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Stan Z. Li

Markov Random Field Modeling in Image Analysis

Third Edition

Springer

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In Memory of My Mother

“An excellent book — very thorough and very clearly written.”

— Stuart Geman

“I have found the book to be a very valuable reference. I am very impressed by both the breadth and depth of the coverage. This must have been a truly monumental undertaking.”

— Charles A. Bouman

Foreword by Anil K. Jain

The objective of mathematical modeling in image processing and computer vision is to capture the intrinsic character of the image in a few parameters so as to understand the nature of the phenomena generating the image. Models are also useful to specify natural constraints and general assumptions about the physical world; such constraints and assumptions are necessary to solve the "inverse" problem of three-dimensional scene interpretation given two-dimensional image(s) of the scene. The introduction of stochastic or random field models has led to the development of algorithms for image restoration, segmentation, texture modeling, classification, and sensor fusion. In particular, Gibbs and Markov random fields for modeling spatial context and stochastic interaction among observable quantities have been quite useful in many practical problems, including medical image analysis and interpretation of remotely sensed images. As a result, Markov random field models have generated a substantial amount of excitement in image processing, computer vision, applied statistics, and neural network research communities.

This monograph presents an exposition of Markov random fields (MRF's) that is likely to be extensively used by researchers in many scientific disciplines. In particular, those investigating the applicability of MRF's to process their data or images are bound to find its contents very useful. The main focus of the monograph, however, is on the application of Markov random fields to computer vision problems such as image restoration and edge detection in the low-level domain, and object matching and recognition in the high-level domain. Using a variety of examples, the author illustrates how to convert a specific vision problem involving uncertainties and constraints into essentially an optimization problem under the MRF setting. In doing so, the author introduces the reader to the various special classes of MRF's, including MRF's on the regular lattice (e.g., auto-models and multilevel logistic models) that are used for low-level modeling and MRF's on relational graphs that are used for high-level modeling.

The author devotes considerable attention to the problems of parameter estimation and function optimization, both of which are crucial in the MRF paradigm. Specific attention is given to the estimation of MRF parameters in the context of object recognition, and to the issue of algorithm selection for MRF-based function optimization. Another contribution of the book is a study on discontinuities, an important issue in the application of MRF's to image analysis. The extensive list of references, high-level descriptions of algorithms, and computational issues associated with various optimization algorithms are some of the attractive features of this book.

On the whole, the contents of this monograph nicely complement the material in Kindermann and Snell's book *Markov Random Fields and Their Applications* and Chellappa and Jain's edited volume entitled *Markov Random Fields: Theory and Applications*. In my opinion, the main contribution of this book is the manner in which significant MRF-related concepts are lucidly illustrated via examples from computer vision.

Anil K. Jain

East Lansing, Michigan

June 8, 1995

Foreword by Rama Chellappa

Noncausal spatial interaction models were first introduced by Peter Whittle in his classic 1954 paper. Whittle applied 2D noncausal autoregressive models for representing spatial data and pointed out the inconsistency of ordinary least-squares methods for parameter estimation. He then proceeded to derive maximum likelihood estimation techniques and hypothesis testing procedures. In a lesser known paper, Rosanov in 1967, discussed the representation of 2D discrete Gaussian Markov random fields (MRF's). The 2D discrete Gaussian MRF models were introduced to the engineering literature in 1972 by John Woods, who used them for 2D spectral analysis. A big impetus to theoretical and practical considerations of 2D spatial interaction models, of which MRF's form a subclass, was given by the seminal works of Julian Besag. Since the early 1980s, MRF's have dominated the fields of image processing, image analysis and computer vision.

Before the first edition of this book appeared, there were only two books on the topic, one by Kindermann and Snell and the other an edited volume by Chellappa and Jain. The former focused on the mathematical representation and the latter was mostly tailored to the needs of advanced researchers. The first edition of this book filled a great void, in that one could readily use it in a classroom. I have done so for more than a decade while teaching image processing and computer vision courses for graduate students. The author greatly succeeded in finding the delicate balance between theory and applications of MRF's and presented all the major topics in a reader-friendly manner.

Due to the overwhelmingly positive response to the first edition the second edition of the book appeared in due course with several additions. The new topics covered include a formal approach to textures analysis using MRF's, Monte Carlo Markov Chain (MCMC)-based algorithms and their variations.

In this third edition, the author has included detailed discussions on graphical models and associated inference techniques and pointed out their relationships to MRF-based approaches. A second example of such an outreach effort is the inclusion of discussions on conditional random field models, which are increasingly being used in the graphics and vision community. Another illustrative example is the discussion related to graph flow algorithms, which have become very popular in many image analysis and computer vision problems. When links to such diverse applications are made, one often loses the focus; the author should be congratulated for maintaining a cohesive picture of all the topics related to MRF's, new and old, and their inter-relationships. Although one cannot be sure if another enlightened author such as Stan Li may not come along and best this book, I am confident that for many years to come this book will be the one that will be read and reread for anything and everything on MRF's. The author has done a tremendous service to the students and researchers who are interested in learning about MRF's.

Rama Chellappa

College Park, Maryland

June 2, 2008

Preface to the Third Edition

Most important advances in MRF modeling made in the past decade or so are included in this edition. Mathematical MRF models are presented in a newly added chapter. The following are the added contents.

- Mathematical MRF Models:
  conditional random field, discriminative random fields,
  strong MRF, $\mathcal{K}$-MRF and Nakagami-MRF,
  MRF's and Bayesian networks (graphical models)
- Low-Level Models:
  stereo vision, spatio-temporal models
- High-Level Models:
  face detection and recognition
- Discontinuities in MRF's:
  total variation (TV) models
- MRF Model with Robust Statistics:
  half-quadratic minimization
- Minimization – Local Methods:
  belief propagation, convex relaxation
- Minimization – Global Methods:
  graph cuts

I would like to thank the following colleagues for assisting me in writing this edition. Linjing Li made a great effort in putting materials together. It would have been very difficult to deliver the manuscript to the publisher without a long delay without Linjing's tremendous help. Xiaotong Yuan and Junyan Wang provided their comments on the additions.

Finally, I would like to thank Rama Chellappa for giving his positive and encouraging feedback on his use of the book in teaching image processing and computer vision courses and for his kindness in writing the foreword to the third edition.

Preface to the Second Edition

Progress has been made since the first edition of this book was published five years ago. The second edition has included the most important progress in MRF modeling in image analysis in recent years, such as Markov modeling of images with "macro" patterns (e.g., the FRAME model), Markov chain Monte Carlo (MCMC) methods, and reversible jump MCMC. Work done by the author in this area after publication of the first edition is also included. The author would like to thank Song Chun Zhu for valuable discussions and suggestions.

Preface to the First Edition

Since its beginning, image analysis research has been evolving from heuristic design of algorithms to systematic investigation of approaches. Researchers have realized: (1) The solution to a vision problem should be sought based on *optimization* principles, either explicitly or implicitly, and (2) *contextual constraints* are ultimately necessary for the understanding of visual information in images. Two questions follow: how to define an optimality criterion under contextual constraints and how to find its optimal solution.

Markov random field (MRF), a branch of probability theory, provides a foundation for the characterization of contextual constraints and the derivation of the probability distribution of interacting features. In conjunction with methods from decision and estimation theory, MRF theory provides a systematic approach for deriving optimality criteria such as those based on the *maximum a posteriori* (MAP) concept. This MAP-MRF framework enables us to systematically develop algorithms for a variety of vision problems using rational principles rather than ad hoc heuristics. For these reasons, there has been increasing interest in modeling computer vision problems using MRF's in recent years.

This book provides a coherent reference to theories, methodologies, and recent developments in solving computer vision problems based on MRF's, statistics, and optimization. It treats various problems in low- and high-level computational vision in a systematic and unified way within the MAP-MRF framework. The main issues of concern are how to use MRF's to encode contextual constraints that are indispensable to image understanding; how to derive the objective function, typically the posterior distribution, for the optimal solution to a problem; and how to design computational algorithms for finding the optimal solution.

As the first thorough reference on the subject, the book has four essential parts for solving image and vision analysis problems using MRF's: (1) introduction to fundamental theories, (2) formulations of various image models in the MAP-MRF framework, (3) parameter estimation, and (4) optimization methods.

Chapter 1 introduces the notion of visual labeling and describes important results in MRF theory for image modeling. A problem is formulated in terms of Bayes labeling of an MRF. Its optimal solution is then defined as the MAP configuration of the MRF. The role of optimization is discussed. These form the basis on which MAP-MRF models are formulated.

Chapter 2 formulates MRF models for low-level vision problems, such as image restoration, reconstruction, edge detection, texture, and optical flow. The systematic MAP-MRF approach for deriving the posterior distribution is illustrated step by step.

Chapter 3 addresses the issue of discontinuities in low-level vision. An important necessary condition is derived for any MRF prior potential function to be adaptive to discontinuities to avoid oversmoothing. This gives rise to the definition of a class of *adaptive interaction functions* and thereby a class of MRF models capable of dealing with discontinuities.

Chapter 4 provides a comparative study on discontinuity adaptive MRF priors and robust M-estimators based on the results obtained in Chapter 3. To tackle the problems associated with M-estimators, a method is presented to stabilize M-estimators w.r.t. the initialization and convergence.

Chapter 5 presents high-level MRF models for object recognition and pose determination. Relational measurements are incorporated into the energy function as high-level constraints. The concept of line process is extended for the separation of overlapping objects and the elimination of outlier features.

Chapter 6 describes various methods for both supervised and unsupervised parameter estimation, including the coding method, pseudo-likelihood, least squares method, and expectation maximization. A simultaneous image labeling and parameter estimation paradigm is also presented that enhances the low-level models in Chapter 2.

Chapter 7 presents a theory of parameter estimation for optimization-based object recognition. Two levels of criteria are proposed for the estimation: correctness and optimality. Optimal parameters are learned from examples using supervised learning methods. The theory is applied to parameter learning for the MRF recognition.

Chapters 8 and 9 present local and global methods, respectively, for energy optimization in finding MAP-MRF solutions. These include various algorithms for continuous, discrete, unconstrained, and constrained minimization as well as strategies for approximating global solutions.

The final version of this manuscript benefited from comments on earlier versions by a number of people. I am very grateful to Anil K. Jain and Kanti V. Mardia for their valuable suggestions. I would like to thank Kap Luk Chan, Lihui Chen, Yi-Ping Hung, Eric Sung, Han Wang, Ming Xie, and Dekun Yang. Their corrections have had a very positive effect on the book. I am particularly indebted to Yunjun Zhang, Weiyun Yau, and Yihong Huang for their proofreading of the whole manuscript. Finally, I owe a deep debt of gratitude to my wife for her understanding, patience, and support.

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