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Mixed and Hybrid Finite Element Methods

With 65 Illustrations

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Preface

When, a few years ago, we began the redaction of this book, we had the naive thought that the theory of mixed and hybrid finite element methods was ripe enough for a unified presentation. We soon realized that things were not so simple and that, if basic facts were known, many obscure zones remained in many applications. Indeed the literature about nonstandard finite element method is still evolving rapidly and this book cannot pretend to be complete. We would rather like to lead the reader through the general framework in which development is taking place.

We have therefore built our presentation around a few classical examples: Dirichlet's problem, Stokes problem, linear elasticity, ... They are sketched in Chapter I and basic methods to approximate them are presented in Chapter IV, following the general theory of Chapter II and using finite element spaces of Chapter III. Those four chapters are therefore the essential part of the book. They are complemented by the following three chapters which present a more detailed analysis of some problems.

Chapter V comes back to mixed approximations of Dirichlet's problem and analyses, in particular, the (lambda)-trick that enables to make the link between mixed methods and more classical non-conforming methods. Chapter VI deals with Stokes problem and Chapter VII with linear elasticity and the Mindlin-Reissner plate model.

The reader should not look here for practical implementation tricks. Our goal was to provide an analysis of the methods in order to understand their properties as thoroughly as possible. We refer, among others, to the recent work of BATHE [A] or HUGHES [A] or to the classical and indispensable book of ZIENKIEWICZ [A] for practical considerations. We are of course strongly indebted to CIARLET [A] which remains the essential reference for the classical theory of finite element methods. Finally we also refer to ROBERTS–THOMAS [A] for another presentation of mixed methods.

This book would never have come to its end without the help, encouragement and criticisms of our friends and colleagues. We must also thank all those who took the time reading the first draft of our manuscript and proposed significant improvements. We hope that the final result will better than what one might expect, according to the quotations thereafter, of the hybrid resulting of a collaboration between Pavia and Québec.

Apris atque sui setosus nascitur hybris. (C. Plinius Caecilius Secondus.)

Mixtumque genus prolesque biformis. (Publius Virgilius Maro.)

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