

Game of Life Cellular Automata

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Editor

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Preface

In 1970 Martin Gardner spelled out the rules of a new solitaire game forged by John Horton Conway.¹ An unparalleled combination of functional simplicity with behavioural complexity made Conway's Game of Life the most popular cellular automaton of all time. We commemorate the Game of Life's 40th birthday with a unique collection of works authored by renowned mathematicians, computer scientists, physicists and engineers. The superstars of science, academy and industry present their visions of the Game of Life cellular automaton, its extensions and modifications, and spatially-extended systems inspired by the Game.

The book covers hot topics in theory of computation, pattern formation, optimization, evolution, non-linear sciences and mathematics. Academics, researchers, hobbyists and students interested in the Game of Life theory and applications will find this monograph a valuable guide to the field of cellular automata and excellent supplementary reading.

Bristol, UK

Andrew Adamatzky

¹"...each cell of the checkerboard (assumed to be an infinite plane) has eight neighboring cells, four adjacent orthogonally, four adjacent diagonally. The rules are:

1. Survivals. Every counter with two or three neighboring counters survives for the next generation.
2. Deaths. Each counter with four or more neighbors dies (is removed) from overpopulation. Every counter with one neighbor or none dies from isolation.
3. Births. Each empty cell adjacent to exactly three neighbors — no more, no fewer — is a birth cell. A counter is placed on it at the next move.

... all births and deaths occur simultaneously."

Martin Gardner, The fantastic combinations of John Conway's new solitaire game "life". Scientific American 223 (October 1970): 120–123.

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