses on this subject.

Typically, results are first developed for linear systems and then extended to nonlinear systems. Complete proofs of most of the results are provided, other proofs are given in sketched form. A few exercises are scattered throughout the text. Since the book focuses on theoretical developments, students interested in applications will need to study other sources. In the course that I taught at the University of Illinois, the students were required to do final projects in which they could apply the theory developed in class to practical problems.

I would like to call special attention to the Notes and References section at the end of the book. It complements the main text by providing many additional comments and pointers to a large body of literature, from research articles on which this book is based to a variety of related topics not covered here. The reader should remember to consult this section often, as references in the main text are kept to a minimum. The literature on the subject is growing so rapidly, however, that the bibliography supplied here will quickly go out of date.

I am indebted to many people who influenced my thinking and offered valuable advice on the material of this book. I would especially like to thank my former advisors: Andrei Agrachev, Roger Brockett, and Steve Morse. This book would not have been possible without the research contributions of João Hespanha. It has also benefited greatly from my interactions with Eduardo Sontag. I am grateful to my colleagues at the University of Illinois for creating a very stimulating environment, and particularly to Tamer Başar who encouraged me to teach a course on hybrid systems and to publish this book. I am thankful for the numerous corrections and comments that I received from students while teaching the course. The support of the National Science Foundation and the DARPA/AFOSR MURI Program is gratefully acknowledged.

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