

Statistical Models of Shape

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Optimisation and Evaluation



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*If fy rhieni, diolch am eich holl gefnogaeth.
Rhodri*

*To my wife Jo, and to the memory of
my father.
Carole*

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me sane.
Chris*

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