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# Rough Set–Based Classification Systems

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*To my wife, Agnieszka.*

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# Symbols

List of the most important symbols used in the book:

$U, U_i$	The universe of discourse, which is a set of objects or states under classification
$V, V_i$	The universe of discourse, which is a set of features values of objects under classification
$Y, Y_i$	Space of a direct output of a classifier
$Z, Z_i = [0, 1]$	Space of object memberships to class level (degree)
$Q$	Set of features of objects under classification
$C \subseteq Q$	Set of condition features
$D \subseteq Q$	Set of decision features
$P \subseteq C$	Set of condition features used in a classification process, with known values
$G \subseteq C$	Set of condition features not used in a classification process, with unknown values
$\mathbb{R}$	Set of real numbers
$\mathbb{R}^n$	$n$ -dimensional space of vectors of real numbers
$X \subseteq U$	Set of objects or states
$\omega_j \subseteq U$	Class of objects
$\Omega$	Set of classes under consideration
$R$	Relation, fuzzy relation
$[x]_R$	Equivalence class (atom) determined by object $x$ and relation $R$
$\tilde{P}$	Indiscernibility relation determined by set $P$
$\overline{RX}$	$R$ -upper approximation of set $X$
$\underline{RX}$	$R$ -lower approximation of set $X$
$\{\underline{RX}, \overline{RX}\}$	Rough set
$A \subseteq U, A \subseteq V, B \subseteq Z$	Fuzzy sets

$A'_i \subseteq V_i$	Fuzzy set obtained in fuzzification of real value $v_i$
$A' = A'_1 \times A'_2 \times \dots \times A'_n \subseteq V$	Fuzzy set obtained in fuzzification of vector $\mathbf{v}$
$A_i^r \subseteq V_i$	Fuzzy set occurring in an antecedent of rule $R^r$ concerning feature $q_i$
$A^r = A_1^r \times A_2^r \times \dots \times A_n^r \subseteq V$	Fuzzy set occurring in consequence of rule $R^r$
$B_j^r \subseteq Z_j$	Fuzzy set occurring in consequence of rule $R^r$ concerning the membership of an object under classification to class $\omega_j$
$B_j^{r'} \subseteq Z_j$	Fuzzy set which is the result of inference based on rule $R^r$ , concerning the membership of an object under classification into class $\omega_j$
$B_j' \subseteq Z_j$	Fuzzy set which is the result of inference, concerning the membership of an object under classification into class $\omega_j$
$x \in U, x_i \in U_i$	Object or state under classification
$q_i \in Q$	Feature of an object or state under classification
$\mathbf{q} = [q_1, q_2, \dots, q_n]$	Vector of features of an object under classification
$c_i \in C$	Condition feature of an object under classification
$\mathbf{c} = [c_1, c_2, \dots, c_n]$	Vector of condition features of an object under classification
$\mathbf{c}_P$	Vector of condition features used in a classification process, with known values
$\mathbf{c}_G$	Vector of condition features not used in a classification process, with unknown values
$d, d_j \in D$	Decision feature
$d, d_j \in Y$	Reference (desired) output value
$v_i \in V_i$	Value of feature $q_i$
$\mathbf{v} = [v_1, v_2, \dots, v_n] \in V$	Vector of feature values of an object under classification
$\mathbf{v}_P \in V_P$	Vector of feature values of an object under classification, used in classification process, with known values
$\mathbf{v}_G \in V_G$	Vector of feature values of an object under classification, not used in a classification process, with unknown values
$y$	Output variable of a fuzzy inference system
$\bar{y}$	Real value which is a representative of fuzzy set $B'$ , the result of defuzzification
$\bar{y}^r$	Real value which is a representative of fuzzy set $B^r$ , the result of defuzzification or the only element of the core of set $B^r$
$\bar{z}_j$	Representative of set $B_j$ in classifiers

$n$	Number of considered condition features of an object under classification, cardinality of set $C$
$i = 1, 2, \dots, n$	Index of single input of a fuzzy system, index of a subsequent condition feature
$n_P$	Number of condition features of an object under classifications used in a classification process, with known values, cardinality of set $P$
$n_G$	Number of condition features of an object under classifications not used in a classification process, with unknown values, cardinality of set $G$
$m$	Number of considered classes, the number of decision features
$\omega_j$	Single class with index $j$
$\tau$	Index of a simple sample in a training or testing sequence
$\tau_{\max}$	Number of samples in a training or testing sequence
$v$	Index of a single group of samples in a training or testing sequence
$\Upsilon$	Number of groups in a training or testing sequence
$t$	Index of a subsystem in an ensemble
$T$	Number of the subsystems in an ensemble
$h_t$	Hypothesis inferred from the subsystem with index $t$
$N$	Number of rules in a decision system
$r = 1, 2, \dots, N$	Index of a single rule
$\mu_A(x)$	Membership function of fuzzy set $A$
$\text{sgn}(a)$	Sign of value $a$
$A \cup B$	Sum of (fuzzy) sets $A$ and $B$
$\bigcup_{i=1}^n A_i$	Sum of $n$ (fuzzy) sets $A_i$
$A \cap B$	Intersection of (fuzzy) sets $A$ and $B$
$\bigcap_{i=1}^n A_i$	Intersection of $n$ (fuzzy) sets $A_i$
$\neg A$	Complement of set $A$
$A \times B$	Cartesian product of (fuzzy) sets $A$ and $B$
$R \circ S$	Composition of fuzzy sets (fuzzy relation) $R$ and $S$
$T(a, b)$	t-norm of $a$ and $b$
$T(a_1, a_2, \dots, a_n) = \bigwedge_{i=1}^n a_i$	t-norm of $n$ values (variables)
$S(a, b)$	t-conorm of $a$ and $b$
$S(a_1, a_2, \dots, a_n) = \bigvee_{i=1}^n a_i$	t-norm of $n$ values (variables)
$N(a)$	Fuzzy negation