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Stability of Time-Delay Systems

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Preface

This book is devoted to the study of stability of time-delay systems, an area long pursued by mathematicians, physical and life scientists, engineers, and economists. Given the history and maturity of the discipline, one may ask what new material could be presented beyond the extensive existing body of work. The answer lies in the enduring and surprising vitality of the subject: in particular, over the last decade, research has yielded a number of notable results. The study of time-delay systems has therefore undergone a significant conceptual and practical leap. Our aim in this book is to present some of the highlights of these advances and to further develop the techniques and tools along recent trends.

By definition, the future evolution of a time-delay system depends not only on its present state but also on its history. This particular cause and effect relationship can be most succinctly captured and has been traditionally modeled by differential-difference equations, or more generally, by functional differential equations. While in practice many dynamical systems may be satisfactorily described by ordinary differential equations alone, for which the system's future evolution depends solely on its current state, there are times when the delay effect cannot be neglected. In short, time delay is hardly a matter of rarity; numerous examples presented in this book and elsewhere attest to its prevalence. And, because of their ubiquitous presence, time-delay systems have been an active area of scientific research in a wide range of natural and social sciences: they are often known as hereditary systems, systems with after-effect, systems with time-lag, and more generally, as a subclass of functional differential equations and infinite-dimensional systems.

The field of time-delay systems had its origins in the 18th century, and it received substantial attention in the early 20th century in works devoted to the modeling of biological, ecological, and engineering systems. Stability of time-delay systems became a formal subject of study in the 1940s, with contributions from such towering figures as Pontryagin and Bellman. Over the years its interest and popularity have grown steadily. In particular, in the last 10 to 15 years there has been a surge in research and a proliferation of new techniques and results. A cursory glance at the large numbers of articles published in international conferences, organized workshops, and archive journals during this period indicates the scale and magnitude of this progress. Advances in numerical methods and control theory, especially the theory of robust stability and control, have had substantial impact on the field. We observe that the small gain theorem made it possible to formulate the stability problem of time-delay systems as one of robust stability and of the structured singular value. Techniques in robust stability analysis of uncertain polynomials find generalizations to uncertain quasipolynomials, which serve as models for uncertain time-delay systems. Efficient numerical algorithms for solving linear matrix inequalities (LMI) have generated substantial interest to pose stability problems as LMI conditions.

Recognizing these advances, we concluded that a book project would be the natural outcome of more than a decade's creative work which we hope will contribute to future studies of the subject. Our primary objective is to present a sufficiently thorough yet focused treatment of the key methods and results of the past 10 years or so, and to systematically organize and discuss in sufficient technical depth many of the results that are otherwise scattered in various journals and conference proceedings. Since the volume of such publications can be overwhelming, we hope this book will be a convenient reference and will provide quick access to important issues and developments in time-delay systems.

Our particular viewpoint is centered on computability, robust stability and robust control. We emphasize conceptual significance over technical details, without sacrificing mathematical rigor. We stress fundamental, intuitive observations behind methods and results, and we try to build links and relationships among concepts that, at first glance, may seem unrelated. In other words, we try to provide an "insider's story" based on our experience. This perspective has guided us in both the selection and presentation of material: while the facts and results are certainly worthwhile, we also hope that the ideas and intuition will be helpful in understanding the nitty-gritty technical details, especially to new readers. On the other hand, we face the unpleasant yet inevitable task of limiting ourselves to some selected topics, excluding materials that to some may be equally or even more important. Examples include classical results, recent developments on stabilization, control, and filtering of time-delay systems, and those on stochastic time-delay systems, to name just several. Of course, this selection merely reflects the authors' viewpoints and preferences rather than the value of the particular issues. Fortunately, we have been able to refer readers to several classic and recently-published works on these topics.

Despite widespread interest, it is still uncommon to find time-delay systems adopted as a regular course at most institutions. For many, the first and perhaps only encounter with the subject is likely to take place in an undergraduate course on systems modeling or automatic control. For this reason, our book has been conceived and written as a monograph aimed at researchers, practicing engineers, and graduate students. We do not presume, although we do not preclude the possibility, that it can serve as a textbook for an advanced graduate course. It is nevertheless appropriate as a self-study text. Most of the book is easily accessible to a typical secondyear graduate student in engineering with basic knowledge of state-space and transfer function descriptions and stability concepts of dynamical systems. A few specialized topics will require more advanced mathematical background. Readers with less preparation may opt to skip the proofs of such results, but quickly move on to the theorem statements. The book can be read according to the reader's interests: for those interested mainly in frequency-domain results, the first four chapters can be read in order. For time-domain techniques, the reader may begin with Chapter 1 and then move directly to Chapters 5 to 7. Most of the material in Chapter 8 can be understood after Chapters 5 and 6. The book has been made sufficiently self-contained, with necessary technical preliminaries integrated into the respective chapters and included in two appendices.

The contents are centered on the theme of stability and robust stability of time-delay systems. Chapter 1 begins with a number of practical examples in which time delays play an important role. It then continues with an introductory exposition of some basic concepts and results essential to stability analysis, such as functional differential equation representation, characteristic quasipolynomials, the Lyapunov–Krasovskii Stability Theorem, and the Razumikhin Theorem.

The next three chapters develop frequency-domain criteria for stability and robust stability of linear time-invariant systems. Chapter 2 focuses on systems with commensurate delays only. It presents both frequency-sweeping and constant matrix tests, which are both necessary and sufficient conditions for delay-dependent and delay-independent stability, and both require computing the generalized eigenvalues of matrix pairs. Chapter 3 studies systems with incommensurate delays. The development is built upon a small gain approach, and the necessary and sufficient condition for stability independent of delay is shown to be equivalent to a problem of computing the structured singular value. The chapter opens with a brief exposure to key concepts found in robust stability analysis, such as the small gain theorem and the structured singular value, and ends with a formal analysis of the computational complexity inherent in the stability problem. Both Chapters 2 and 3 treat systems modeled either by state-space descriptions or quasipolynomials, but with no consideration of system uncertainty. Uncertain time-delay systems are addressed in Chapter 4. More specifically, this chapter examines uncertain quasipolynomials of systems with incommensurate delays, that is, families of multivariate polynomials whose coefficients are permitted to vary in a prescribed set; notable examples in this class include interval and diamond quasipolynomials. In much the same spirit as in robust control, this chapter develops robust stability conditions that ensure the stability of the entire family of quasipolynomials, by checking the so-called edge, and in some instances only the vertices of the quasipolynomial family.

book is based on such work. We are equally grateful to Chaouki Abdallah, Jean-Michel Dion, Luc Dugard, Michael Fan, Didier Georges, Jacob Kogan, Brad Lehman, James Louisell, Olivier Sename, Gilead Tadmor, Sophie Tarboriech, Onur Toker, Li Qiu, Erik Verriest, and Kemin Zhou. We benefited from their ideas and wisdom through numerous discussions. They contributed either directly to the book via joint work, or indirectly by making suggestions and imparting to us their discerning taste. We thank Birkhäuser for the opportunity to publish the book, and its fine editorial staff for their patience and technical support. They have leniently accommodated several "delays" which we hope have paid off. We thank Rex Pierce and Gang Chen, who each helped prepare a number of computer-generated graphs. Finally, we gratefully acknowledge the US National Science Foundation for its financial support during the period of this project, under the direction of Kishan Baheti and Rose Gombay, respectively.

In closing we ask the reader for the privilege of a moment of reflection on our collaborative process. It may seem somewhat odd that the authors never had a record of working together before initiating such a major, time-consuming project. The idea was first initiated by KG at the 1997 American Control Conference held at Albuquerque, New Mexico, during a casual conversation between KG and JC. Writing a book together seemed at first a remote possibility. It remained so until the summer of 1998, when at a robust control workshop held in Sienna, Italy, VK and JC rekindled the idea. The picturesque setting of Tuscany's rolling hills and stunning sunsets seemed to promise a happy ending, and the project then took off. The three authors had discussions via emails and phone calls, conceiving the contents and carving out a game plan. It would take another four years to implement the plan, and after numerous drafts and iterations, the project was finally brought to its present form. We have attempted in good faith to communicate our views and passion for the subject. The book, in retrospect, is a triumph of will and friendship. We thus thank each other as well.

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Notation

Sets and Operators

$\mathbb{R}$, $\mathbb{R}^n$, $\mathbb{R}^{n\times n}$	The sets of real numbers, n component real vectors, and n by n real matrices
$\mathbb{C}$, $\mathbb{C}^n$, $\mathbb{C}^{n\times n}$	The sets of complex numbers, n component complex vectors, and n by n complex matrices
$\mathbb{D}$	The open unit disk $\{s \mid s \in \mathbb{C}, \ \lvert s\rvert < 1\}$
$\mathbb{R}_+$	The set of positive real numbers
$\mathbb{T}$	The unit circle $\{s \mid s \in \mathbb{C}, \ \lvert s\rvert = 1\}$
$\mathbb{Z}$	The set of integers
$\bar{\mathbb{S}}$, $\mathbb{S}^\circ$, $\partial\mathbb{S}$	The closure, interior, and boundary of $\mathbb{S}$, where $\mathbb{S}$ is any set. For example, $\bar{\mathbb{D}}$ is the closed unit disk.
$\mathbb{S}^n$	$\{(s_1, s_2, ..., s_n) \mid s_i \in \mathbb{S}, \ i = 1, 2, ..., n\}$, $\mathbb{S}$ any set
$\mathbb{C}_+$	The set $\{w \in \mathbb{C} \mid \mathrm{Re}(w) > 0\}$
L_2	The set of real square integrable functions
$\mathcal{L}_2$	The set of of functions of $j\omega$ integrable for ω
$\mathcal{H}_\infty$	The set of transfer functions $H(s)$, with $\lvert\lvert H\rvert\rvert_\infty < \infty$
$\mathrm{Re}(w)$, $\mathrm{Im}(w)$	The real and imaginary parts of $w \in \mathbb{C}$
w^*	The complex conjugate of w
$\lvert\lvert\cdot\rvert\rvert$	Vector or matrix norm.
$\mathcal{C}[a, b]$	The set of $\mathbb{R}^n$-valued continuous functions on $[a, b]$
$\mathcal{C}$	$\mathcal{C}[-r, 0]$
$\lvert\lvert\phi\rvert\rvert_c$	The continuous norm $\max\limits_{a\leq\xi\leq b} \lvert\lvert\phi(\xi)\rvert\rvert$ for $\phi \in \mathcal{C}[a, b]$
$\dot{x}$	Derivative of x with respect to time t, $\frac{dx}{dt}$
$\mathcal{L}(\cdot), \mathcal{L}^{-1}(\cdot)$	Laplace transform and the inverse Laplace transform

Matrix-related notation

A^T, A^*, A^H	Transpose, component-wise complex conjugate, and Hermitian (conjugate) transpose of matrix A.
$\lvert A\rvert$	Matrix formed by taking absolute value of each entry of A
$A > 0$, $A \geq 0$	The matrix A is positive (semi)definite ($<$ or $\leq$ similar)
$A > B$, $A \geq B$	$A - B > 0$, $A - B \geq 0$
$\lambda(A), \lambda_i(A)$	An eigenvalue and the ith eigenvalue of matrix A

$\lambda(A,B)$	A generalized eigenvalue of matrix pair (A,B)
$\lambda_{\max}(A), \lambda_{\min}(A)$	The maximum and the minimum eigenvalue of Hermitian matrix A
$\rho(A)$	Spectral radius of matrix A, $\max_i \lvert\lambda_i(A)\rvert$
$\sigma(A)$	Spectrum (the set of all the eigenvalues) of matrix A
$\sigma(A,B)$	Spectrum of matrix pair (A,B)
$\sigma_i(A)$	The ith singular value of A
$\sigma_{\max}(A)$ or $\bar{\sigma}(A)$	The maximum singular value of A
$\sigma_{\min}(A)$ or $\underline{\sigma}(A)$	The minimum singular value of A
$\nu(A)$	Matrix measure of matrix A
$A \otimes B$	Kronecker product of A and B
$A \oplus B$	Kronecker sum of A and B, $A \otimes I + I \otimes B$