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Advanced Topics in Fuzzy Graph Theory

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John N. Mordeson would like to dedicate the book to the volunteers of St. James Parish and St. Vincent de Paul Society.

Sunil Mathew would like to dedicate the book to his mother Mary, wife Sonia, and children Ragam and Rahul.

Foreword

At the end of the nineteenth century, G. Cantor introduced his famous set theory that has been widely accepted as the language of science in general and of mathematics in particular and that was based on Aristotle's binary logic with two truth values (true and false). For a given universe of discourse X (e.g., students in the first bachelor mathematics), each crisp property (e.g., male students, students taller than 1m82, students with more than 80% in the final year of secondary school) leads to a crisp partition of the universe into the (sub)set A of elements of X that satisfy this property and the remaining (sub)set complement A of elements of X that do not satisfy this property. Using the logical operations of negation, disjunction, and conjunction, Cantor defined the set-theoretical operations of complementation, union, and intersection, leading to new sets (e.g., the set of male students that are taller than 1m82). However, it is not possible to model imprecise, vague properties such as tall, clever, slim built, and excellent result on the calculus course in the first semester. In order to represent such qualitative, imprecise predicates, one has to specify boundaries. For example, a student is considered tall as soon as his or her length exceeds 1m85. As a consequence of this artificial boundary, the student that measures 1m84 will be classified as not tall while practically there is almost no difference between 1m84 and 1m85. In the same way, a guy of 2m15 will also belong to the class of tall people! In this respect, we have to cite the so-called Poincare paradox: Poincare observed that a human cannot determine by hand a difference between a package of 100gr and one of 101gr, so for a human observer both packages have the same weight, i.e., they are equal with respect to weight. Similarly, there is no difference between 101gr and 102gr, and hence, applying the transitivity property of the equality relation leads to the conclusion that there is no difference between 100gr and 102gr. It is easy to see that continuing this reasoning from classical logic induces the silly conclusion that all packages have the same weight. Similar paradoxes were already known by the old Greeks during the fifth century B.C., namely the Sorites and Falakros paradox.

I would like to stress the inevitable consequences of binary logic and classical set theory on mathematics. As soon as the classifier $\{/\}$ has been introduced—intuitively or axiomatically—the basic concept of a relation between two sets can be defined. Then—too soon to my opinion—the concept of a relation has been narrowed into a functional one of shortly a function. In particular, a function with the natural numbers as domain is known as a sequence. The class of sequences is again divided into two disjoint subsets—the convergent and the non-convergent sequences. This ongoing bivalent partitioning is very rough as illustrated by the following example. Consider the sequence (1, −2, 3, −4, 5, …); it is clearly a non-convergent one. Similarly, the sequence (0.01, −0.01, 0.01, −0.01, 0.01, …) is also not convergent. So both sequences belong to the same class of non-convergent real sequences while there is a huge difference between them: The first one is totally hopeless with respect to being convergent, while the second one is almost convergent to zero. So if convergence could be introduced as a gradual notion (with more than two degrees), then the second sequence certainly would get a high degree.

As illustrated so far, there was a big need for mathematical models to represent and process imprecise and uncertain information. Till 1965, only probability theory and error calculus were partly able to satisfy the need to handle a special kind of uncertainty, namely randomness. As stated by Zadeh: Probability theory is insufficiently expressive to serve as the language of uncertainty. It has no facilities to describe fuzzy predicates such as small, young, much larger than nor fuzzy quantifiers such as most, many, a few… nor fuzzy probabilities such as likely, not very likely… nor linguistic modifiers such as very and more-or-less. We had to wait till 1965 when L. Zadeh launched his seminal paper "Fuzzy sets." The concept of a fuzzy set allows to have besides membership and non-membership, intermediate, or partial degrees of membership. So instead of black-or-white decisions, a gradual transition from membership to non-membership has been introduced. I want to say that I prefer the term "mathematics of fuzziness" instead of "fuzzy mathematics" because there is nothing fuzzy or blurry in this kind of mathematics!

When I started my research in fuzzy set theory in 1976, only a few hundreds of papers were published on this theory and hence a very manageable number to start research in this promising domain. Nowadays, there are more than 115,000 papers in the INSPEC database with fuzzy in the title and around 30,000 papers in the MATH.SCI.NET database! After a short dissemination period for Zadeh's brilliant concept of fuzzy sets, mathematicians became aware of the enormous possibilities of this theory for extending the existing mathematical apparatus, especially with regard to the applications, since this concept embraces the elasticity of natural language and human's qualitative summarization capabilities. Probably the first domain of mathematics that underwent the coloring process was general topology. Indeed already in 1969, C. L. Chang published his seminal paper "Fuzzy topological spaces." Very soon after, that straightforward fuzzyfications of the classical mathematical concepts based upon union and intersection were given birth: fuzzy groups, fuzzy vector spaces, fuzzy metric spaces, fuzzy geometries, fuzzy relational calculus, fuzzy graphs… Most of those papers written in the seventies appeared in the *Journal of Mathematical Analysis and Its Applications*. Nowadays, there are more than 30 international journals with fuzzy in the title! Starting from the eighties, a new period started in the development of the mathematics of fuzziness:

Due to the introduction of the notions of triangular norm and co-norm by Schweizer and Sklar in the framework of probabilistic metric spaces, an explosion of the possible generalizations of the binary mathematical structures took place, leading to the introduction of the T–S fuzzy concepts with T a triangular norm and S a triangular co-norm. Because of the overwhelming literature on fuzzy sets and related so-called soft computing models such as L-fuzzy sets, rough sets, flou sets, intuitionistic fuzzy sets, type2 fuzzy sets, interval-valued fuzzy sets, probabilistic sets, twofold fuzzy sets, gray sets, fuzzy rough sets, soft sets, toll sets, and vague sets and the increasing number of researchers, there is a big need for good textbooks and monographs on the basic issues as well as on state-of-the-art volumes.

Already in the first textbook on fuzzy set theory, A. Kaufmann has launched in 1973 the notion of a fuzzy graph, later on fine-tuned by A. Rosenfeld, R. T. Yeh, and S. Bang such that the level sets of a fuzzy graph are crisp graphs, a frequently desired property of a fuzzyfied concept or structure. The authors of this book have substantially contributed to the development and the flourishing of the theory of fuzzy graphs as can be checked from the many co-authored papers in the references. The present volume *Advanced Topics in Fuzzy Graph Theory* written by two famous experts in fuzzy mathematics and fuzzy graphs in particular concentrates on a few advanced research items on fuzzy graphs: connectivity and its relation to Wiener indices and distances, t-norm fuzzy graphs and their operations, and finally fuzzy graphs based on dialectic synthesis. For newcomers in the field, the authors provided a short introduction on fuzzy sets. I am very happy to see that the authors besides strong theoretical contributions also added several interesting applications on current hot problems such as human trafficking and immigration flows in Europe and the USA. I want to congratulate the authors Prof. John Mordeson and Prof. Sunil Mathew for their excellent work and giving us the opportunity to learn more about the amazing theory of fuzzy graphs.

Ghent, Belgium Etienne E. Kerre
June 2018

Preface

Inspired by Lotfi Zadeh's seminal work on fuzzy logic, Azriel Rosenfeld developed cornerstone papers in fuzzy abstract algebra and fuzzy graph theory. This book is the third book of the authors on fuzzy graph theory. The book is motivated by the authors' desire to apply fuzzy mathematics to the problems of human trafficking, illegal immigration, and modern slavery.

In Chap. 1, we present basic results on fuzzy sets, relations, and graphs that are needed for the remainder of the book.

In Chap. 2, we concentrate on connectivity concepts of fuzzy graphs. Our work deals with vertex connectivity, average fuzzy connectivity, and critical blocks. In particular, we focus mainly on constructions of t-connected graphs, average fuzzy vertex connectivity, and uniformly connected fuzzy graphs. We apply our results to the study of human trafficking across the Mexican border into the USA.

In Chap. 3, we concentrate on connectivity and Wiener indices in fuzzy graphs. We study the relationship between the Weiner index and the connectivity index. We also introduce the notion of average connectivity index of a fuzzy graph. We apply our results to the problems of human trafficking, internet routing, and illegal immigration.

In Chap. 4, we focus on distance and connectivity in fuzzy graphs. We consider geodesic blocks and monophonic blocks in the first part. Three different distances, namely geodetic distance, sum distance, and strong sum distance, are introduced, and several properties are analyzed.

In Chap. 5, we generalize the definition of a fuzzy graph by using an arbitrary t-norm in place of the t-norm minimum. We do this because a t-norm is sometimes better than minimum in real world situations. We illustrate this with real-world applications to human trafficking and modern slavery. We also develop the notion of a generalized fuzzy relation using a t-norm other than minimum. We consider operations on fuzzy graphs involving t-norms. We define the composition of fuzzy relations using t-norms, t-co-norms, and aggregation operators rather than minimum and maximum. We show when results using minimum carryover when using an arbitrary t-norm and provide examples when it does not. Our data involving trafficking is taken from the Global Slavery Index and the Walk Free Foundation.

We develop a measure on the susceptibility of trafficking in persons for networks by using t-norms other than minimum. We also develop a connectivity index for a fuzzy network. In one application, a high connectivity index means a high susceptibility to trafficking. In addition, we use norm functions and median functions to model study situations involving modern slavery.

In Chap. 6, we are interested in developing a new type of fuzzy graph. This graph is based on the groundbreaking work of Trillas and Garcia Honrado on dialectic synthesis. Dialectic synthesis is concerned with a method of reasoning by means of the triplet Thesis–Antithesis–Synthesis triad. We show this method can be used in fuzzy graph theory and applied to the problems of human trafficking, modern slavery, and illegal immigration.

The book is dependent on the journals New Mathematics and Natural Computation, Information Sciences, and Fuzzy Sets and Systems for their support of our work involving applications to human trafficking, illegal immigration, and modern slavery.

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Dr. John N. Mordeson is Professor Emeritus of Mathematics at Creighton University. He received his B.S., M.S., and Ph.D. from Iowa State University. He is a member of Phi Kappa Phi. He is the president of the Society for Mathematics of Uncertainty. He has published 17 books and 200 journal articles. He is on the editorial board of numerous journals. He has served as an external examiner of Ph.D. candidates from India, South Africa, Bulgaria, and Pakistan. He has refereed for numerous journals and granting agencies. He is particularly interested in applying mathematics of uncertainty to combat the problem of human trafficking.

Dr. Sunil Mathew is a Faculty Member in the Department of Mathematics, NIT Calicut, India. He has acquired his masters from St. Joseph's College Devagiri, Calicut, and Ph.D. from NIT Calicut, India, in the area of fuzzy graph theory. He has published more than 100 research papers and written four books. He is a member of several academic bodies and associations. He is editor and reviewer of several international journals. He has an experience of 20 years in teaching and research. His current research topics include fuzzy graph theory, bio-computational modeling, graph theory, fractal geometry, and chaos.