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Stable Mutations for Evolutionary Algorithms

Springer

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To my wife
Beata
and our children
Maria, Adam, and Tomasz

Contents

1 Introduction 1
References 7

2 Foundations of Evolutionary Algorithms 9
2.1 Basic Concepts 10
2.2 Standard Evolutionary Algorithms 12
2.2.1 Genetic Algorithms 13
2.2.2 Genetic Programming 14
2.2.3 Evolutionary Programming 16
2.2.4 Evolutionary Strategies 16
2.2.5 Evolutionary Search with Soft Selection 19
2.3 Summary 21
References 21

3 Stable Distributions 23
3.1 Stable Random Value 23
3.1.1 Definition of Stable Distributions 23
3.1.2 Chosen Properties of Stable Distributions 26
3.1.3 Simulation of α-Stable Random Variables 28
3.1.4 Stable Random Variable and the Evolutionary Algorithm 29
3.2 Stable Random Vector 33
3.2.1 Definition of the Stable Random Vector 33
3.2.2 Selected Properties of the Stable Random Vector 34
3.2.3 Non-isotropic Stable Random Vector 36
3.2.4 Isotropic Distributions 40
3.2.5 Isotropic Distribution Based on an α-Stable Generator 41
3.2.6 Isotropic Stable Random Vector 43
3.3 Summary 47
References 48

4 Non-isotropic Stable Mutation 49
4.1 Dead Surrounding Effect 50
4.1.1 Dead Surrounding Versus Extremum Localization 51
4.1.2 Saddle Crossing 55
4.1.3 Optimization of Multimodal Functions 60
4.2 Symmetry Effect 66
4.2.1 Local Convergence of $(1+1)ES_{N;\alpha}$ 66
4.2.2 Saddle Crossing 69
4.2.3 Symmetry Effect Versus Global Extremum Searching 71
4.3 Summary 75
References 75

5 Isotropic Stable Mutation 77
5.1 Dead Surrounding Effect 77
5.1.1 Probability Density Function of the Norm of the Isotropic Stable Vector 77
5.1.2 Soft Selection Versus the Dead Surrounding 80
5.2 Local Convergence Analysis 81
5.2.1 Progress Rate 81
5.2.2 Convergence Analysis for $n = 3$ 83
5.3 Exploration Versus Exploitation 86
5.3.1 Local Convergence of the $(1+1)\mathrm{ES}_{I;\alpha}$ Strategy 86
5.3.2 Saddle Crossing: The $(1,2)\mathrm{ES}_{I;\alpha}$ Strategy 89
5.3.3 Conclusions 91
5.4 Robustness Analysis of Stable Mutation 91
5.4.1 General Searching Space 92
5.4.2 Isotropic Stable Mutation Effectiveness in $\mathscr{F}$ 93
5.4.3 Simulation Experiments 95
5.5 Adaptation of Stable Mutation Parameters 97
5.5.1 Adaptation Strategy for $(1+1)\mathrm{ES}_{I;\alpha}$ 97
5.6 Summary 105
References 105

6 Stable Mutation with the Discrete Spectral Measure 107
6.1 Stable Distributions with the DSM 107
6.2 Selection of the Optimal Stable Distribution 110
6.3 DSM Adaptation 112
6.3.1 DSM Estimation 115
6.3.2 Simulation Example 116
6.4 Summary 119
References 120

7 Isotropic Mutation Based on an α-Stable Generator 123
7.1 Dead Surrounding Effect 123
7.2 Local Convergence Analysis 125
7.2.1 Local Convergence of the $(1+1)\mathrm{ES}_{S,\alpha}$ Strategies 125
7.2.2 Local Convergence of the $(1+\lambda)\mathrm{ES}_{S,\alpha}$ Strategies 126
7.3 Exploitation and Exploration: $\mathrm{ESTS}_{S,\alpha}$ 128
7.3.1 Precision of Local Extremum Localization 128
7.3.2 Saddle Crossing 130
7.3.3 Exploitation Contra Exploration: The Pareto Front 131
7.4 Global Optimization 131
7.5 Summary 134
References 134

8 Mutation Based on Directional Distributions 135
8.1 Motivation 135
8.2 Directional Distributions 139
8.2.1 Von Mises–Fisher Distributions 139
8.3 Simulation Experiments 143
8.3.1 Conducting the Experiment 146
8.3.2 Results 147
8.4 Summary 153
References 153

9 Conclusions 155
References 158

Appendix A: Saddle Crossing Problem 159

Appendix B: Benchmark Functions 161

Abbreviations

DSM	Discrete spectral measure
EP	Evolutionary programming
$\mathrm{EP}_{N;\alpha}$	EP with mutation based on $NS\alpha S(\sigma)$ distribution
$\mathrm{EP}_{I;\alpha}$	EP with mutation based on $IS\alpha S(\sigma)$ distribution
$\mathrm{EP}_{S;\alpha}$	EP with mutation based on $S\alpha SU(\sigma)$ distribution
ES	Evolutionary strategy
ESSS	Evolutionary search with soft selection
$\mathrm{ESSS}_{N;\alpha}$	ESSS with mutation based on $NS\alpha S(\sigma)$
$\mathrm{ESSS}_{I;\alpha}$	ESSS with mutation based on $IS\alpha S(\sigma)$
$\mathrm{ESSS}_{S;\alpha}$	ESSS with mutation based on $S\alpha SU(\sigma)$
ESSS_{α}-DM	ESSS with directional distribution $M(\mu, \kappa)$
ESTS	Evolutionary search with tournament selection
$\mathrm{ESTS}_{N;\alpha}$	ESTS with mutation based on $NS\alpha S(\sigma)$
$\mathrm{ESTS}_{I;\alpha}$	ESTS with mutation based on $IS\alpha S(\sigma)$
$\mathrm{ESTS}_{S;\alpha}$	ESTS with mutation based on $S\alpha SU(\sigma)$
ESTS_{α}-DM	ESTS with directional distribution $M(\mu, \kappa)$
GA	Genetic algorithm
GP	Genetic programming
i.i.d.	Independent and identical distribution
$(\eta + \lambda)$ ES	ES with a succession of the best η individuals from joined base and offspring populations
$(\eta + \lambda)\,\mathrm{ES}_{N;\alpha}$	$(\eta + \lambda)$ ES with mutation based on $NS\alpha S(\sigma)$
$(\eta + \lambda)\,\mathrm{ES}_{I;\alpha}$	$(\eta + \lambda)$ ES with mutation based on $IS\alpha S(\sigma)$
$(\eta + \lambda)\,\mathrm{ES}_{S;\alpha}$	$(\eta + \lambda)$ ES with mutation based on $S\alpha SU(\sigma)$
(η, λ) ES	ES with a succession of the best η individuals from λ offspring
$(\eta, \lambda)\,\mathrm{ES}_{N;\alpha}$	(η, λ) ES with mutation based on $NS\alpha S(\sigma)$
$(\eta, \lambda)\,\mathrm{ES}_{I;\alpha}$	(η, λ) ES with mutation based on $IS\alpha S(\sigma)$
$(\eta, \lambda)\,\mathrm{ES}_{S;\alpha}$	(η, λ) ES with mutation based on $S\alpha SU(\sigma)$
SGA	Simple GA

Symbols

$\mathbb{R}$	Set of real numbers
$\mathscr{G}$	Genetic universe
$\mathscr{D}$	Phenotype universe
η	Number of individuals in a population
$P(t)$	Population in the t-th iteration
$\mathscr{P}_\eta(\mathscr{G})$	Set of all possible populations, i.e., multisets of η elements from $\mathscr{G}$
a_k^t	k-th individual from the population $P(t)$
$\Phi(\cdot)$	Fitness function
$A, B, \ldots, X, Y, Z$	Random variables
$A \stackrel{d}{=} B$	Random variables A and B are of the same distribution
$X_{i:\lambda}$	i-th random variable of the ordered statistics of λ variables
α	Stability index (tail index)
β	Skewness parameter
σ	Scale parameter
μ	Localization parameter
$\varphi(\cdot)$	Characteristic function
$p(\cdot)$	Probability density function
$E(\cdot)$	Expectation value
$Var(\cdot)$	Variance
$U(a, b)$	Continuous uniform distribution on (a, b)
$N(\mu, \sigma^2)$	Normal (Gaussian) distribution
$C(\mu, \sigma)$	Cauchy distribution
$Levy(\mu, \sigma)$	Lévy distribution
$S_\alpha(\sigma, \beta, \mu)$	α-stable distribution
$S\alpha S(\sigma)$	Symmetric α-stable distribution
Γ_s	Spectral measure
$\mathbf{N}(\mu, \mathbb{C})$	Multidimensional elliptic Gaussian distribution
$NS\alpha S(\sigma)$	Multidimensional non-isotropic symmetric α-stable distribution
$IS\alpha S(\sigma)$	Multidimensional isotropic α-stable distribution

$S\alpha SU(\sigma)$	Multidimensional isotropic distribution with an α-stable generator
$\mathscr{F}$	General search space
$MF(\mu, \kappa)$	Von Mises–Fisher distribution
$M(\mu, \kappa)$	Directional distribution