Mining Periodic Patterns with a MDL Criterion

Esther Galbrun¹, Peggy Cellier², Nikolaj Tatti^{1,4}, Alexandre Termier², and Bruno Crémilleux³

 ¹ Department of Computer Science, Aalto University, Finland {esther.galbrun,nikolaj.tatti}@aalto.fi
 ² Univ. Rennes, {INSA, Inria}, CNRS, IRISA, France {peggy.cellier,alexandre.termier}@irisa.fr
 ³ Normandie Univ., UNICAEN, ENSICAEN, CNRS - UMR GREYC, France bruno.cremilleux@unicaen.fr
 ⁴ F-Secure, Finland

Abstract. The quantity of event logs available is increasing rapidly, be they produced by industrial processes, computing systems, or life tracking, for instance. It is thus important to design effective ways to uncover the information they contain. Because event logs often record repetitive phenomena, mining periodic patterns is especially relevant when considering such data. Indeed, capturing such regularities is instrumental in providing condensed representations of the event sequences.

We present an approach for mining periodic patterns from event logs while relying on a Minimum Description Length (MDL) criterion to evaluate candidate patterns. Our goal is to extract a set of patterns that suitably characterises the periodic structure present in the data. We evaluate the interest of our approach on several real-world event log datasets.

Keywords: Periodic patterns · MDL · Sequence mining.

1 Introduction

Event logs are among the most ubiquitous types of data nowadays. They can be machine generated (server logs, database transactions, sensor data) or human generated (ranging from hospital records to life tracking, a.k.a. quantified self), and are bound to become ever more voluminous and diverse with the increasing digitisation of our lives and the advent of the Internet of Things (IoT). Such logs are often the most readily available sources of information on a system or process of interest. It is thus critical to have effective and efficient means to analyse them and extract the information they contain.

Many such logs monitor repetitive processes, and some of this repetitiveness is recorded in the logs. A careful analysis of the logs can thus help understand the characteristics of the underlying recurrent phenomena. However, this is not an easy task: a log usually captures many different types of events. Events related to occurrences of different repetitive phenomena are often mixed together as well as with noise, and the different signals need to be disentangled to allow analysis. This can be done by a human expert having a good understanding of the domain and of the logging system, but is tedious and time consuming.

Periodic pattern mining algorithms [17] have been proposed to tackle this problem. These algorithms can discover periodic repetitions of sets or sequences of events amidst unrelated events. They exhibit some resistance to noise, when it takes the form of slight variations in the inter-occurrence delay [2] or of the recurrence being limited to only a portion of the data [16]. However, such algorithms suffer from the traditional plague of pattern mining algorithms: they output too many patterns (up to several millions), even when relying on condensed representations [15].

Recent approaches have therefore focused on optimising the quality of the extracted *pattern set* as a whole [5], rather than finding individual high-quality patterns. In this context, the adaptation of the Minimal Description Length (MDL) principle [18,8] to pattern set mining has given rise to a fruitful line of work [21,4,20,3]. The MDL principle is a concept from information theory based on the insight that any structure in the data can be exploited to compress the data, and aiming to strike a balance between the complexity of the model and its ability to describe the data.

The most important structure of the data on which we focus here, i.e. of event logs, is the periodic recurrence of some events. For a given event sequence, we therefore want to identify a set of patterns that captures the periodic structure present in the data, and we devise a MDL criterion to evaluate candidate pattern sets for this purpose. First, we consider a simple type of model, representing event sequences with cycles over single events. Then, we extend this model so that cycles over distinct events can be combined together. By simply letting our patterns combine not only events but also patterns recursively, we obtain an expressive language of periodic patterns. For instance, it allows us to express the following daily routine:

Starting Monday at 7:30 AM, wake up, then, 10 minutes later, prepare coffee, repeat every 24 hours for 5 days, repeat this every 7 days for 3 months

as a pattern consisting of two nested cycles, respectively with 24 hours and 7 days periods, over the events "waking up" and "preparing coffee".

In short, we propose a novel approach for mining periodic patterns using a MDL criterion. The main component of this approach—and our main contribution—is the definition of an expressive pattern language and the associated encoding scheme which allows to compute a MDL-based score for a given pattern collection and sequence. We design an algorithm for putting this approach into practise and perform an empirical evaluation on several event log datasets. We show that we are able to extract sets of patterns that compress the input sequences and to identify meaningful patterns.

We start by reviewing the main related work, in Section 2. In Section 3, we introduce our problem setting and a simple model consisting of cycles over single events, which we extend in Section 4. In Section 5, we look at how patterns can be combined and compare costs. We present an algorithm for mining periodic patterns that compress in Section 6 and evaluate our proposed approach over several event log datasets in Section 7. We reach conclusions in Section 8.

This report extends our conference publication [6] with technical details, numerous examples, and additional experiments.

2 Related Work

The first approaches for mining periodic patterns [17,10,9] were designed to augment traditional itemset and sequence mining techniques with the capacity to identify events whose occurrences are regularly spaced in time. They used extremely constrained definitions of the periodicity. In [17], *all* occurrences must be regularly spaced; In [10,9], some missing occurrences are permitted but all occurrences must follow the same regular spacing. As a result, these approaches are extremely sensitive to even small amounts of noise in the data. Ma *et al.* [16] later proposed a more robust approach, which can extract periodic patterns in the presence of gaps of arbitrary size in the data: the recurrence can be interrupted and restarted, possibly with a different spacing. Such perturbations are frequent in real data.

The above approaches require time to be discretized as a preprocessing (time steps of hour or day length, for example), smoothing out small changes in interoccurrence delays and limiting the search for the correct period to a predetermined range. These approaches might be too coarse grained, however, and are dependent on the discretization. Several solutions have been proposed to directly discover candidate periods from raw timestamp data, using the Fast Fourier Transform [2] or statistical models [14,22].

All of the above approaches are susceptible to producing a huge number of patterns, making the exploitation of their results difficult. The use of a *condensed representation* for periodic patterns [15] allows to significantly reduce the number of patterns output, without loss of information, but falls short of satisfactorily addressing the problem.

Considering pattern mining more in general, to tackle this pervasive issue of the overwhelming number of patterns extracted, research has focused on extracting *pattern sets* [5]: finding a (small) set of patterns that together optimise some interest criterion. One such criterion is based on the Minimum Description Length (MDL) principle [7]. Simply put, it states that the best model is the one that compresses the data best. Following this principle, the KRIMP algorithm [21] was proposed, to select a subset of frequent itemsets that yields the best lossless compression of a transactional database. This algorithm was later improved [19] and the approach extended to analyse event sequences [20,13,3]. Along a somewhat different approach, Kiernan and Terzi proposed to use MDL to summarize event sequences [12].

To the best of our knowledge, the only existing method that combines periodic pattern mining and a MDL criterion was proposed by Heierman *et al.* [11]. This approach considers a single regular episode at a time and aims to select the best occurrences for this pattern, independently of other patterns. Instead, we use a MDL criterion in order to select a good collection of periodic patterns.

3 Preliminary Notation and Problem Definition

Next, we formally define the necessary concepts and formulate our problem, focusing on simple cycles. But first, let us clarify some of the notation we use throughout.

Lists are represented by enumerating their elements in order of occurrence, enclosed between \langle and \rangle , as in $\langle i_1, i_2, \ldots \rangle$ for instance, with $\langle \rangle$ denoting the empty list. We use \oplus to represent the concatenation of lists, as in

$$\langle a, b, c \rangle = \langle a \rangle \oplus \langle b, c \rangle$$
 and $\langle i_1, i_2, \dots, i_9 \rangle = \bigoplus_{k \in [1..9]} \langle i_k \rangle$.

Given a list L, L[k] returns the element at k^{th} position (indexing starts at 1).

We also use a simplified notation for lists, especially when using them as indices. Lists and single elements are then denoted respectively as upper-case and lower-case letters or numbers, and concatenation is simply represented by concatenating the corresponding letters. In this notation, we use 0 to represent the empty list. For instance, the indices in B_0 , B_X and B_{Xy} represent an empty list, a list X, and element y concatenated to the list X, respectively.

All logarithms are to base 2.

Symbols used are listed on the last page of this report.

A timestamped event sequence as input data. Our input data is a collection of timestamped occurrences of some events, which we call an *event sequence*. The events come from an alphabet Ω and will be represented with lower case letters. We assume that an event can occur only once per time step, so the data can be represented as a list of timestamp–event pairs, such as

$$S_1 = \langle (2,c), (3,c), (6,a), (7,a), (7,b), (19,a), (30,a), (31,c), (32,a), (37,b), (42,a), (48,c), (54,a) \rangle .$$

Whether timestamps represent days, hours, seconds, or something else depends on the application, the only requirement is that they be expressed as positive integers. We denote as $S^{(\alpha)}$ the event sequence S restricted to event α , that is, the subset obtained by keeping only occurrences of event α . For instance, we can represent $S_1^{(\alpha)}$, the event sequence above restricted to event a, simply as a list of timestamps:

$$S_1^{(a)} = \langle 6, 7, 19, 30, 32, 42, 54 \rangle$$
.

We denote as |S| the number of timestamp–event pairs contained in event sequence S, i.e. its *length*, and $\Delta(S)$ the time spanned by it, i.e. its *duration*. That is, $\Delta(S) = t_{\text{end}}(S) - t_{\text{start}}(S)$, where $t_{\text{end}}(S)$ and $t_{\text{start}}(S)$ represent the largest and smallest timestamps in S, respectively. Observe that $|S^{(\alpha)}|$ equals the number of occurrences of α in the original sequence, and that $\Delta(S^{(\alpha)}) \leq \Delta(S)$. In the example above we have $|S_1| = 13$, $|S_1^{(\alpha)}| = 7$, $\Delta(S_1) = 52$ and $\Delta(S_1^{(\alpha)}) = 48$.

Cycles as periodic patterns. Given such an event sequence, our goal is to extract a representative collection of cycles. A *cycle* is a periodic pattern that takes the form of an ordered list of occurrences of an event, where successive occurrences appear at the same distance from one another. We will not only consider perfect cycles, where the inter-occurrence distance is constant, but will allow some variation.

A cycle is specified by indicating:

- the repeating event, called the *cycle event* and denoted as α ,
- the number of repetitions of the event, called the *cycle length* and denoted as r
- the inter-occurrence distance, called the *cycle period* and denoted as p, and
- the timestamp of the first occurrence, called the *cycle starting point* and denoted as τ .

Cycle lengths, cycle periods and cycle starting points take positive integer values (we choose to restrict periods to be integers for simplicity and interpretability). More specifically, we require r > 1, p > 0 and $\tau \ge 0$.

In addition, since we allow some variation in the actual inter-occurrence distances, we need to indicate an offset for each occurrence in order to be able to reconstruct the original subset of occurrences, that is, to recover the original timestamps. For a cycle of length r, this is represented as an ordered list of r-1 signed integer offsets, called the *cycle shift corrections* and denoted as E. Hence, a cycle is a 5-tuple $C = (\alpha, r, p, \tau, E)$.

Note that since the cycles we consider here involve one event each, we can process the occurrences of each event separately. In other words, we can split the original sequence S into subsequences $S^{(\alpha)}$, one for each event α , and handle them separately.

A cycle's cover. For a given cycle $C = (\alpha, r, p, \tau, E)$, with $E = \langle e_1, \ldots, e_{r-1} \rangle$ we can recover the corresponding occurrences timestamps by reconstructing them recursively, starting from τ : $t_1 = \tau$, $t_k = t_{k-1} + p + e_{k-1}$. Note that this is different from first reconstructing the occurrences while assuming perfect periodicity as $\tau, \tau + p, \tau + 2p, \ldots, \tau + (r-1)p$, then applying the corrections, because in the former case the corrections actually accumulate.

Then, we overload the notation and denote the time spanned by the cycle as $\Delta(C)$, that is

$$\Delta(C) = t_r - t_1$$

= $(t_{r-1} + p + e_{r-1}) - \tau$
= $((t_{r-2} + p + e_{r-2}) + p + e_{r-1}) - \tau$
= $(r - 1)p + e_1 + \dots + e_{r-1}$.

Denoting as $\sigma(E)$ the sum of the shift corrections in E, $\sigma(E) = \sum_{e \in E} e$, we have

$$\Delta(C) = (r-1)p + \sigma(E) \; .$$

Note that this assumes that the correction maintains the order of the occurrences. This assumption is reasonable since an alternative cycle that maintains the order can be constructed for any cycle that does not.

We denote as cover(C) the corresponding set of reconstructed timestamp–event pairs

$$cover(C) = \{(t_1, \alpha), (t_2, \alpha), \dots, (t_r, \alpha)\}.$$

We say that a cycle covers an occurrence if the corresponding timestamp–event pair belongs to the reconstructed subset cover(C).

Since we represent time in an absolute rather than relative manner and assume that an event can only occur once at any given timestamp, we do not need to worry about overlapping cycles nor about an order between cycles. Given a collection of cycles representing the data, the original list of occurrences can be reconstructed by reconstructing the subset of occurrences associated with each cycle, regardless of order, and taking the union. We overload the notation and denote as $cover(\mathcal{C})$ the set of reconstructed timestamp–event pairs for a collection \mathcal{C} of cycles $\mathcal{C} = \{C_1, \ldots, C_m\}$, that is

$$cover(\mathcal{C}) = \bigcup_{C \in \mathcal{C}} cover(C)$$

For a sequence S and cycle collection C we call *residual* the timestamp–event pairs not covered by any cycle in the collection:

$$residual(\mathcal{C}, S) = S \setminus cover(\mathcal{C})$$
.

We associate a cost to each individual timestamp–event pair $o = (t, \alpha)$ and each cycle C, respectively denoted as L(o) and L(C), which we will define shortly. Then, we can reformulate our problem of extracting a representative collection of cycles as follows:

Problem 1. Given an event sequence S, find the collection of cycles C minimising the cost

$$L(\mathcal{C}, S) = \sum_{C \in \mathcal{C}} L(C) + \sum_{o \in residual(\mathcal{C}, S)} L(o)$$

Code lengths as costs. This problem definition can be instantiated with different choices of costs. Here, we propose a choice of costs motivated by the MDL principle. Following this principle, we devise a scheme for encoding the input event sequence using cycles and individual timestamp–event pairs. The cost of an element is then the length of the code word assigned to it under this scheme, and the overall objective of our problem becomes finding the collection of cycles that results in the shortest encoding of the input sequence, i.e. finding the cycles that compress the data most. In the rest of this section, we present our custom encoding scheme.

For each type of information, we need to determine the most appropriate way to encode it, given the type of patterns we are interested in finding. The following should always be kept in mind

In MDL we are NEVER concerned with actual encodings; we are only concerned with code length functions. (Peter D. Grünwald 2004)

Outline of code systems. Given a collection of symbols Z that we might need to transmit, such as, in our case the alphabet of events over which our data sequence is expressed or the range of values that the periods might take, and a particular symbol z, all we are interested is the length of the code assigned to z, which we denote as L(z), not the actual code.

Different code systems can be used, but we focus on those that possess the *prefix* property, meaning that there will not be any two code words in the system such that one is a prefix of the other, making such code uniquely decodable.

For a collection of symbols Z, where each symbol z is associated with an occurrence frequency fr(z), the optimal prefix code is such that $L(z) = -\log(fr(z))$. However, this requires that the receptors knows the occurrence frequencies.

Prequential coding allows to obtain a code that is almost optimal, without knowing the frequencies. Such a code will assign shorter codes to, and hence favour, frequently occurring values.

Fixed-length codes, as the name indicates, assign codes of equal length to all values, and hence do not favour any value. Each value is encoded with a code of length $\log(|Z|)$.

Universal codes allow to encode non-negative integers, assigning shorter codes to smaller numerical values. In particular, the code length assigned to z is $l_{\mathbb{N}}(z) = \log^*(z) + \log(c_0)$, where c_0 is a constant which must be adjusted to ensure that the Kraft inequality is satisfied, i.e. such that

$$\sum_{z\in\mathbb{N}}2^{-l_{\mathbb{N}}(z)}\leq 1.$$

How much small values are favoured compared to larger ones can be adjusted. To avoid wasting bits on unused values large values, c_0 can be adjusted to ensure that Kraft inequality is not only satisfied but holds with strict equality. That is, given some upper bound v on the values to encode, we denote as l_v the code length obtained with an adjusted c_0 so that

$$\sum_{z \in [1..v]} 2^{-l_v(z)} = 1$$

Choosing the most appropriate encoding for cycles. For each cycle we need to specify its event, length, period, starting point and shift corrections, that is

$$L(C) = L(\alpha) + L(r) + L(p) + L(\tau) + L(E)$$
.

It is important to look more closely at the range in which each of these pieces of information takes value, at what values—if any—should be favoured, and at how the values of the different pieces depend on one another.

Clearly, a cycle over event α cannot have a length greater than $|S^{(\alpha)}|$. On the other hand, if it has length r, it cannot have a period greater than $\Delta(S^{(\alpha)})/(r-1)$. Furthermore, once τ is known, the period is further restricted to $(t_{\text{end}}(S^{(\alpha)})-\tau)/(r-1)$. And vice-versa, if we first fix the period, it creates limitations on the values the length can take, which in turn affects the values the starting point can take. So, we see a clear dependency between these values. Also note that the maximum values for the period and the starting point depend on the time span of the sequence, while the maximum value for the length depends on the number of occurrences of the event. To avoid wasting bits, it might be useful to normalise the time scale to the smallest encountered time step.

Encoding with fixed-length codes. A somewhat naive approach to encode a cycle is to use fixed-length codes for the event, length, period and starting point, and an adjusted universal code for the shift corrections. The magnitude of an individual shift correction can be anywhere between 0 and $\Delta(S)$. So if we let $m = \Delta(S) + 1$,

we can use a code word of length $l_m(|e|+1)$ to indicate the absolute value of shift correction e and add one bit to indicate its direction. Since we can easily determine that the length of a cycle can be no larger than |S| and that, neglecting the shift corrections, its period and starting point can take values no larger than $\Delta(S)/2$ and $\Delta(S)$, respectively, we get

$$\begin{split} L(C) = & L(\alpha) + L(r) + L(p) + L(\tau) + L(E) \\ = & \log(|\Omega|) + \log(|S|) + \log(\Delta(S)/2) + \log(\Delta(S)) \\ & + \sum_{e \in E} (l_m(|e|+1) + 1) \; . \end{split}$$

Optimising the encoding. But we can do better, by exploiting the dependencies between the pieces of information. To encode the cycles' events, we can use either fixed-length coding, as above, or codes based on the events' frequency in the original sequence. In the first case the length of the code word representing the event is constant across all cycles, regardless of the event and only depends on the size of the alphabet. In the second case, events that occur more frequently in the event sequence will receive shorter code words:

$$L(\alpha) = -\log(fr(\alpha)) = -\log(\frac{|S^{(\alpha)}|}{|S|}).$$

This requires that we transmit the number of occurrences of each event in the original event sequence. To optimise the overall code length, the length of the code word associated to each event should actually depend on the frequency of the event in the selected collection of cycles. However, this would require keeping track of these frequencies and updating the code lengths dynamically. Instead, we use the frequencies of the events in the input sequence as a simple proxy.

Once the cycle event α and its number of occurrences are known, we can encode the cycle length with a code word of length

$$L(r) = \log(\left|S^{(\alpha)}\right|) ,$$

resulting in the same code length for large numbers of repetitions as for small ones. Recall that

$$\Delta(C) = (r-1)p + \sigma(E) .$$

Clearly, a cycle spans at most the time of the whole sequence, i.e. $\Delta(C) \leq \Delta(S)$. Hence

$$p \leq \left\lfloor \frac{\Delta(S) - \sigma(E)}{r - 1} \right\rfloor,$$

so that knowing the cycle length, the shift corrections, and the sequence time span, we can encode the cycle period with a code word of length

$$L(p) = \log\left(\left\lfloor\frac{\Delta(S) - \sigma(E)}{r - 1}\right\rfloor\right).$$

Note that the code word for the period of a cycle will be shorter if the cycle has greater length (since there are more repetitions, the period cannot be as long).

Next, knowing the cycle length and period as well as the sequence time span, the starting point τ can take any value between $t_{\text{start}}(S)$ and $t_{\text{end}}(S) - \Delta(C) = t_{\text{end}}(S) - \sigma(E) - (r-1)p$. Hence, we can specify the value of the starting point with a code word of length

$$L(\tau) = \log(\Delta(S) - \sigma(E) - (r-1)p + 1)$$

Note that if the cycle spans a larger part of the sequence, the range of the starting point is more restricted, and so it can be represented with a shorter code word.

Finally, we encode the shift corrections as follows: each correction e is represented by |e| ones, prefixed by a single bit to indicate the direction of the shift, with each correction separated from the previous one by a zero. For instance, $E = \langle 3, -2, 0, 4 \rangle$ would be encoded as 01110111000011110 with value digits, separating digits and sign digits, in italics, bold and normal font, respectively (the sign bit for zero is arbitrarily set to 0 in this case). As a result, the code length for a sequence of shift corrections E is

$$L(E) = 2 |E| + \sum_{e \in E} |e|$$
.

Putting everything together, we can write the cost of a cycle C as

$$\begin{split} L(C) = &L(\alpha) + L(r) + L(p) + L(\tau) + L(E) \\ = &\log(|S|) + \log\left(\left\lfloor\frac{\Delta(S) - \sigma(E)}{r - 1}\right\rfloor\right) \\ &+ \log(\Delta(S) - \sigma(E) - (r - 1)p + 1) \\ &+ 2\left|E\right| + \sum_{e \in E} |e| \ . \end{split}$$

On the other hand, the cost of an individual occurrence $o = (t, \alpha)$ is simply the sum of the cost of the corresponding timestamp and event:

$$L(o) = L(t) + L(\alpha) = \log(\Delta(S) + 1) - \log(\frac{|S^{(\alpha)}|}{|S|})$$

Note that if our goal was to actually encode the input sequence, we would need to transmit the smallest and largest timestamps $(t_{\text{start}}(S) \text{ and } t_{\text{end}}(S))$, the size of the event alphabet $(|\Omega|)$, as well as the number of occurrences of each event $(|S^{(\alpha)}|$ for each event α) of the event sequence. We should also transmit the number of cycles in the collection $(|\mathcal{C}|)$, which can be done, for instance with a code word of length $\log(|S|)$. However, since our goal is to compare collections of cycles, we can simply ignore this, as it represents a fixed cost that remains constant for any chosen collection of cycles.

Finally, consider that we are given an ordered list of occurrences $\langle t_1, t_2, \ldots, t_l \rangle$ of event α , and we want to determine the best cycle with which to cover all these occurrences at once. Some of the parameters of the cycle are determined, namely the

repeating event α , the length r, and the timestamp of the first occurrence τ . All we need to determine is the period p that yields the shortest code length for the cycle. In particular, we want to find p that minimises L(E). The shift corrections are such that $E_k = (t_{k+1}-t_k)-p$ (cf. the definition of a cycle's cover). If we consider the list of inter-occurrence distances $d_1 = t_2 - t_1, d_2 = t_3 - t_2, \ldots, d_{l-1} = t_l - t_{l-1}$, the problem of finding p that minimises L(E) boils down to minimising $\sum_{d_i} |d_i - p|$. This is achieved by letting p equal the geometric median of the inter-occurrence distances, which, in the one-dimensional case, is simply the median. Hence, for this choice of encoding for the shift corrections, the optimal cycle covering a list of occurrences can be determined by simply computing the inter-occurrences distances and taking their median as the cycle period.

4 Defining Tree Patterns

So far, our pattern language is restricted to cycles over single events. In practise, however, several events might recur regularly together and repetitions might be nested with several levels of periodicity. To handle such cases, we now introduce a more expressive pattern language, that consists of a hierarchy of cyclic blocks, organised as a tree.

Instead of considering simple cycles specified as 5-tuples $C = (\alpha, r, p, \tau, E)$ we consider more general patterns specified as triples $P = (T, \tau, E)$, where T denotes the tree representing the hierarchy of cyclic blocks, while τ and E respectively denote the starting point and shift corrections of the pattern, as with cycles.

Pattern trees. Each *leaf node* in a pattern tree represents a simple block containing one event. Each *intermediate node* represents a cycle in which the children nodes repeat at a fixed time interval. In other words, each intermediate node represents cyclic repetitions of a sequence of blocks. The root of a pattern tree is denoted as B_0 . Using list indices, we denote the children of a node B_X as B_{X1} , B_{X2} , etc. We denote the ordered list of the children of node B_X as $\Gamma(B_X)$, that is,

$$\Gamma(B_X) = \langle B_{X1}, B_{X2}, \ldots \rangle .$$

All children of an intermediate node except the left-most child are associated to their distance to the preceding child, called the *inter-block distance*. This distance for node B_{Xi} is denoted as d_{Xi} , i.e. d_{Xi} represents the time that separates occurrences of node $B_{X(i-1)}$ and node B_{Xi} . Inter-block distances take non-negative integer values. Each intermediate node B_X is associated with the period p_X and length r_X of the corresponding cycle. Each leaf node B_Y is associated with the corresponding occurring event α_Y .

An example of an abstract pattern tree is shown in Fig. 1. Some concrete pattern trees that we will use as examples are shown in Fig. A.7–A.9. We call *height* and *width* of the pattern tree—and by extension of the associated pattern—respectively the number of edges along the longest branch from the root to a leaf node and the number of leaf nodes in the tree.

For a given pattern, we can construct a tree of event occurrences by expanding the pattern tree recursively, that is, by appending to each intermediate node the

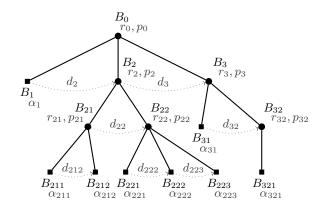


Fig. 1. Abstract pattern tree.

corresponding number of copies of the associated subtree, recursively. We call this expanded tree the *expansion tree* of the pattern, as opposed to the contracted *pattern* tree that more concisely represents the pattern.

When a pattern tree is expanded, several copies of a node can be generated as a result of repetitions in possibly nested cycles. Each node in an expansion is identified with a pair (n, L), where n is the node of the pattern tree that generated the expansion node, and L is a list indicating the specific combination of repetitions of ancestors that produced it.

The expansion tree of the pattern tree of Fig. 1 is shown in Fig. 2. Node $(B_0, \langle \rangle)$ is the root of the expansion tree, $(B_0, \langle 1 \rangle)$ is the node generated as the first repetition of pattern node B_0 , and $(B_{21}, \langle 2, 3 \rangle)$ is the node generated from node B_{21} in the third repetition of pattern node B_2 nested within the second repetition of pattern node B_0 .

The notation used to identify nodes in pattern trees and expansion trees allows to easily navigate the trees. In particular, the left-most leaf among the descendants of a given node B_X can be obtained by going down the left-most branch, looking at nodes B_{X1} , B_{X11} , etc. until reaching a leaf. We denote that node, the left-most leaf descendant of B_X as $\gamma_{\mathbf{L}}(B_X)$. Similarly, we denote as $\gamma_{\mathbf{L}}((n,L))$ the left-most leaf descendant of node (n, L) in the expansion tree, which is such that $\gamma_{\mathbf{L}}((n, L)) =$ $(\gamma_{\mathbf{L}}(n), L')$, where $L' = L \oplus \langle 1, 1 \dots \rangle$, that is, L' is the list L trailing with ones. That is, in addition to selecting always the left-most child, we always select the first repetition of a node when travelling the expansion tree until reaching a leaf. Note that $\gamma_{\mathbf{L}}(B_X) = B_X$ and $\gamma_{\mathbf{L}}((B_X, L)) = (B_X, L)$ if B_X itself is a leaf node.

We use the recursive notation $\{r = r_X, p = p_X\}(B_{X1} - d_{X2} - B_{X2}...)$ to represent a block B_X . With this notation, T_1 from Fig. A.7 is represented as

$$\{r=4, p=2\}(a)$$

and T_7 from Fig. A.9 as

$$\{r=3, p=10\}(b-3-\{r=4, p=1\}(a)-1-c)$$
.

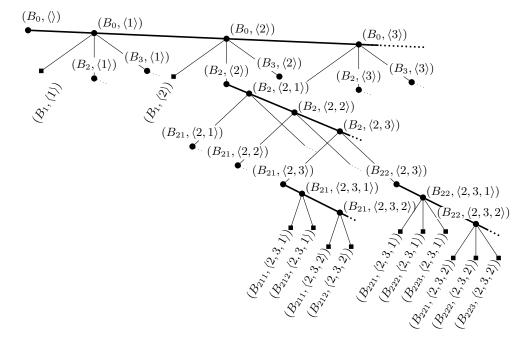


Fig. 2. Expansion of the pattern tree from Fig. 1.

Reconstructing a pattern's cover. We can enumerate the event occurrences of a pattern by traversing its expansion tree and recording the encountered leaf nodes. The expansion tree is traversed in a depth-first left-to-right manner, first travelling through all children in a repetition of a block before moving on to the next repetition. For instance, the traversal of the expansion tree shown in Fig. 2, starts from the root node $(B_0, \langle \rangle)$ and first reaches $(B_0, \langle 1 \rangle)$. Then, children nodes $(B_1, \langle 1 \rangle), (B_2, \langle 1 \rangle)$ and $(B_3, \langle 1 \rangle)$, and their descendants, should be traversed before travelling to the next repetition of $B_0, (B_0, \langle 2 \rangle)$. Simply put, pattern edges (represented as thin lines in Fig. 2) take priority over repetition edges (represented as thick lines).

We define the following recursive function:

$$\Theta(B_X, l) = \begin{cases} \langle B_X, l \rangle & \text{if } B_X \text{ is a leaf,} \\ \bigoplus_{k \in [1..r_X]} \bigoplus_{B_{Xi} \in \Gamma(B_X)} \Theta(B_{Xi}, l \oplus \langle k - 1 \rangle) \\ & \text{otherwise.} \end{cases}$$

The list of leaf nodes encountered in the expansion tree during the traversal can be obtained as $\Theta(T) = \Theta(B_0, \langle \rangle)$.

Using a similar recursive function, following the same traversal of the expansion tree, we can construct the perfect event occurrences. That is, we can recursively construct the list of uncorrected timestamps-events pairs produced by a pattern tree T, which we denote as $occs^*(T) = occs^*(B_0)$.

13

For this purpose, we first define a function $shift(S, t_s)$ that shifts a set of event occurrences S by a specified value t_s , that is,

$$shift(S, t_s) = \{(t_i + t_s, \alpha_i), \quad \forall (t_i, \alpha_i) \in S\}.$$

For instance

$$shift(\langle (2,c), (3,c), (6,a), (7,a) \rangle, -1) \\ = \langle (1,c), (2,c), (5,a), (6,a) \rangle.$$

Overloading the notation, we let $occs^*(B_X)$ denote the list of occurrences associated with B_X . If B_X is a leaf, $occs^*(B_X)$ is a one-element list

$$occs^*(B_X) = \langle (0, \alpha_X) \rangle.$$

If B_X is an intermediate node, we let $O(B_X)$ denote the concatenation of the lists of occurrences of its children, each one shifted by the accumulated inter-block distances:

$$O(B_X) = \bigoplus_{B_{Xi} \in \Gamma(B_X)} shift(occs^*(B_{Xi}), \sum_{1 < j \le i} d_{Xj}) .$$

Then the list of occurrences is obtained by concatenating r_X copies of $O(B_X)$, shifted according to the period p_X :

$$occs^*(B_X) = \bigoplus_{k \in [1..r_X]} shift(O(B_X), (k-1) \cdot p_X).$$

Finally, if the starting point of pattern P is τ , we have $occs^*(P) = shift(occs^*(T), \tau)$.

The occurrences appear in the list in the order in which they are generated during the expansion, which does not necessarily match the order of the timestamps. More specifically, if the sequence of timestamps in $occs^*(T)$ is not monotone, we say that the pattern tree T (and the associated pattern P) is *interleaved*. If a pattern tree is not interleaved, all events constituting a repetition of a block must occur at latest when an event of the following repetition occurs. If several events occur at the same time, we say that the pattern tree has *overlaps*. For example, pattern trees T_3 and T_4 cover the same occurrences, but T_4 is interleaved while T_3 is not. Both patterns T_6 and T_7 have overlaps, but T_7 is interleaved while T_6 is not.

We denote as o_i the i^{th} event occurrence generated by T, and let $occs^*(o_i)$ be the corresponding timestamp–event pair and $\Theta(o_i)$ be the corresponding expansion leaf node, i.e. mapping o_i to the elements at position i in $occs^*(T)$ and $\Theta(T)$, respectively.

As for the simple cycles, we will not only consider perfect patterns but will allow some variations. For this purpose, a list of shift corrections E is provided with the pattern, which contains a correction for each occurrence except the first one, i.e. $|E| = |occs^*(P)| - 1.$

By applying the shift corrections in E to the perfect occurrences in $occs^*(P)$, we can generate the list of corrected occurrences for pattern P, denoted as occs(P). The corrections are listed in E in the same order as the leaf nodes are encountered in the expansion tree. Therefore, the correction associated to occurrence o_i is the element

Algorithm 1 CoCo: Collect occurrence corrections.

Require: An occurrence *o* Ensure: A set of occurrences whose corrections apply to o 1: if $o = (B_0, \langle \rangle)$ then \triangleright Root of pattern 2: $\omega \leftarrow \emptyset$ 3: if $o = (B_{Xy}, Uv)$ then $\omega \leftarrow \{\gamma_{\mathbf{L}}((B_{Xy'}, Uv)), y' < y\}$ ▷ Left-siblings 4: $\omega \leftarrow \omega \cup \{\gamma_{\mathbf{L}}((B_X, Uv')), v' < v\}$ \triangleright Previous repetitions 5: $\omega \leftarrow \omega \cup \operatorname{CoCo}((B_X, U))$ 6: \triangleright Recurse for parent 7: return ω

at position i-1 in E, i.e. E[i-1], which we also denote as $E(o_i)$ or E((n, L)), where (n, L) is the corresponding expansion node. For ease of notation we let $E(o_1) = 0$, since the left most occurrence o_1 has no correction.

However, as for simple cycles, corrections accumulate over successive occurrences, and we cannot recover the list of corrected occurrences occs(P) by simply adding the individual corrections to the elements of $occs^*(P)$. Instead, we first have to compute the accumulated corrections for each occurrence. In addition to its own correction, the corrections that should be applied to an occurrence come from the offsets of its left siblings in multi-events blocks and the offsets of previous repetitions in cycles the occurrence belongs to.

Algorithm 1 shows the procedure—named CoCo—that can be used to collect the occurrences whose individual corrections impact occurrence o (recall that $\gamma_{\mathbf{L}}$ () returns the left-most leaf descendant of a node). Then, the correction to be applied to the timestamp of o is

$$\epsilon(o) = E(o) + \sum_{o_k \in \text{CoCo}(o)} E(o_k) \; .$$

The corrected occurrence timestamps can thus be reconstructed by shifting the perfect timestamp by the corresponding correction, i.e. $occs(o_i) = occs^*(o_i) + \epsilon(o_i)$.

Encoding the patterns. To transmit a pattern, we need to encode its pattern tree, as well as its starting point and shift corrections. Furthermore, to encode the pattern tree, we consider separately its event sequence, its cycle lengths, its top-level period, and the other values, as explained below.

First we encode the event in the leaves of the pattern tree, traversing the tree from left to right, depth-first, enclosing blocks between parenthesis. The string representing the events in the pattern tree is defined recursively as follows:

$$\zeta(B_X) = \begin{cases} `\alpha_X' & \text{if } B_X \text{ is a leaf,} \\ `(` \oplus \left(\bigoplus_{B_Y \in \Gamma(B_X)} \zeta(B_Y) \right) \oplus `)' \text{ otherwise.} \end{cases}$$

We denote as A the string $\zeta(B_0)$ for the top-level block of the tree of a pattern, representing its event sequence. We encode each symbol s in the string A using a code of length L(s), where L(s) depends on the frequency of s, adjusted to take into account the additional symbols '(' and ')', used to delimit blocks. In particular, we set the code length for the extended alphabet as

$$L(`(') = L(`)') = -\log(\frac{1}{3})$$

for the block delimiters, and

$$L(`)`) = -\log\left(\frac{\left|S^{(\alpha)}\right|}{3\left|S\right|}\right)$$

for the original events.

Next, we encode the cycle lengths, i.e. the values r_X associated to each intermediate node B_X encountered while traversing the tree depth-first and from left to right, as a sequence of values, and denote this sequence R. For a block B_X the number of repetitions of the block cannot be larger than the number of occurrences of the least frequent event participating in the block. Formally, the cycle length r_X of a block B_X , can take at most a value $\rho(B_X)$ defined recursively as follows:

$$\rho(B_X) = \begin{cases} \left| S^{(\alpha_X)} \right| & \text{if } B_X \text{ is a leaf,} \\ \min_{B_Y \in \Gamma(B_X)} \rho(B_Y) \text{ otherwise,} \end{cases}$$

We can thus encode the sequence of cycle lengths R with code of length

$$L(R) = \sum_{r_X \in R} L(r_X) = \sum_{r_X \in R} \log \left(\rho(B_X) \right) \,.$$

Knowing the cycle lengths R and the structure of the pattern tree from its event sequence A, we can deduce the total number of events covered by the pattern, $N(B_0)$, using the following formula

$$N(B_X) = \begin{cases} 1 & \text{if } B_X \text{ is a leaf,} \\ r_X \cdot \sum_{B_Y \in \Gamma(B_X)} N(B_Y) \text{ otherwise.} \end{cases}$$

The shift corrections for the pattern consist of the correction to each event occurrence except the first one (assumed not to require correction). This ordered list of $N(B_0) - 1$ values can be transmitted using the same encoding as for the simple cycles.

In simple cycles, we had a unique period characterising the distances between occurrences. Instead, with these more complex patterns, we have a period p_X for each intermediate node B_X , as well as an inter-block distance d_X for each node B_X that is not the left-most child of its parent.

First, we transmit the period of the root node of the pattern tree, B_0 . In a similar way as with simple cycles, we can deduce the largest possible value for p_0 from r_0 and E. Since we do not know when the events within the main cycle occur, we assume what would lead to the largest possible value for p_0 , that is, we assume that all the events within each repetition of the cycle happen at once, so that each repetition spans no time at all. The corrections that must be taken into account are those applying to the left-most leaf of each repetition of the main cycle. These are exactly the corrections accumulated in $\epsilon(o_{za})$ where o_{za} is the first occurrence of the last repetition of the main cycle, i.e. $o_{za} = \gamma_{\mathbf{L}}((B_0, \langle r_0 \rangle))$.

	$\Delta^*(B_0)$	$(T_8, 0, 0)$
$\Delta^*(B_1) = \delta^*(B_0) \Delta^*_{\max}(B_1) = \delta^*_{\max}(B_0)$		
$\Delta^*(B_{12})\Delta^*_{\max}(B_{12}) \underbrace{\delta^*(B_1)\delta^*_{\max}(B_1)}_{\bullet}$		
b a a a a c b a a a c b a a a a	a c	b
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	-7	33
d_{12} d_{13} d_{12} d_{13} d_{12} d_{13}	13	d_{12}
p_1 p_1 p_1		$\xrightarrow{p_1} p_0$

Fig. 3. Pattern $(T_8, 0, 0)$ partially shown on timeline (maximum time spans assume interleaving is not allowed).

Thus we have

$$L(p_0) = \log\left(\left\lfloor \frac{\Delta(S) - \epsilon(o_{za})}{r_0 - 1} \right\rfloor\right)$$

Once the main period is known, we can use the same principle as for simple cycles to transmit the starting point and we have

$$L(\tau) = \log(\Delta(S) - \epsilon(o_{za}) - (r_0 - 1)p_0 + 1) .$$

We denote as $\Delta^*(B_X)$ the time spanned by the entire cycle of block B_X , that is, the time spanned by the r_X repetitions of the block. We denote as $\delta^*(B_X)$ the time spanned by a single repetition of the block. Note that here we consider the perfect occurrences of the block, before applying the corrections. In this case all repetitions span the same time, which might no longer be true after correction. In Fig. 3 we provide a timeline schema of the first occurrences of pattern $(T_8, 0, \mathbf{0})$, i.e. the pattern consisting of the pattern tree T_8 from Fig. A.9, with starting point 0 and no shift corrections. We indicate the time spanned by different blocks and their maximum value assuming interleaving is not allowed.

Suppose we know $\Delta^*(B_X)$. Then, in order for r_X repetitions (equally long, but potentially spanning no time at all) to happen within time $\Delta^*(B_X)$, p_X must satisfy $p_X \leq \lfloor \Delta^*(B_X)/(r_X-1) \rfloor$ and can therefore be represented with a code word of length

$$L(p_X) = \log\left(\left\lfloor\frac{\Delta^*(B_X)}{r_X - 1}\right\rfloor\right).$$

If we do not allow interleaving, each repetition can span at most $\lfloor \Delta^*(B_X)/r_X \rfloor$, and also no longer than p_X . On the other hand, if we do allow interleaving, each

17

repetition can have a time span of at most $\Delta^*(B_X) - r_X + 1$. Thus, the maximum time span of a repetition is

$$\delta_{\max}^*(B_X) = \begin{cases} \Delta^*(B_X) - r_X + 1 \\ \text{if interleaving is allowed,} \\ \min(p_X, \lfloor \Delta^*(B_X)/r_X \rfloor) & \text{otherwise.} \end{cases}$$

Obviously, the sum of the distances between the children of the block cannot be larger than the time span of a repetition. Therefore, we can represent the distances between the children of B_X with code words such that

$$\sum_{B_{Xi} \in \Gamma(B_X), i > 1} L(d_{Xi}) = (|\Gamma(B_X)| - 1) \cdot \log \left(\delta_{\max}^*(B_X) + 1\right) \,.$$

We can then determine the maximum span of each child of a block. If interleaving is allowed, the child can span as much time as is left in the time span of its parent after accounting for the distances of the left siblings:

$$\Delta_{\max}^*(B_{Xi}) = \delta_{\max}^*(B_X) - \sum_{1 \le j \le i} d_{Xj}.$$

Alternatively, if interleaving is not allowed, all events of the child must occur before the first event of the next sibling:

$$\Delta_{\max}^*(B_{Xi}) = \begin{cases} \delta_{\max}^*(B_X) - \sum_{j \neq i} d_{Xj} \\ & \text{if } B_{Xi} \text{ is the right-most child,} \\ d_{X(i+1)} & \text{otherwise.} \end{cases}$$

Note that $d_{X(i+1)}$ is not defined if B_{Xi} is the right-most child of the block.

Applying the formulas above recursively allows to compute the length of the code words needed to represent all the periods and inter-block distances in the tree, for a known value $\delta^*(B_0)$.

Looking at the last occurrence of the main cycle $(B_0, \langle r_0 \rangle)$, we have

$$\tau + (r_0 - 1)p_0 + \delta^*(B_0) + \epsilon(o_{zz}) \le t_{\text{end}}(S)$$
,

and hence

$$\delta_{\max}^*(B_0) = t_{\text{end}}(S) - \epsilon(o_{zz}) - (r_0 - 1)p_0 - \tau$$

where $\epsilon(o_{zz})$ denotes the accumulated corrections that apply to the event having the largest uncorrected timestamp.

If interleaving is not allowed, that event is the right-most leaf node of the expansion tree, i.e. the last element in the occurrence list. Besides, if interleaving is not allowed, we also have $\delta^*(B_0) \leq p_0$.

On the other hand, if interleaving is allowed the event having the largest uncorrected timestamp is not necessarily the last one in the list of occurrences (see $occs^*(T_6)$ in Fig. A.8 for instance). Since it depends on periods and inter-block distances within the block, which have not been specified at that point, we cannot determine which event has the largest timestamp. Hence, we compute $\epsilon(o_i)$ for

all occurrences o_i that correspond to the right most child of a block and take the minimum (possibly a negative value) as $\epsilon(o_{zz})$.

To compute the periods and inter-block distances, we can use the actual value $\delta^*(B_0)$, which we first need to transmit explicitly after the value of τ , with a code word of length log $(\delta^*_{\max}(B_0) + 1)$. Instead, we could use the upper-bound on $\delta^*_{\max}(B_0)$, which we do not need to transmit. It is probably more economical to transmit the value explicitly.

We denote as D the collection of all the periods (except p_0) and inter-block distances in the tree (as well as $\delta^*(B_0)$, if necessary), that need to be transmitted to fully describe the pattern. The corresponding code length is

$$L(D) = \sum_{v \in D} L(v) \; ,$$

where the code length of each element can be computed using the formulas presented above.

To put everything together, the code used to represent a pattern $P = (T, \tau, E)$ has length

$$L(P) = L((T, \tau, E))$$

= $L(A) + L(R) + L(p_0) + L(D) + L(\tau) + L(E)$.

From simpler patterns to more complex ones. Let us have a look at what happens to the encoding of a simple cycle, when using this more complex encoding scheme to represent it. Consider a simple cycle $C = (\alpha, r, p, \tau, E)$. Using the more complex encoding it can be represented as $P = (T, \tau, E)$, where the cycle is represented using a more general pattern formalism $T = \{r = r, p = p\}(\alpha)$. Both encodings are very similar, with $R = \langle r \rangle$, $p_0 = p$ and $D = \langle \rangle$, $A = (\alpha)$. The code word representing the cycle length, L(r), depends only on the frequency of occurrence of the event, which is fixed. The corrections accumulated for the first occurrence of the last repetition of the main cycle are equal to the sum of the corrections in E, hence $\epsilon(o_{za}) = \sigma(E)$, so that the length of the code words representing the cycle period and starting point also remain the same. The corrections are the same and encoded the same way under both encodings. The only difference comes from the different way to encode the event, which is longer under the more complex encoding, to accommodate for the additional symbols which allow to represent (nested) event sequences. That is, for any event α , its code length under the more complex pattern encoding $L_P(\alpha)$ is larger than its code length under the simpler cycle encoding, $L_C(\alpha)$, due to the over-head of having block delimiters.

Note that the actual value of τ does not impact the code length of a pattern. If we consider two cycles

$$C_1 = (\alpha_1, r_1, p_1, \tau_1, E_1)$$
 and $C_2 = (\alpha_2, r_2, p_2, \tau_2, E_2)$

such that $\tau_1 \neq \tau_2$ but all other values are equal, then $L(C_1) = L(C_2)$. Simply put, translation does not affect the cost of a cycle or pattern.

On the other hand, the values of the corrections, through $\epsilon(o_{za})$ impact the length of the code words representing the starting point and the main period. For

this reason, given two cycles with the same length and period but with different corrections (i.e. such that $r_1 = r_2$ and $p_1 = p_2$, but $E_1 \neq E_2$), the code words representing their respective periods and starting points will differ (i.e. we will have $L(r_1) = L(r_2)$ but $L(p_1) \neq L(p_2)$ and $L(\tau_1) \neq L(\tau_2)$).

5 Combining patterns and comparing costs

Recall that for a given input sequence S, our goal is to find a collection of patterns C that minimises the cost

$$L(\mathcal{C}, S) = \sum_{P \in \mathcal{C}} L(P) + \sum_{o \in residual(\mathcal{C}, S)} L(o)$$

It is useful to compare the cost of different patterns, or sets of patterns, on a subset of the data, i.e. compare $L(\mathcal{C}', S')$ for different sets of patterns \mathcal{C}' and some subsequence $S' \subseteq S$. In particular, we might compare the cost of a pattern P to the cost of representing the same occurrences separately. This means comparing

$$L(\{P\}, cover(P)) = L(P)$$
 and $L(\emptyset, cover(P)) = \sum_{o \in cover(P)} L(o)$.

If $L(\{P\}, cover(P)) < L(\emptyset, cover(P))$, we say that pattern P is cost-effective. In addition, we compare patterns in terms of their cost-per-occurrence ratio defined, for a pattern P, as

$$\frac{L(P)}{|cover(P)|} \; ;$$

and say that a pattern is more *efficient* when this ratio is smaller.

Furthermore, in order to reduce the number of candidate patterns considered and to retain only the most promising ones, we use a procedure called FILTERCANDIDATES that takes as input a collection of patterns \mathcal{K} together with some integer k and returns only those patterns from \mathcal{K} that are among the top-k most efficient ones for some occurrence they cover.

A natural way to build patterns is to start with the simplest patterns, i.e. cycles over single events, and combine them together into more complex, possibly multilevel multi-event patterns. Therefore, we now look at how the cost of patterns relates to the cost of the building blocks they are constructed from. We start by looking at the cost of covering k occurrences $(k \ge 3)$ with a simple cycle as compared to representing them separately. In other words, we look in more details at what it takes for a cycle to be cost-effective.

Simple cycles vs. residuals. Assume we have a candidate cycle C of length $k \geq 3$, covering k occurrences of event α , and we want to check whether this cycle is cost-effective, i.e. compare the cost of representing this k-subsequence with C to the cost of representing it with individual occurrences

$$L(\{C\}, cover(C)) = L(C)$$
 and $L(\emptyset, cover(C)) = \sum_{o \in cover(C)} L(o)$.

The cost of representing the individual occurrences separately is

$$L(\emptyset, cover(C)) = k \cdot (L(t) + L(\alpha)) = k \left(\log(\Delta(S) + 1) - \log(\frac{|S^{(\alpha)}|}{|S|}) \right)$$

and the cost for representing the same occurrences with cycle C is

$$L(C) = L(\alpha) + \beta + L(r) + L(p) + L(\tau) + L(E) ,$$

where β denotes the length of the code for one pair of block delimiters. The cost of corrections in the cycle is

$$L(E) = 2(k-1) + \sum_{e \in E} |e|$$

and the code length of the period and starting point of a cycle satisfy, respectively,

$$L(p) < \log(\frac{\Delta(S) + 1}{k - 1})$$
 and $L(\tau) < L(t)$,

so that

$$L(C) < L(\alpha) + \beta + L(r) + \log\left(\frac{\Delta(S) + 1}{k - 1}\right) + L(t) + 2k - 2 + \sum_{e \in E} |e| .$$

If we let

$$W(k) = (k-1)(L(t) + L(\alpha)) - \beta - L(r) - \log\left(\frac{\Delta(S) + 1}{k-1}\right) - 2k + 2$$

= (k-2) log(\Delta(S) + 1) + (k-1)L(\alpha) - \beta - log(\Beta^{(\alpha)}) + log(k-1) - 2k + 2,

we have

$$\sum_{e \in E} |e| < W(k) \implies L(\{C\}, cover(C)) < L(\emptyset, cover(C)) .$$

In other words, if the sum of the absolute shift corrections in a cycle C of length k is less than W(k), then the cost of representing the occurrences with C is smaller than the cost of representing them separately.

Furthermore, we can state the following:

Lemma 1. Given a sequence S, if C is a cycle of length k over event α with corrections E satisfying $\sum_{e \in E} |e| < W(k)$, and if extending C to cover one further occurrence of event α does not increase the sum of the absolute corrections by more than $\log(\Delta(S) + 1) - 2$, then the cost of representing the k + 1 occurrences with the extended cycle is smaller than the cost of representing them separately, i.e. the extended cycle remains cost-effective.

Proof. Assume we have a cycle C with corrections E, satisfying $\sum_{e \in E} |e| < W(k)$. Let C' be the cycle obtained by extending C to cover one further occurrence, i.e. C' is a cycle of length k + 1, and let E' be the associated corrections. Since

$$W(k+1) - W(k) = \log(\Delta(S) + 1) + L(\alpha) + \log(k/(k-1)) - 2 > \log(\Delta(S) + 1) - 2,$$

we have

$$\begin{split} \sum_{e \in E'} |e| &- \sum_{e \in E} |e| \leq \log(\varDelta(S) + 1) - 2 \\ \Longrightarrow \sum_{e \in E'} |e| &- \sum_{e \in E} |e| < W(k + 1) - W(k) \\ \Longrightarrow \sum_{e \in E'} |e| < W(k + 1) - W(k) + \sum_{e \in E} |e| \\ \Longrightarrow \sum_{e \in E'} |e| < W(k + 1) \\ \Longrightarrow L(\{C'\}, cover(C')) < L(\emptyset, cover(C')) . \end{split}$$

For a simple criterion to decide whether to extend a cycle we compare the magnitude of the new correction to $\log(\Delta(S) + 1) - 2$.

Vertical combination: Nesting cycles. First, let us consider a practical example. Imagine that the following sequence is part of the input:

$$S_2 = \langle (2, a), (5, a), (7, a), (8, a), (13, a), (15, a), (20, a), (21, a), (26, a), (29, a), (32, a), (33, a) \rangle.$$

We can represent this sequence with simple cycles, using three patterns over pattern tree T_1 from Fig. A.7 with starting points 2, 13, and 26, respectively.

Using this notation, the first option is to represent the sequence with the collection

$$\begin{aligned} \mathcal{C}_1 &= \{ P_{1,1}, P_{1,2}, P_{1,3} \} \\ &= \{ (T_1, 2, \langle 1, 0, -1 \rangle), (T_1, 13, \langle 0, 3, -1 \rangle), (T_1, 26, \langle 1, 1, -1 \rangle) \} . \end{aligned}$$

Alternatively, we can represent the sequence using four patterns over pattern tree T_2 from Fig. A.7 with starting points 2, 5, 7 and 8, respectively:

$$\begin{aligned} \mathcal{C}_2 &= \{ P_{2,1}, P_{2,2}, P_{2,3}, P_{2,4} \} \\ &= \{ (T_2, 2, \langle -2, 0 \rangle), (T_2, 5, \langle -3, 1 \rangle), \\ (T_2, 7, \langle 0, -1 \rangle), (T_2, 8, \langle 0, -1 \rangle) \} \end{aligned}$$

But it can also be represented as a single pattern containing two nested cycles, namely as patterns over pattern trees T_3 or T_4 from Fig. A.7, respectively, depending whether the inner cycle is T_1 or T_2 . So, we can represent the sequence with a single pattern, with either

$$\mathcal{C}_3 = \{P_{3,1}\} = \{(T_3, 2, \langle 1, 0, -1, -2, 0, 3, -1, 0, 1, 1, -1 \rangle)\}, \text{ or } \\ \mathcal{C}_4 = \{P_{4,1}\} = \{(T_4, 2, \langle -2, 0, 1, -3, 1, 0, 0, -1, -1, 0, -1 \rangle)\}.$$

Note that with this type of pattern combining two nested cycles over the same event, the list of corrections for the combined pattern is a simple combination of corrections for the basic cycles:

$$E_{3,1} = E_{1,1} \oplus \langle E_{2,1}[1] \rangle \oplus E_{1,2} \oplus \langle E_{2,1}[2] \rangle \oplus E_{1,3}$$

a) GROWHORIZONTALLY:

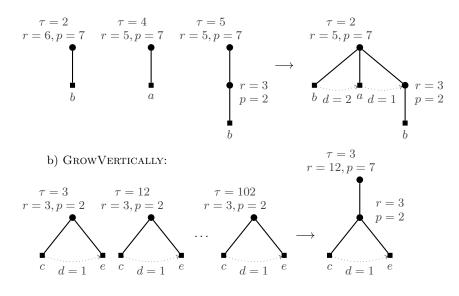


Fig. 4. Examples of growing patterns through combinations.

where $E_{x,y}$ is the list of shift corrections for pattern $P_{x,y}$ and $E_{x,y}[i]$ is the correction at position i in that list.

Let us look at the code lengths for these different patterns. For this example, we have

$$t_{\text{start}}(S_2) = 0, t_{\text{end}}(S_2) = 34, \ \Delta(S_2) = 34,$$

and $\left| S_2^{(a)} \right| = 12.$

We list the code lengths for the different elements in Tables A.2–A.4. In Fig. A.10 we provide a timeline schema of the occurrences of $P_{3,1}$ as well as of the occurrences of $(T_3, 0, \mathbf{0})$ and $(T_4, 0, \mathbf{0})$, i.e. the occurrences of pattern trees T_3 and T_4 with starting point 0 and no corrections.

Now, let us turn to the general case. Assume that we have a pattern tree T_I which occurs multiple times in the event sequence. In particular, assume that it occurs at starting points $\tau_1, \tau_2, \ldots, \tau_{r_J}$ (where the starting points are ordered) and that this sequence of starting points itself can be represented as a cycle of length r_J and period p_J . In other words, if we denote as α the left-most event of T_I , i.e. the event associated to the starting point of T_I , the sequence consisting of the starting points of the different occurrences of T_I can be represented by a pattern (T_J, τ_1, E_J) where $T_J = \{r = r_J, p = p_J\}(\alpha)$ is a cycle of length r_J and period p_J over event α , with shift corrections

$$E_J = \langle (\tau_i - \tau_{i-1}) - p_J \text{ for } i \in [2, r_J] \rangle.$$

In such a case, the occurrences of T_I might be combined together and represented as a nested pattern tree $T_N = \{r = r_J, p = p_J\}(T_I)$. We refer to such a combination as vertical combination, since it produces patterns of greater depth than the original ones. GROWVERTICALLY is the procedure which takes as input a collection C_I of patterns over a tree T_I , i.e. $C_I = \{(T_I, \tau_1, E_{I,1}), \dots, (T_I, \tau_{r_J}, E_{I,r_J})\}$ and returns the nested pattern, covering the same timestamp–event pairs, obtained by combining them together as depicted in Fig. 4(b).

This situation is illustrated in Fig. A.12.

Lemma 2. Let $C_I = \{(T_I, \tau_1, E_{I,1}), \dots, (T_I, \tau_{r_J}, E_{I,r_J})\}$ be a collection of patterns consisting of r_J occurrences of the same pattern tree T_I and $P_N = \text{GROWVERTICALLY}(C_I)$ be the nested pattern obtained by combining the patterns in C_I . If the cycle P_J over the starting points of the patterns in C_I satisfies

$$L(P_J) < (r_J - 1) \cdot L((T_I, \tau_1, \langle \rangle)) ,$$

then

$$L(\{P_N\}, cover(\mathcal{C}_I)) < L(\mathcal{C}_I, cover(\mathcal{C}_I))$$
.

Proof. The code length of the event sequence in T_N , i.e. $A_N = `(T_I)$ ' equals the code length to encode the event sequence in T_I plus the code length for one pair of block delimiters and satisfies

$$L(A_N) < L(A_{I,r_J}) + L(A_J).$$

Once nested, the time spans in T can only become more constrained, so that $L(D_N) \leq L(D_{I,r_J})$. The shift corrections for the nested pattern can be written as

$$E_N = E_{I,1} \oplus \langle E_J[1] \rangle \oplus E_{I,2} \oplus \langle E_J[2] \rangle \dots \langle E_J[r_J - 1] \rangle \oplus E_{I,r_J},$$

so that

$$L(E_N) = L(E_J) + \sum_{i \in [1, r_J]} L(E_{I,i}) .$$

For the remaining elements, we have

$$L(R_N) = L(R_{I,r_J}) + L(R_J)$$
$$L(p_{0N}) = L(p_{0J})$$
$$L(\tau_N) = L(\tau_J)$$

Hence, the following holds for the code length of the nested pattern P_N when compared to the code length for the inner patterns $P_{I,i}$ and the outer pattern P_J :

$$L(P_N) < L(P_J) + L(A_{I,r_J}) + L(R_{I,r_J}) + L(D_{I,r_J}) + \sum_{i \in [1,r_J]} L(E_{I,i}).$$

We can then compare the code length of the outer pattern to the code length of the structure of all but one of the inner patterns P_J , that is

$$\begin{split} &L(P_J) < (r_J - 1) \cdot L((T_I, \tau_1, \langle \rangle)) \\ \implies &L(P_J) + L(A_{I,r_J}) + L(R_{I,r_J}) + L(D_{I,r_J}) + \sum_{i \in [1,r_J]} L(E_{I,i}) \\ &< (r_J - 1) \cdot L((T_I, \tau_1, \langle \rangle)) + L(A_{I,r_J}) + L(R_{I,r_J}) + L(D_{I,r_J}) + \sum_{i \in [1,r_J]} L(E_{I,i}) \\ \implies &L(P_N) = L(\{P_N\}, cover(\mathcal{C}_I)) < \sum_{i \in [1,r_J]} (T_{I,i}, \tau_{I,i}, E_{I,i}) = L(\mathcal{C}_I, cover(\mathcal{C}_I)) \;. \end{split}$$

Horizontal combination: Concatenating cycles. Again, let us first consider a practical example. Imagine that the following sequence is part of the input:

$$S_3 = \langle (2,b), (5,a), (7,c), (13,b), (18,a), (21,c), (26,b), (30,a), (31,c) \rangle .$$

We can represent this sequence with single cycles of length 3 and period 13, over events b, a, and c and with starting points 2, 5, and 7, respectively. The cycle over a corresponds to pattern tree T_2 from Fig. A.7, the other two cycles correspond to similar pattern trees but over event b and c, so we denote them respectively as T_{2b} and T_{2c} . This corresponds to the following collection:

$$\mathcal{C}_5 = \{ P_{5,1}, P_{5,2}, P_{5,3} \}$$

= { ($T_{2b}, 2, \langle -2, 0 \rangle$), ($T_2, 5, \langle 0, -1 \rangle$), ($T_{2c}, 7, \langle 1, -3 \rangle$) }

We can also use a more complex pattern tree, concatenating the three events. This corresponds to using pattern tree T_5 from Fig. A.8:

$$\begin{aligned} \mathcal{C}_6 &= \{ P_{6,1} \} \\ &= \{ (T_5, 2, \langle 0, 1, -2, 2, 2, 0, 1, 0 \rangle) \} \; . \end{aligned}$$

Let us look at the code lengths for these different patterns. For this example, we have

$$\begin{aligned} t_{\text{start}}(S_3) &= 0, \, t_{\text{end}}(S_3) = 34, \, \varDelta(S_3) = 34, \\ \text{and} \quad \left| S_3^{(a)} \right| &= \left| S_3^{(b)} \right| = \left| S_3^{(c)} \right| = 3. \end{aligned}$$

We list the code lengths for the different elements in Tables A.5–A.6. In Fig. A.11 we provide a timeline schema of the occurrences of $P_{6,1}$ as well as of the occurrences of $(T_5, 0, \mathbf{0})$.

Given a collection of patterns that occur close to one another and share similar periods, we might want to combine them together into a concatenated pattern by merging the roots of their respective trees. We refer to such a combination as *horizontal combination*, since it produces patterns of greater width than the original ones.

To understand what this means in terms of cost, we focus on the basic case where we have two patterns P_I and P_J , such that $T_I = \{r = r, p = p_I\}(T)$ and $T_J = \{r = r, p = p_J\}(T')$, both patterns have top-level blocks of the same length r, and with starting points $\tau_I \leq \tau_J$. We compare the cost of these two patterns to the code length for the pattern that concatenates them, that is, pattern P_N with $T_N = \{r = r, p = p_N\}(T - d_N - T')$ covering the same event occurrences in the original sequence. ℓ and ℓ' denote the number of occurrence in one repetition of the top-level block of patterns P_I and P_J respectively, that is $|occs^*(T)| = \ell$ and $|occs^*(T')| = \ell'$. This situation is illustrated in Fig. A.13.

Since the shift corrections are applied relatively within a block, concatenating T and T' only impacts the first event occurrence of each repetition of the top-level block in either pattern, i.e. the left-most leaf in T and in T'. We must look at the timestamps of occurrences of the first event in T and in T', let's denote the timestamp of the i^{th} occurrence of these events as $t(o_{i,1})$ and $t(o'_{i,1})$ respectively.

Looking at the position at which these occurrences are produced by the different patterns, we have

$$E_I(o_{i,1}) = E_I[(i-1)\ell] \qquad E_J(o'_{i,1}) = E_J[(i-1)\ell'] E_N(o_{i,1}) = E_N[(i-1)(\ell+\ell')] \qquad E_N(o'_{i,1}) = E_N[i\ell+(i-1)\ell'] .$$

Per (T_I, τ_I, E_I) we have

$$t(o_{1,1}) = \tau_I$$
, (1)

$$t(o_{2,1}) = \tau_I + p_I + E_I(o_{2,1}) , \qquad (2)$$

$$t(o_{3,1}) = \tau_I + 2p_I + E_I(o_{3,1}) + E_I(o_{2,1}) , \qquad (3)$$

and per (T_N, τ_N, E_N)

$$t(o_{1,1}) = \tau_N$$
, (4)

$$t(o_{2,1}) = \tau_N + p_N + E_N(o_{2,1}) , \qquad (5)$$

$$t(o_{3,1}) = \tau_N + 2p_N + E_N(o_{3,1}) + E_N(o_{2,1}) .$$
(6)

Hence, from eq. 1 and eq. 4 we get

$$\tau_N = \tau_I.$$

And generalising from eq. 2 and eq. 5 we get

$$E_N(o_{i,1}) = (p_I - p_N) + E_I(o_{i,1}).$$

And therefore, we let $p_N = p_I$ so that $E_N(o_{i,j}) = E_I(o_{i,j})$ for all event occurrences of P_I .

Furthermore, we have per (T_J, τ_J, E_J)

$$t(o_{1,1}') = \tau_J , (7)$$

$$t(o'_{2,1}) = \tau_J + p_J + E_J(o'_{i,2}) , \qquad (8)$$

$$t(o'_{3,1}) = \tau_J + 2p_J + E_J(o'_{i,3}) + E_J(o'_{i,2}) , \qquad (9)$$

and per (T_N, τ_N, E_N)

$$t(o'_{1,1}) = \tau_N + d_N + E_N(o'_{1,1}), \qquad (10)$$

$$t(o'_{2,1}) = t(o_{2,1}) + d_N + E_N(o'_{2,1}), \qquad (11)$$

$$t(o'_{3,1}) = t(o_{3,1}) + d_N + E_N(o'_{3,1}).$$
(12)

Hence, from eq. 7 and eq. 10 we get

$$d_N = (\tau_J - \tau_I) - E_N(o'_{1,1})$$
.

and therefore we let $d_N = (\tau_J - \tau_I)$. From eq. 8 and eq. 11 we get

$$\tau_J + p_J + E_J(o'_{2,1})$$

= $\tau_N + p_N + E_N(o_{2,1}) + d_N + E_N(o'_{2,1}) ,$
= $\tau_N + p_N + E_N(o_{2,1}) + (\tau_J - \tau_I) - E_N(o'_{1,1}) + E_N(o'_{2,1}) ,$

and hence

$$(p_J - p_N) + E_J(o'_{2,1}) = E_N(o'_{2,1}) - E_N(o'_{1,1}) + E_N(o_{2,1})$$

More generally, we have

$$(p_J - p_N) + E_J(o'_{i,1}) = E_N(o'_{i,1}) - E_N(o'_{(i-1),1}) + E_N(o_{i,1}) + E_N(o_{i,1})$$

and using $p_N = p_I$ and $E_N(o_{i,1}) = E_I(o_{i,1})$:

$$E_N(o'_{i,1}) = (p_J - p_I) + E_I(o_{i,1}) - E_J(o'_{i,1}) + E_N(o'_{(i-1),1}) .$$

In the best case, the patterns are well aligned, in the sense that $E_I(o_{i,1}) = E_J(o'_{i,1})$, so then, summing up the shift corrections above, which are the only ones that differ between the old patterns and the new one, we get

$$\sum_{i \in [1,r-1]} \left| E_N(o'_{i,1}) \right| = \frac{r(r-1)}{2} \left| p_J - p_I \right| \; .$$

We use this as a filter for patterns to concatenate requiring that

$$\sum_{i \in [1, r-1]} \left| E_N(o'_{i,1}) \right| \le \sum_{i \in [1, r-1]} \left| E_J(o'_{i,1}) \right| ,$$

i.e.

$$|p_J - p_I| \le \frac{2}{r(r-1)} \sum_{i \in [1,r-1]} |E_J(o'_{i,1})|$$
.

This can be interpreted as requiring that the difference in period between the two concatenated patterns does not produce shift corrections larger than in the original patterns.

GROWHORIZONTALLY is the procedure which takes as input a collection of patterns and returns the pattern obtained by concatenating them together in order of

27

Algorithm 2 Mining periodic patterns that compress.

Require: A multi-event sequence S, a number k of top candidates to keep **Ensure:** A collection of patterns \mathcal{P} 1: $\mathcal{I} \leftarrow \text{EXTRACTCYCLES}(S, k)$ 2: $\mathcal{C} \leftarrow \emptyset; \mathcal{V} \leftarrow \mathcal{I}; \mathcal{H} \leftarrow \mathcal{I}$ 3: while $\mathcal{H} \neq \emptyset$ or $\mathcal{V} \neq \emptyset$ do 4: $\mathcal{V}' \leftarrow \text{COMBINEVERTICALLY}(\mathcal{H}, \mathcal{P}, S, k)$ 5: $\mathcal{H}' \leftarrow \text{COMBINEHORIZONTALLY}(\mathcal{V}, \mathcal{P}, S, k)$ 6: $\mathcal{C} \leftarrow \mathcal{C} \cup \mathcal{H} \cup \mathcal{V}; \mathcal{V} \leftarrow \mathcal{V}'; \mathcal{H} \leftarrow \mathcal{H}'$ 7: $\mathcal{P} \leftarrow \text{GREEDYCOVER}(\mathcal{C}, S)$ 8: return \mathcal{P}

increasing starting points as depicted in Fig. 4(a). More specifically, let the input collection be $\{P_i\}$, where each pattern is a cycle of length r_i and period p_i over a pattern tree T_i (possibly a single event) with starting point τ_i , and assume that the patterns in the collection are indexed in order of increasing starting points, i.e. in the order in which they occur in the data. The resulting pattern tree T_N is a cycle of length $r_N = \min(r_i)$ and period $p_N = p_1$ over the concatenation of T_1, T_2, \ldots , where the distance between T_{i-1} and T_i is set to $d_i = \tau_i - \tau_{i-1}$, and with $\tau_N = \tau_1$.

6 Algorithm for Mining Periodic Patterns that Compress

We are now ready to present our main algorithm for mining a collection of periodic patterns that compresses the input sequence. As outlined in Algorithm 2, our proposed algorithm consists of three stages: (i) extracting cycles (line 1), (ii) building tree patterns from cycles (lines 2–6) and (iii) selecting the final pattern collection (line 7). We now present each stage in turn.

Extracting cycles. The first stage of the algorithm consists in extracting cycles (line 1). The algorithm used for the initial mining of cycles is given as Algorithm 3. Considering each event in turn, we use two different routines to mine cycles from the sequence of timestamps obtained by restricting the input sequence to the event of interest, combine and filter their outputs to generate the set \mathcal{I} of initial candidate patterns.

The first routine, EXTRACTCYCLESDP (line 4), uses dynamic programming. Indeed, if we allow neither gaps in the cycles nor overlaps between them, finding the best set of cycles for a given sequence corresponds to finding an optimal segmentation of the sequence, and since our cost is additive over individual cycles, we can use dynamic programming to solve it optimally [1].

The second routine, EXTRACTCYCLESTRI (line 5), extracts cycles using a heuristic which allows for gaps and overlappings. It collects triples (t_0, t_1, t_2) such that $||t_2 - t_1| - |t_1 - t_0|| \leq \ell$, where ℓ is set so that the triple can be beneficial when used to construct longer cycles. Triples are then chained into longer cycles. A triple (t_{-1}, t_0, t_{+1}) , can be seen as an elementary cycle with a single shift correction $e = |(t_0 - t_{-1}) - (t_{+1} - t_0)|$. Since we are looking for triples that could produce cost-effective cycles, we only keep triples for which $e < \log(\Delta(S) + 1) - 2$, following

Algorithm 3 EXTRACTCYCLES: Mines simple cycles from the data sequence.

Require: A sequence S **Ensure:** A collection of cycles C1: $C \leftarrow \emptyset$ 2: $l_{\max} \leftarrow \log(\Delta(S) + 1) - 2$ 3: **for each** event $\alpha \in \omega$ **do** 4: $C \leftarrow C \cup \text{EXTRACTCYCLESDP}(S^{(\alpha)})$ 5: $C \leftarrow C \cup \text{EXTRACTCYCLESTRI}(S^{(\alpha)}, l_{\max})$ 6: FILTERCANDIDATES(C, S, k)7: **return** C

Lemma 1. Triples (t_{-1}, t_0, t_{+1}) and (t'_{-1}, t'_0, t'_{+1}) are chained together if $t_0 = t'_{-1}$ and $t_{+1} = t'_0$, producing $(t_{-1}, t_0, t_{+1}, t'_{+1})$, and so on.

Finally, the set C of cost-effective cycles obtained by merging the output of the two routines is filtered with FILTERCANDIDATES, to keep only the k most efficient patterns for each occurrence (line 6) for a user-specified k, and returned.

Building tree patterns from cycles. The second stage of the algorithm builds tree patterns, starting from the cycles produced in the previous stage. That is, while there are new candidate patterns, the algorithm performs combination rounds, trying to generate more complex patterns through vertical and horizontal combinations. If desired, this stage can be skipped, thereby restricting the pattern language to simple cycles.

In a round of vertical combinations performed by COMBINEVERTICALLY (line 4), each distinct pattern tree represented among the new candidates in \mathcal{H} is considered in turn. Patterns over that tree are collected and EXTRACTCYCLESTRI is used to mine cycles from the corresponding sequence of starting points. This time, the threshold used to mine the cycles is derived from the cost of the considered pattern tree, in accordance with Lemma 2. For each obtained cycle, a nested pattern is produced by combining the corresponding candidates using GROWVERTICALLY (see Fig. 4(b)). The set of candidates produced through these vertical combinations is filtered, and returned as \mathcal{V}' . The procedure COMBINEVERTICALLY for generating candidate patterns by means of vertical combinations is shown in Algorithms 4.

In a round of horizontal combinations performed by COMBINEHORIZONTALLY (line 5), pairs of candidates such that (i) at least one of the two patterns was produced in the previous round, and (ii) their starting points are closer than the period of the earliest occurring of the two patterns are considered for concatenation. A graph G is constructed, with vertices representing candidate patterns and with edges connecting pairs of candidates $\mathcal{K} = \{P_I, P_J\}$ for which the concatenated pattern $P_N = \text{GROWHORIZONTALLY}(\mathcal{K})$ satisfies $L(\{P_N\}, cover(\mathcal{K})) < L(\mathcal{K}, cover(\mathcal{K}))$. A new pattern is then produced for each clique of G, by applying GROWHORIZONTALLY to the corresponding set of candidate patterns. The set \mathcal{H}' of new patterns is then filtered and returned. The procedure COMBINEHORIZONTALLY for generating candidate patterns by means of horizontal combinations is shown in Algorithms 5.

To limit the number of concatenations generated and evaluated when testing pairs of patterns, we require that the periods of two patterns be similar enough not

Algorithm 4 COMBINEVERTICALLY: Combine patterns vertically.

Require: A collection of new candidate patterns \mathcal{H} , and other candidate patterns \mathcal{C} , a sequence S, a number k of top candidates to keep

Ensure: A collection of patterns resulting from vertical combinations \mathcal{V}' 1: $\mathcal{V}' \leftarrow \emptyset$

2: for each distinct $T_c \in \mathcal{H}$ do

3: $\mathcal{C} \leftarrow \{(T_x, \tau_x, E_x) \in \mathcal{H} \cup \mathcal{C}, \text{ such that } T_x = T_c\}$

4: $l_{\max} \leftarrow L((T_1, \tau_1, \langle \rangle))$

5: for each cycle $(r, p, O) \in \text{EXTRACTCYCLESTRI}(\{\tau_x \in \mathcal{C}\}, l_{\max})$ do

- 6: $\mathcal{K} \leftarrow \{(T_y, \tau_y, E_y) \in \mathcal{C}, \text{ such that } \tau_y \in O\}$
- 7: $K \leftarrow \text{GROWVERTICALLY}(\mathcal{K})$
- 8: **if** $L({K}, cover(\mathcal{K})) < L(\mathcal{K}, cover(\mathcal{K}))$ **then**

9:
$$\mathcal{V}' \leftarrow \mathcal{V}' \cup \{K\}$$

- 10: $\mathcal{V}' \leftarrow \text{FILTERCANDIDATES}(\mathcal{V}', S, k)$
- 11: return \mathcal{V}'

Algorithm 5 COMBINEHORIZONTALLY: Combine patterns horizontally.

Require: A collection of new candidate patterns \mathcal{V} , and other candidate patterns \mathcal{C} , a sequence S, a number k of top candidates to keep

Ensure: A collection of patterns resulting from horizontal combinations \mathcal{H}'

1: $\mathcal{H}' \leftarrow \emptyset; G \leftarrow \emptyset$ 2: $\mathcal{C} \leftarrow$ pattern pairs $(P_a, P_b) \in (\mathcal{V} \cup \mathcal{C})^2$, such that $(P_a \in \mathcal{V} \text{ or } P_b \in \mathcal{V})$ and $\tau_b \leq \tau_a + p_{0a}$ 3: for each pair of patterns $\mathcal{K} = (P_a, P_b) \in \mathcal{C}$ do 4: $K \leftarrow \text{GrowHORIZONTALLY}(\mathcal{K})$ 5: if $L(\{K\}, cover(\mathcal{K})) < L(\mathcal{K}, cover(\mathcal{K}))$ then 6: $\mathcal{H}' \leftarrow \mathcal{H}' \cup \{K\}$ 7: $G \leftarrow G \cup \{(a, b)\}$ 8: $\mathcal{H}' \leftarrow \mathcal{H}' \cup \{\text{GRowHORIZONTALLY}(\mathcal{K}) \text{ for each clique } \mathcal{K} \text{ in the graph } G\}$ 9: $\mathcal{H}' \leftarrow \text{FILTERCANDIDATES}(\mathcal{H}', S, k)$

```
10: return \mathcal{H}'
```

to produce shift corrections larger than in the patterns of the pair, as discussed in Section 5.

Note that if we obtain, as a result from a horizontal combination, a pattern a the following shape

$$\{r = r_0, p = p_0\}(\{r = r_1, p = p_1\}(T_a) - d - \{r = r_1, p = p_1\}(T_b))$$

we will factorise it into

$$\{r = r_0, p = p_0\} (\{r = r_1, p = p_1\} (T_a - d - T_b)),$$

if it results in shorter code length, as is often the case.

Selecting the final pattern collection. Selecting the final set of patterns to output among the candidates in C is very similar to solving a weighted set cover problem. Each candidate pattern can be seen as a set containing the occurrences it covers and associated to a weight representing its code length. A singleton set

is associated to each occurrence whose weight is the cost of encoding that occurrence as a residual. Therefore, the selection is done using a simple variant of the greedy algorithm for this problem, denoted as GREEDYCOVER (line 7), that works as follows. Initially, the set \mathcal{P} of selected patterns is empty. Let \mathcal{O} be the set of event occurrences covered so far, also initially empty. In each round, the pattern P with smallest value of $L(P)/|occs(P) \setminus \mathcal{O}|$ among remaining candidates, i.e. the most efficient when considering only uncovered occurrences, is selected. If P is costeffective for the remaining uncovered occurrences, it is added to \mathcal{P} , \mathcal{O} is updated and the selection proceeds to the next round. Otherwise the selection stops and \mathcal{P} is returned.

7 Experiments

In this section, we evaluate the ability of our algorithm to find patterns that compress the input event sequences. We make the code and the prepared datasets publicly available.⁵ To the best of our knowledge, no existing algorithm carries out an equivalent task and we are therefore unable to perform a comparative evaluation against competitors. To better understand the behaviour of our algorithm, we first performed experiments on synthetic sequences. We then applied our algorithm to real-world sequences including process execution traces, smartphone applications activity, and life-tracking. We evaluate our algorithm's ability to compress the input sequences and present some examples of extracted patterns.

For a given event sequence, the main objective of our algorithm is to mine and select a good collection of periodic patterns, in the sense that the collection should allow to compress the input sequence as much as possible. Therefore, the main measure that we consider in our experiments is the *compression ratio*, defined as the ratio between the length of the code representing the input sequence with the considered collection of patterns and the length of the code representing the input sequence with an empty collection of patterns, i.e. using only individual event occurrences, given as a percentage. For a given sequence S and collection of patterns C the compression ratio is defined as

$$\%L = 100 \cdot L(\mathcal{C}, S) / L(\emptyset, S)$$

with smaller values associated to better pattern collections.

7.1 Mining synthetic sequences

We begin by probing the behaviour of our algorithm on synthetic sequences containing planted periodic patterns.

First we generate sequences that contain a single pattern. Each pattern consists of a basis of one to three events, repeated in a cycle, in two nested cycles or in three nested cycles, that is building pattern trees of depth 1, 2 and 3 respectively. The simplest basis consists of event a, with the period of the inner cycle being either greater than five (specifically, in [5,9]) or greater than 10 (specifically, in [10,24]).

⁵ https://github.com/nurblageij/periodic-patterns-mdl

31

To build more complex patterns, we use event a followed by event b at distance 4, i.e. (a - 4 - b), as well as event a followed by event c at distance 1, followed by event d at distance 2, i.e. (a - 1 - c - 2 - d).

Each resulting perfect synthetic sequence can then be perturbed with *shift noise*, i.e. by displacing the occurrences by a few time steps either forward or backward, or with *additive noise*, i.e. by adding sporadic occurrences. Displacement noise is parameterised, on one hand, by the maximum absolute shift by which the occurrences might be displaced and, on the other hand, by the fraction of occurrences that are displaced. We refer to these two parameters as the *level* and the *density* of the noise, respectively. For additive noise, we insert occurrences of event *a* at random timestamps. This type of noise has a single parameter, *density*, fixing the number of of sporadic occurrences as compared to the number of occurrences of the event in the unperturbed sequence. The generated sequences contain from about fifty up to over two thousand occurrences.

In each round, we mine each generated sequence in turn for periodic patterns, check whether the planted pattern was recovered exactly and compare the length of the code for encoding the perturbed sequence using either the planted pattern, denoted as L_H , or those that have been selected by the algorithm, denoted as L_F . The first round of experiments is run on sequences with only shift noise. The second and third rounds of experiments are run on sequences with additive noise of density 0.1 and density 0.5 respectively. The fourth round is run on sequences with only shift noise, but letting the occurrences of the planted pattern interleave, unlike in the three previous rounds.

In Fig. A.14–A.17, we plot the compression ratio achieved by the planted pattern versus the compression ratio achieved by the pattern collection selected by the algorithm for each of the twenty sequences generated with each considered combination of parameters, for the four rounds respectively. A different take on the same results is presented in Fig. A.18–A.21, where we show the distribution of $\% L_F - \% L_H$ among the twenty sequences generated with each combination of parameters as boxplots, for the four rounds respectively. A value of $\% L_F - \% L_H = 0$ means that the patterns selected by our algorithm achieve the same compression as the planted patterns, while positive (resp. negative) values of $\% L_F - \% L_H$ correspond to selected patterns achieving longer (resp. shorter) code length than with planted patterns. On the left next to each boxplot, we indicate the number of sequences for which the planted pattern was recovered exactly.

Next, we consider sequences containing multiple planted patterns. For this purpose, we consider the pool of sequences generated in each of the four rounds with single patterns above and generate new sequences by selecting between two and five sequences from the pool and combining them together. The patterns can be combined either with or without overlap, that is, either letting a sequence start before or after the preceding sequence ends. The results for the runs over these synthetic sequences containing multiple planted patterns are presented in Fig. A.23.

We see from Fig. A.18 that when no spurious occurrences are inserted the planted pattern is recovered exactly in most cases for simple patterns of depth one, while the performance deteriorates and fewer planted patterns are recovered for more complex patterns and greater depths, as also visible from Fig. A.14. This is expected since

recovering multi-event patterns requires that the corresponding cycles are properly recovered in the first stage of the algorithm for each of the events that make up the pattern. Even in the absence of noise, the algorithm might miss the planted pattern, e.g. because it merges successive nested repetition of a cycle that appear close to each other. When the sequences involve interleaving (Fig. A.17 and A.21) the algorithm behaves in a similar way, except for the more complex basis with depths two and three, which are expectedly impacted more strongly by interleaving, resulting in more degraded performances.

Spurious occurrences break the planted patterns which are no longer recovered by the algorithm. With low density of additive noise the algorithm often selects patterns very similar to the planted one but covering also the spurious occurrences, using shift corrections to accommodate them (Fig. A.19). This is typical of the dynamic programming cycle mining, which is able to find cycles with many repetitions but does not allow to skip any occurrence, which are thus incorporated at the cost of increased corrections. When the density of noise becomes fairly large, the inserted occurrences might actually generate new patterns that can result in shorter code length than the planted pattern, as can be observed in Fig. A.20. Indeed, except for the patterns over single event a with long periods, the difference in compression ratios is negative in the majority of cases.

When several planted patterns are combined without overlap, the algorithm is able to recover them all exactly in roughly half of the cases for patterns taken from pools with no additive noise, with or without interleaving (44 and 51%, respectively, see Fig. A.23). In most cases the patterns selected by the algorithm yield a longer code length than the planted patterns, except in the presence of dense additive noise.

Note that the requirement that the planted pattern(s) should be recovered exactly is very strict, as it means that the pattern(s) selected by the algorithm should cover the exact same occurrences as the planted ones, with the exact same pattern tree. Closer inspection of the results reveals that the algorithm is able to recover large fragments of the planted patterns in most cases. More specifically, in cases where it fails to recover planted patterns with height greater than one, the algorithm is in general able to identify cycles that constitute large fragments of different repetitions of the inner cycle of the pattern, but merely omitting a few occurrences in these fragment prevents the algorithm from combining them into vertical patterns of greater height. Designing a procedure that is able to build on the extracted fragments from different repetitions to recover the omitted occurrences could make the retrieval of this type of patterns more robust, but is clearly not trivial.

7.2 Mining real-world sequences

Next, we apply our algorithm to real-world datasets.

Datasets. Our first two datasets come from a collaboration with STMicroelectronics and are execution traces of a set-top box based on the STiH418 SoC⁶ running STLinux. Both traces are a log of system actions (interruptions, context switches and

⁶ STiH418 description: http://www.st.com/resource/en/data_brief/stih314.pdf

33

system calls) taken by the KPTrace instrumentation system developed at STMicroelectronics. The **3zap** dataset corresponds to 3 successive changes of channel ("zap"), while the **bugzilla** dataset corresponds to logging a display blackout bug into the bug tracking system of ST. Each dataset contains two traces, one for either of the two cores of the box, named respectively **3zap-0** and **3zap-1**, on one hand, **bugzilla-0** and **bugzilla-1**, on the other hand. For our analysis of these traces, we do not consider timestamps, only the succession of events.

The ubiqLog dataset was obtained from the UCI Machine learning repository.⁷ It contains traces collected from the smartphones of users over the course of two months. For each of 31 users (we excluded those whose data was not encoded using Hindu-Arabic numerals), we obtain a sequence recording what applications are run on that user's smartphone. We either consider absolute timestamps with a granularity of one minute or only the succession of events, and denote the corresponding collections of sequences respectively as ubiqLog-abs and ubiqLog-rel.

The samba dataset consists of a single sequence recording the emails identifying the authors of commits on the git repository of the samba network file system⁸ from 1996 to 2016. We consider timestamps with a granularity of one day. User commits are instantaneous. We aggregated together users that appeared fewer than 10 times as "other".

The sacha dataset contains records from the quantified awesome life log⁹ recording the daily activities of its author between November 2011 and January 2017. The daily activities are associated to start and end timestamps, and are divided between categories organised into a hierarchy. Categories with fewer than 200 occurrences were aggregated to their parent category. Each resulting category is represented by an event. Adjacent occurrences of the same event were merged together. We either consider absolute timestamps with a granularity of one minute or only the succession of events, and denote the corresponding sequences respectively as sacha-abs and sacha-rel. Further, we investigate what happens when we coarsen the time granularity, from the original one minute to 15 minutes, 30 minutes, 1 hour, half a day and a full day. The corresponding sequences are denoted sacha-abs-G15, sacha-abs-G30, sacha-abs-G60, sacha-abs-G720 and sacha-abs-G1440, respectively.

When considering absolute timestamps for occurrences involving non-instant processes (e.g. daily activities, running applications), each process might be associated with three different events representing its start, its end, and the process happening for a duration smaller than the time granularity respectively. When considering only the succession of events or, in other words, focusing on the order in which things happen rather than the specific times, we only consider the starting time of the process and each process is hence associated with only one event.

Tables A.7–A.10 present the statistics of the sequences used in our experiments. We indicate the length (|S|) and duration $(\Delta(S))$ of each sequence, the size of its alphabet $(|\Omega|)$, as well as the median and maximum length of the event subsequences $(|S^{(\alpha)}|)$. We also indicate the code length of the sequence when encoded with an

⁷ https://archive.ics.uci.edu/ml/datasets/UbiqLog+(smartphone+lifelogging)

⁸ https://git.samba.org/

⁹ http://quantifiedawesome.com/records

empty collection of patterns $(L(\emptyset, S))$, as well as the running time of the algorithm (RT, in seconds) for mining and selecting the patterns, as well as for the first stage of mining cycles for each separate event.

Measures. Beside the code length and the compression ratio achieved with the selected pattern collections, we consider several other characteristics. For a given pattern collection \mathcal{C} , we denote the set of residuals $residual(\mathcal{C}, S)$ simply as \mathcal{R} and look at what fraction of the code length is spent on them, denoted as $L : \mathcal{R} = \sum_{o \in \mathcal{R}} L(o)/L(\mathcal{C}, S)$. Note that when the pattern collection is empty $L:\mathcal{R} = 1$, since only residuals are used, and hence the code length results entirely from residuals. $|\mathcal{R}|$ and $|\mathcal{C}|$ are the number of residuals (individual event occurrences) and the number of patterns in the collection, respectively. We also look at the number of patterns of different types in \mathcal{C} , specifically, (i) simple cycles, i.e. patterns with both width and height equal to 1, (ii) vertical patterns, having a width of 1 and a height strictly greater than 1, (iii) horizontal patterns, having a height of 1 and a width strictly greater than 1. Finally, we look at the fraction of patterns in \mathcal{C} that cover strictly more than three occurrences, i.e.

$$c_{>3} = \left| \left\{ P \in \mathcal{C}, \left| cover(P) \right| > 3 \right\} \right| / \left| \mathcal{C} \right| \ ,$$

where cover(P) denotes the set of timestamp–event pairs covered by a pattern P, and the median and maximum cover size of patterns in C.

Results. To better understand the role of the pattern combinations, in addition to looking at the final collection of patterns returned by the algorithm (denoted as C_F), we also consider intermediate collections of patterns, namely a collection selected among simple cycles mined during the initial phase of the algorithm (denoted as C_S), a collection selected among simple cycles and patterns resulting from the first round of horizontal combinations (denoted as C_H), from the first round of vertical combinations (denoted as C_V) and from both, or in other words among the candidate patterns obtain at the end of the first round of combinations (denoted as C_{V+H}).

Table A.11 shows the results for application trace log sequences 3zap-0, 3zap-1, bugzilla-0, bugzilla-1 and samba. Table A.12 shows the results for sacha sequences when considering timestamps with different time granularities, as well as when considering only the event succession. Tables A.13–A.17 show the results for the sequences from the ubiqLog-abs dataset, while tables A.18–A.22 show the results for the sequences from the ubiqLog-rel dataset.

For each sequence and pattern collection we indicate the compression ratio (%L), the code length $(L_{\mathcal{C}})$, the fraction of code used for residual $(L:\mathcal{R})$, the number of residuals $(|\mathcal{R}|)$ and of patterns $(|\mathcal{C}|)$, the number of simple, vertical, horizontal and two-dimensional patterns (s, v, h, and m, respectively), the fraction of patterns covering more than three occurrences $(c_{>3})$ as well as the median (c^{M}) and the maximum (c^+) cover size of patterns in the collection.

Table 1 shows aggregated results for the ubiqLog-abs and ubiqLog-rel datasets, where we indicate the range of values taken for the different sequences in each subset. Fig. A.24–A.27 show the compression ratios achieved for sequences from the different datasets.

0.18, 85.52 0.17, 85.52 0.08, 84.33 0.08, 84.33 0.06, 84.33 6.05, 64.94	[0.23, [0.24, [0.24, [0.24, [0.24, [0.24,	0.60] 0.60] 0.60] 0.60] 0.60] ut	[41, [41, [31, [31, [31, [31, [31, [31, [31, [3	9468 9445 3113 3107	5]/[0, 5]/ [0 7]/ [0 2]/ [0	, 0] / , 57]/ , 0] / , 4] / , 2] /	' [0, 1] (5, 22) (5,	0] / 256]/ 252]/	$\begin{bmatrix} 0, \ 0 \\ 0, \ 0 \\ 0, \ 0 \end{bmatrix}$	$\begin{bmatrix} 17, \\ 17, 2 \\ 17, 2 \\ 17, 2 \end{bmatrix}$	388] 2328] 2328]
$\begin{array}{c} 0.17, 85.52 \\ 0.08, 84.33 \\ 0.08, 84.33 \\ 0.06, 84.33 \\ \hline \\ 6.05, 64.94 \end{array}$	[0.23, [0.24, [0.24, [0.24, [0.24, [0.24,	0.60] 0.60] 0.60] 0.60] ut	[41, [31, [31, [31,	9445 3113 3107 3102	5]/[0, 5]/ [0 7]/ [0 2]/ [0	57]/ , 0] / , 4] / , 2] /	' [0, 1] (5, 22) (5,	0] / 256]/ 252]/	$\begin{bmatrix} 0, \ 0 \\ 0, \ 0 \\ 0, \ 0 \end{bmatrix}$	$\begin{bmatrix} 17, \\ 17, 2 \\ 17, 2 \\ 17, 2 \end{bmatrix}$	388] 2328] 2328]
$0.08, 84.33 \\ 0.08, 84.33 \\ 0.06, 84.33 \\ \hline \\ 6.05, 64.94 \\ \hline$	3] [0.24, 3] [0.24, 5] [0.24, 5] [0.24,	0.60] 0.60] 0.60] ut	[31, [31, [31,	3113 3107 3102	B]/ [0 7]/ [0 8]/ [0	, 0] / (, 4] / (, 2] / ('[5, 22]'[5, 22]	256]/ 252]/	[0, 0]	$\begin{bmatrix} 17, 2\\ 17, 2\\ 17, 2 \end{bmatrix}$	2328] 2328]
$0.08, 84.33 \\ 0.06, 84.33 \\ \hline 6.05, 64.94 \\ \hline$	b] [0.24, b] [0.24, b] [0.24,	0.60] 0.60] ut	[31, [31,	$3107 \\ 3102$	7]/ [0 2]/ [0	, 4] / , 2] /	'[5, 22]	252]/	[0, 0]	[17, 2]	2328]
0.06, 84.33 6.05, 64.94	6] [0.24,	0.60] ut	[31,	3102	2]/[0	, 2] /					
6.05, 64.94	<u> </u>	ub	. ,				'[5, 22]	233]/	[0, 1]	[17, 2]	2328]
	[] [0.12,		piqL	og-r	el (;	91)					
	[] [0.12,			0	(51)					
	1 [0 12	-	-				-		-		-
		-		-					L /	J L /	1
• · · · ·		60		•		, o	· • ·	1200	-	, ,	0
Ă		40		Å	•		<u>م</u>		-	? •	O
	, in	30-		ç	• 🛃	Δ.	A 3	600	_	^	_
	1 1	20		8			×	400	-		
2	1	10-					*				Å Å
100000 1500 S]	0 0	1000			0000 4	0000	0		5000 10000 S	15000
102	1.03		- 04					45	-		90
	5.91, 63.48 5.91, 63.48 5.91, 63.48	5.91, 63.48] [0.12, 5.91, 63.48] [0.12, 5.91, 63.48] [0.12, 5.91, 63.48] [0.12, 0.12,00,00,000,000,000,000,000,000,000,000	5.91, 63.48 [0.12, 0.41] 5.91, 63.48 [0.12, 0.41] 5.91, 63.48 [0.12, 0.41] $\begin{array}{c} & & & & \\ & & & & \\ & & & & \\ & & & & $	5.91, 63.48 [0.12, 0.41] [9, 2 5.91, 63.48 [0.12, 0.41] [9, 2] [9,	5.91, 63.48 [0.12, 0.41] [9, 2083 5.91, 63.48 [0.12, 0.41] [9, 2083 5.91, 63.48 [0.12, 0.41] [9, 2083 5.91, 63.48 [0.12, 0.41] [9, 2083 600 1000000 1500000 1000000 1500000 1000000 200 1000000 200	5.91, 63.48 [0.12, 0.41] [9, 2083] / [0 5.91, 63.48 [0.12, 0.41] [9, 2083] / [0 5.91, 63.48 [0.12, 0.41] [9, 2083] / [0 600000000000000000000000000000000000	$5.91, 63.48 \ [0.12, 0.41] \ [9, 2083] / [0, 0] / 5.91, 63.48 \ [0.12, 0.41] \ [9, 2083] / [0, 2] / 5.91, 63.48 \ [0.12, 0.41] \ [9, 2083] / [0, 2] / 5.91, 63.48 \ [0.12, 0.41] \ [9, 2083] / [0, 2] / \begin{bmatrix} & & & & & & & \\ & & & & & & \\ & & & & $	$5.91, 63.48 \ [0.12, 0.41] \ [9, 2083] / [0, 0] / [0, 3]$ $5.91, 63.48 \ [0.12, 0.41] \ [9, 2083] / [0, 2] / [0, 3]$ $5.91, 63.48 \ [0.12, 0.41] \ [9, 2083] / [0, 2] / [0, 3]$ $60 \ 10000 \ 1500000 \ 1500000 \ 1500000 \ 15000\ 1500000 \ 1500000 \ 15000\ 1500000 \ 150000 \ 150000\$	$5.91, 63.48 \ [0.12, 0.41] \ [9, 2083] / [0, 0] / [0, 339] / 5.91, 63.48 \ [0.12, 0.41] \ [9, 2083] / [0, 2] / [0, 334] / 5.91, 63.48 \ [0.12, 0.41] \ [9, 2083] / [0, 2] / [0, 334] / 5.91, 63.48 \ [0.12, 0.41] \ [9, 2083] / [0, 2] / [0, 334] / \begin{bmatrix} & & & & & & & & & & & & & & & & & & &$	$5.91, 63.48 \begin{bmatrix} 0.12, 0.41 \end{bmatrix} \begin{bmatrix} 9, 2083 \end{bmatrix} / \begin{bmatrix} 0, 0 \end{bmatrix} / \begin{bmatrix} 0, 339 \end{bmatrix} / \begin{bmatrix} 0, 0 \end{bmatrix} \\ 5.91, 63.48 \begin{bmatrix} 0.12, 0.41 \end{bmatrix} \begin{bmatrix} 9, 2083 \end{bmatrix} / \begin{bmatrix} 0, 2 \end{bmatrix} / \begin{bmatrix} 0, 334 \end{bmatrix} / \begin{bmatrix} 0, 0 \end{bmatrix} \\ 5.91, 63.48 \begin{bmatrix} 0.12, 0.41 \end{bmatrix} \begin{bmatrix} 9, 2083 \end{bmatrix} / \begin{bmatrix} 0, 2 \end{bmatrix} / \begin{bmatrix} 0, 334 \end{bmatrix} / \begin{bmatrix} 0, 0 \end{bmatrix} \\ 5.91, 63.48 \begin{bmatrix} 0.12, 0.41 \end{bmatrix} \begin{bmatrix} 9, 2083 \end{bmatrix} / \begin{bmatrix} 0, 2 \end{bmatrix} / \begin{bmatrix} 0, 334 \end{bmatrix} / \begin{bmatrix} 0, 1 \end{bmatrix} \\ 0 \end{bmatrix} $	$5.91, 63.48 \begin{bmatrix} 0.12, 0.41 \end{bmatrix} \begin{bmatrix} 9, 2083 \end{bmatrix} / \begin{bmatrix} 0, 0 \end{bmatrix} / \begin{bmatrix} 0, 339 \end{bmatrix} / \begin{bmatrix} 0, 0 \end{bmatrix} \begin{bmatrix} 158, 355 \\ 5.91, 63.48 \end{bmatrix} \begin{bmatrix} 0.12, 0.41 \end{bmatrix} \begin{bmatrix} 9, 2083 \end{bmatrix} / \begin{bmatrix} 0, 2 \end{bmatrix} / \begin{bmatrix} 0, 334 \end{bmatrix} / \begin{bmatrix} 0, 0 \end{bmatrix} \begin{bmatrix} 158, 355 \\ 5.91, 63.48 \end{bmatrix} \begin{bmatrix} 0.12, 0.41 \end{bmatrix} \begin{bmatrix} 9, 2083 \end{bmatrix} / \begin{bmatrix} 0, 2 \end{bmatrix} / \begin{bmatrix} 0, 334 \end{bmatrix} / \begin{bmatrix} 0, 1 \end{bmatrix} \begin{bmatrix} 158, 355 \\ 15, 91, 63.48 \end{bmatrix} \begin{bmatrix} 0.12, 0.41 \end{bmatrix} \begin{bmatrix} 9, 2083 \end{bmatrix} / \begin{bmatrix} 0, 2 \end{bmatrix} / \begin{bmatrix} 0, 334 \end{bmatrix} / \begin{bmatrix} 0, 1 \end{bmatrix} \begin{bmatrix} 158, 355 \\ 10, 20 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 &$

Table 1. Aggregated results for ubiqLog sequences.

Fig. 5. Running times for sequences from the different datasets, in hours (left) and zoomedin in minutes (middle) and seconds (right).

We see that the algorithm is able to find sets of patterns that compress the input event sequences. The compression ratio varies widely depending on the considered sequence, from a modest 84% for some sequences from ubiqLog-abs to a reduction of more than two thirds, for instance for samba. To an extent, the achieved compression can be interpreted as an indicator of how much periodic structure is present in the sequence (at least of the type that can be exploited by our proposed encoding and detected by our algorithm). In some cases, as with samba, the compression is achieved almost exclusively with simple cycles, but in many cases the final selection contains a large fraction of horizontal patterns (sometimes even about two thirds), which bring a noticeable improvement in the compression ratio (as can be seen in Fig. A.26, for instance). Vertical patterns, on the other hand, are much more rare, and proper two-dimensional patterns, and even more so the 3zap sequences. This agrees with the intuition that recursive periodic structure is more likely to be found in execution logs tracing multiple recurrent automated processes.

In most cases, a large proportion of the selected patterns cover more than the minimum three timestamp–event pairs. Some of the largest patterns cover several hundreds or a few thousand occurrences, depending on the length of the input sequence, obviously, as well as the strength of its periodic structure). Obviously, the more occurrences a pattern covers, the more efficient it is, assuming it can be represented concisely.

From Table A.12 we can see that the chosen time granularity has a strong impact on the extracted patterns. With the finest time granularity, i.e. 1 minute time step (sacha-abs-G1), few patterns are found because the activities need to reoccur with minute regularity and any deviation must be accounted in the shift corrections. Therefore periodic patterns are not very efficient and only little compression is achieved. When increasing the time granularity to 15 minutes, 30 minutes and to 1 hour (respectively sacha-abs-G15, sacha-abs-G30 and sacha-abs-G60) allows to be more forgiving of small deviations the exact times when activities happen, resulting in more efficient patterns found. This is evidenced by a sharp decrease in the fraction of simple cycles $(s/|\mathcal{C}|)$ and increase in the fraction of patterns covering more than three occurrences $(c_{>3})$ and the maximum cover size (c^+) . Further coarsening the time granularity, to a half day and a full day (sacha-abs-G720 and sacha-abs-G1440) the fraction of simple cycles among the selected pattern increases again, but this time each one covers a large number of occurrences. At such level of granularity, the time and order in which the activities are carried out during the day no longer matter, only which activities are performed on any given day. Finally, with type of data considering the succession of activities rather than absolute timestamps (sacha-rel) might allow to identify fairly different patterns, since activities in a pattern are no longer separated by a time span but by the number of other activities performed in between. However, in this context, this can result in patterns that are difficult to understand, since they cannot be easily mapped back to time points and hence calendar dates and hours of the days cannot be used when interpreting the patterns. Hence, the choice of using succession or absolute timestamps, and, in the latter case, of choosing the granularity of the time step, has to be made by the analyst in consideration of the context and the time scale that is of interest.

In some cases (e.g. bugzilla-0 in Table A.11, sacha-abs-G60 in Table A.12 and several ubiqLog sequences), the collection of patterns selected from the final set of candidates, C_F , achieves worse compression than collections selected from intermediate sets of candidates, despite the fact that the intermediate candidate sets are subsets of the final one. This is due to the fact that the pattern selection, which is in essence a weighted set cover problem is solved greedily (see Section 6), and a local decision of choosing a more efficient pattern produced in later combination rounds, might eventually result in degraded compression. However, the degradation is fairly limited and one might simply decide to replace the final solution by an intermediate one, when the candidates produced later on do not appear to contribute to shortening the code length.

Fig. 5 shows the running times for sequences from the different datasets. Circles and squares, coloured according to achieved compression ratio, indicate the running time of the algorithm for sequences from the ubiqLog dataset and from other datasets, respectively. Each such marker is connected to a triangle indicating the

37

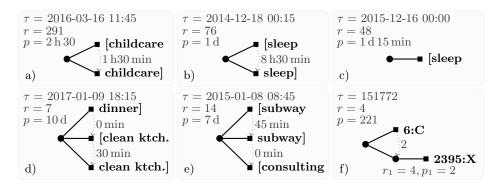


Fig. 6. Example patterns from sacha-abs-G15 (a-e) and 3zap-0 (f).

running time for the combination rounds. Larger triangles correspond to sequences for which more simple cycles are extracted during the initialisation phase. Darker triangles correspond to sequences for which the maximum cover size among these simple cycles is larger. The running times vary greatly, from only a few seconds to several hours. Naturally, mining longer sequences tends to require longer running times. However, directly observable characteristics of the sequence, such as its size, the size of its alphabet, relative frequencies of the events, etc. are not the only factors impacting the running time. The number and length of the cycles extracted in the first stage have a major effect on the time required by the combination rounds, i.e. the second stage, which take the bulk of the overall running time. Indeed, if the initial candidates contain many long cycles, many more tests will be needed when trying to combine them into more complex patterns.

Example patterns. Finally, we present some examples of patterns obtained from the sacha-abs-G15 and 3zap-0 sequences, in Fig. 6. The start and end of an activity A are denoted as "[A" and "A]" respectively. The patterns from the sacha-abs-G15 sequence are simple and rather obvious, but they make sense when considering everyday activities. The fact that we are able to find them is a clear sign that the method is working. The 3zap-0 pattern is a typical system case: the repetition of a context switch (6:C) followed by several activations of a process (2395:X). Further examples can be found in Tables A.23 and A.24. In 3zap-0 patterns, event names consist of a numerical part, indicating the process id, and one or two letter indicating the action. Upper and lower case letters represent the start and end of an action, respectively. The most common actions are interruption (I), context switch (C), system call (X), user function call (U).

Most of the discovered patterns are fairly simple. We suspect that this is due to the nature of the data: there are no significantly complex patterns in these event log sequences. In any case, the expressivity of our proposed pattern language comes at no detriment to the simpler, more common patterns, but brings the potential benefit of identifying sequences containing exceptionally regular structure. 38 Galbrun *et al.*

8 Conclusion

In this paper, we propose a novel approach for mining periodic patterns with a MDL criterion, and an algorithm to put it into practise. Through our experimental evaluation, we show that we are able to extract sets of patterns that compress the input event sequences and to identify meaningful patterns.

An analyst parsing a log might have some intuition about what periods are more meaningful, as well as relations and dependencies between events, depending on the generating process. For instance, we expect days and weeks to strongly structure life tracking logs, while patterns with periods of, say, 21 hours or 17 days would be considered less intuitive. How to take such prior knowledge into account is an interesting question to explore.

Making the algorithm more robust to noise and making it more scalable using for instance parallelisation, are some pragmatic directions for future work, as is adding a visualisation tool to support the analysis and interpretation of the extracted patterns in the context of the event log sequence.

Acknowledgements. The authors thank Hiroki Arimura and Jilles Vreeken for valuable discussions. This work has been supported by Grenoble Alpes Metropole through the Nano2017 Itrami project, by the QCM-BioChem project (CNRS Mastodons) and by the Academy of Finland projects "Nestor" (286211) and "Agra" (313927).

References

- 1. R. Bellman. On the approximation of curves by line segments using dynamic programming. *Communications of the ACM*, 4(6), 1961.
- C. Berberidis, I. P. Vlahavas, W. G. Aref, M. J. Atallah, and A. K. Elmagarmid. On the discovery of weak periodicities in large time series. In *PKDD'02*, pages 51–61, 2002.
- A. Bhattacharyya and J. Vreeken. Efficiently summarising event sequences with rich interleaving patterns. In SDM'17, pages 795–803. SIAM, 2017.
- F. Bonchi, M. van Leeuwen, and A. Ukkonen. Characterizing uncertain data using compression. In SDM'11, pages 534–545. SIAM, 2011.
- L. De Raedt and A. Zimmermann. Constraint-based pattern set mining. In SDM'07, pages 237–248. SIAM, 2007.
- 6. E. Galbrun, P. Cellier, N. Tatti, A. Termier, and B. Crémilleux. Mining periodic patterns with a MDL criterion. In *ECML-PKDD'18*, 2018.
- P. Grünwald. Model selection based on minimum description length. Journal of Mathematical Psychology, 44(1):133–152, 2000.
- 8. P. Grünwald. The Minimum Description Length Principle. MIT Press, 2007.
- J. Han, G. Dong, and Y. Yin. Efficient mining of partial periodic patterns in time series database. In *ICDE'99*, pages 106–115, 1999.
- J. Han, W. Gong, and Y. Yin. Mining segment-wise periodic patterns in time-related databases. In *KDD*'98, pages 214–218, 1998.
- 11. E. O. Heierman, III and D. J. Cook. Improving home automation by discovering regularly occurring device usage patterns. In *ICDM'03*, pages 537–540, 2003.
- J. Kiernan and E. Terzi. Constructing comprehensive summaries of large event sequences. ACM Trans. Knowl. Discov. Data, 3(4):21:1–21:31, 2009.
- H. T. Lam, F. Moerchen, D. Fradkin, and T. Calders. Mining compressing sequential patterns. In SDM'12, pages 319–330. SIAM, 2012.

- Z. Li, J. Wang, and J. Han. Mining event periodicity from incomplete observations. In KDD'12, pages 444–452. ACM, 2012.
- P. Lopez-Cueva, A. Bertaux, A. Termier, J.-F. Méhaut, and M. Santana. Debugging embedded multimedia application traces through periodic pattern mining. In *Int. Conf.* on Embedded Software, EMSOFT'12, 2012.
- S. Ma and J. L. Hellerstein. Mining partially periodic event patterns with unknown periods. In *ICDE'01*, pages 205–214. IEEE Computer Society, 2001.
- B. Özden, S. Ramaswamy, and A. Silberschatz. Cyclic association rules. In *ICDE'98*, pages 412–421. IEEE Computer Society, 1998.
- 18. J. Rissanen. Modeling by shortest data description. Automatica, 14(5):465-471, 1978.
- K. Smets and J. Vreeken. Slim: Directly mining descriptive patterns. In SDM'12, pages 236–247. SIAM, 2012.
- 20. N. Tatti and J. Vreeken. The long and the short of it: Summarising event sequences with serial episodes. In *KDD'12*, pages 462–470. ACM, 2012.
- J. Vreeken, M. van Leeuwen, and A. Siebes. Krimp: Mining itemsets that compress. Data Min Knowl Discov, 23(1):169–214, 2011.
- Q. Yuan, W. Zhang, C. Zhang, X. Geng, G. Cong, and J. Han. Pred: Periodic region detection for mobility modeling of social media users. In WSDM'17, pages 263–272. ACM, 2017.

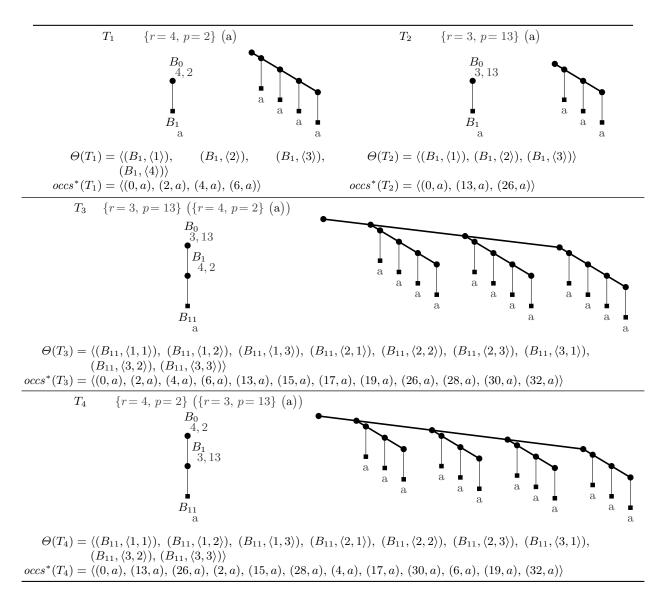


Fig. A.7. Pattern trees T_1-T_4 : Pattern and expansion trees, lists of leaf nodes and of perfect occurrences.

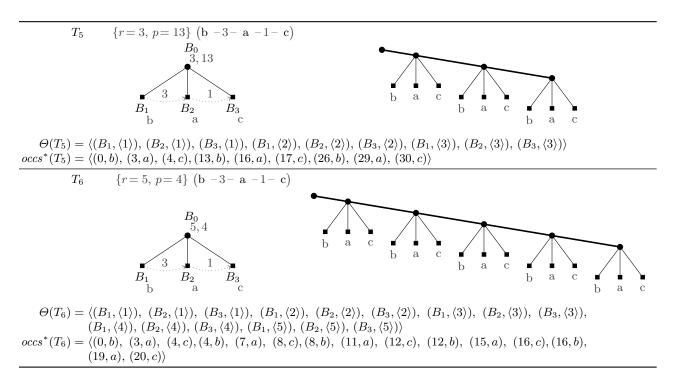


Fig. A.8. Pattern trees T_5-T_6 : Pattern and expansion trees, lists of leaf nodes and of perfect occurrences.

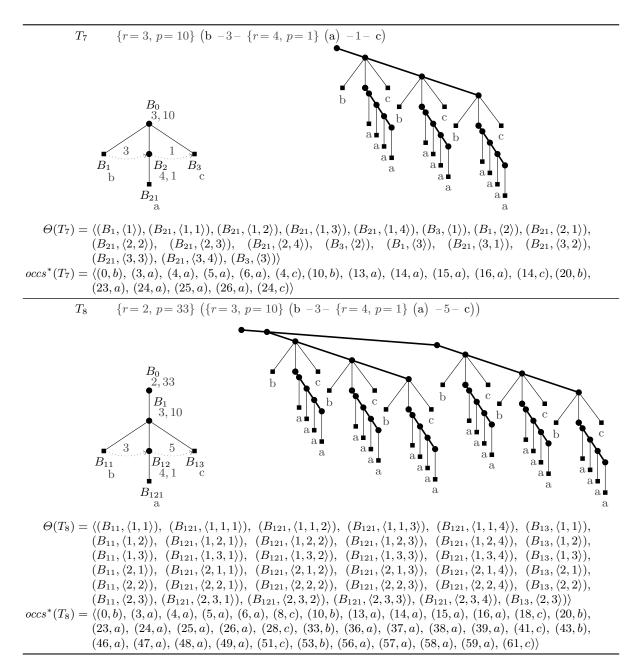


Fig. A.9. Pattern trees T_7-T_8 : Pattern and expansion trees, lists of leaf nodes and of perfect occurrences.

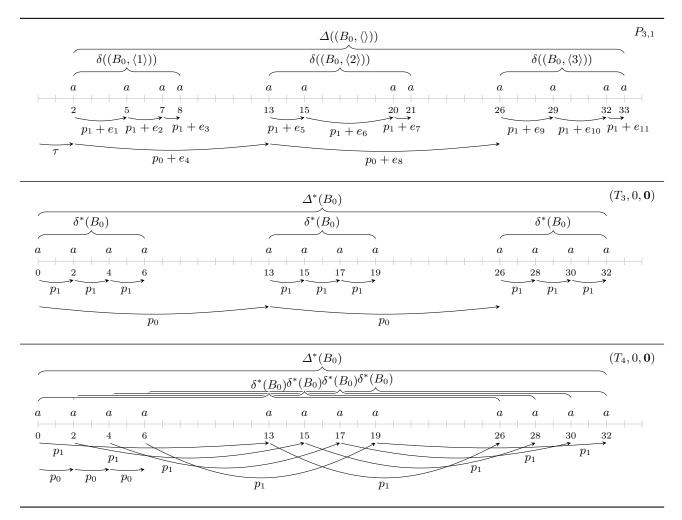


Fig. A.10. Patterns $P_{3,1}$, $(T_3, 0, 0)$ and $(T_4, 0, 0)$ shown on timelines.

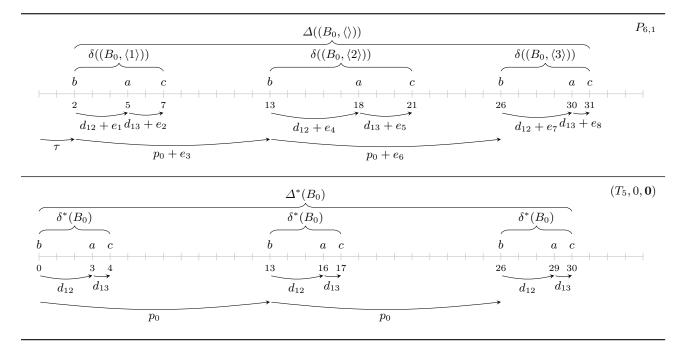


Fig. A.11. Pattern $P_{6,1}$ and $(T_5, 0, 0)$ shown on timeline.

(\mathcal{C}_1								76.681
-	1	P _{1,1}	24.657	I	P _{1,2}	26.417	-	$P_{1,3}$	25.607
A	(a)		4.755	(a)		4.755	(a)		4.755
E	(1, 0, -1)	\rangle	8.000	(0, 3, -1)	>	10.000	(1, 1, -1)	\rangle	9.000
r_0	4	$\log(12)$	= 3.585	4	$\log(12)$) = 3.585	4	$\log(12)$	2) = 3.585
p_0	2	$\log(11)$	= 3.459	2	$\log(10)$) = 3.322	2	$\log(1)$	1) = 3.459
au	2	$\log(29)$	= 4.858	13	$\log(27)$) = 4.755	26	$\log(2)$	(8) = 4.807

Table A.2. Code lengths for the example pattern collection $\mathcal{C}_1.$

Table A.3. Code lengths for the example pattern collection $\mathcal{C}_2.$

(22											87.437
_		$P_{2,1}$	21.969		$P_{2,2}$	23.969		$P_{2,3}$	20.749		$P_{2,4}$	20.749
A	(a)		4.755	(a)		4.755	(a)		4.755	(a)		4.755
E	(-2, 0)	\rangle	6.000	$\langle -3, 1 \rangle$		8.000	$\langle 0, -1 \rangle$	•	5.000	$\langle 0, -1 \rangle$	>	5.000
r_0	3	$\log(12)$	= 3.585	3	log(12) =	= 3.585	3	$\log(12)$	= 3.585	3	$\log(12) =$	3.585
p_0	13	$\log(18)$	= 4.170	13	log(18) =	= 4.170	13	$\log(17)$	= 4.087	13	$\log(17) =$	4.087
au	2	$\log(11)$	= 3.459	5	log(11) =	= 3.459	7	$\log(10)$	= 3.322	8	log(10) =	3.322

	\mathcal{C}_3	59.724	\mathcal{C}_4	63.920
-		$P_{3,1}$ 59.724	1	P _{4,1} 63.920
A	((a))	7.925	((a))	7.925
E	$\langle 1, 0,$.> 33.000	$\langle -2, 0,$.) 32.000
r_0	3	$\log(12) = 3.585$	4	$\log(12) = 3.585$
r_1	4	$\log(12) = 3.585$	3	$\log(12) = 3.585$
p_0	13	$\log(18) = 4.170$	2	$\log(11) = 3.459$
au	2	$\log(11) = 3.459$	2	$\log(29) = 4.858$
δ^*	6	$\log(8) = 3.000$	26	$\log(28) = 4.807$
p_1	2	$\log(2) = 1.000$	13	$\log(13) = 3.700$

Table A.4. Code lengths for the example pattern collections \mathcal{C}_3 and $\mathcal{C}_4.$

	C_5								65.443
-		$P_{5,1}$	21.554		$P_{5,2}$	20.334	Ĺ	$P_{5,3}$	23.554
A	(b)		6.340	(a)		6.340	(c)		6.340
E	$\langle -2, 0 \rangle$	\rangle	6.000	$\langle 0, -1 \rangle$)	5.000	$\langle 1, -3 \rangle$		8.000
r_0	3	$\log(3)$	= 1.585	3	$\log(3) =$	1.585	3	$\log(3) =$	1.585
p_0	13	$\log(18)$	= 4.170	13	$\log(17) =$	4.087	13	$\log(18) =$	4.170
au	2	$\log(11)$	= 3.459	5	$\log(10) =$	3.322	7	$\log(11) =$	3.459

Table A.5. Code lengths for the example pattern collection C_5 .

	\mathcal{C}_6	53.538
-		$P_{6,1}$ 53.538
A	(b a c)) 12.680
E	$\langle 0, 1, \ldots$.) 24.000
r_0	3	$\log(3) = 1.585$
p_0	13	$\log(18) = 4.170$
au	2	$\log(11) = 3.459$
d_{12}	3	$\log(4) = 2.000$
d_{13}	1	$\log(4) = 2.000$
δ^*	4	$\log(8) = 3.000$

Table A.6. Code lengths for the example pattern collection \mathcal{C}_6 .

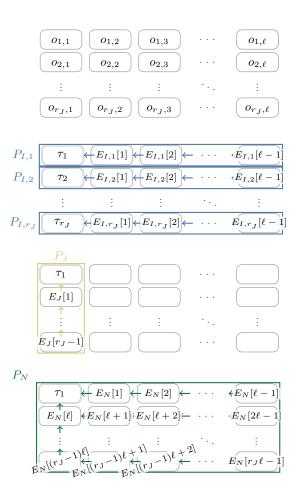
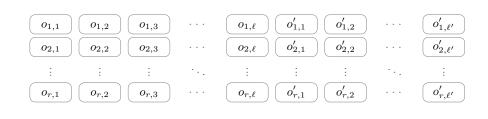


Fig. A.12. Vertical combination: Combining patterns $P_{I,1}, \ldots, P_{I,\tau_J}$ into nested pattern P_N . Rounded rectangles represent event occurrences. Each colored rectangle represents a pattern and encloses the occurrence covered by the pattern. Arrows link occurrences to the preceding occurrences relative to which their timestamp is computed.



P_I	P_J	
$\tau_I \leftarrow E_I[1] \leftarrow E_I[2] \leftarrow \cdots$	$\leftarrow E_I[\ell-1] \qquad \qquad$	$\epsilon E_J[\ell'-1]$
$E_{I}[\ell] \leftarrow E_{I}[\ell+1] \leftarrow E_{I}[\ell+2] \leftarrow \cdots$	$\underbrace{E_I[2\ell-1]} \underbrace{E_J[\ell']} \underbrace{E_J[\ell'+1]} \cdots$	$E_J[2\ell'-1]$
		:
$E_{1} \underbrace{ \begin{bmatrix} (r-1)^{\ell} \\ E_{1} \end{bmatrix} }_{E_{1}} \underbrace{ \begin{bmatrix} r-1)^{\ell} \\ E_{1} \end{bmatrix} }_{E_{1}} \underbrace{ \begin{bmatrix} r-1 \\ $	$ \underbrace{ E_I[r\ell-1]}_{E_J} \underbrace{ \begin{bmatrix} r-1 \\ \ell \end{bmatrix}}_{L_J} \underbrace{ \begin{bmatrix} r-1 \\ \ell \end{bmatrix}}_{L_J}$	$E_J[r\ell'-1]$

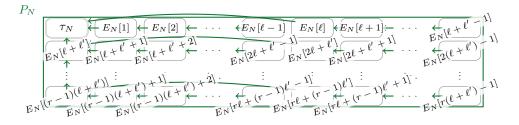


Fig. A.13. Horizontal combination: Concatenating patterns P_I and P_J into new pattern P_N . Rounded rectangles represent event occurrences. Each colored rectangle represents a pattern and encloses the occurrence covered by the pattern. Arrows link occurrences to the preceding occurrences relative to which their timestamp is computed.

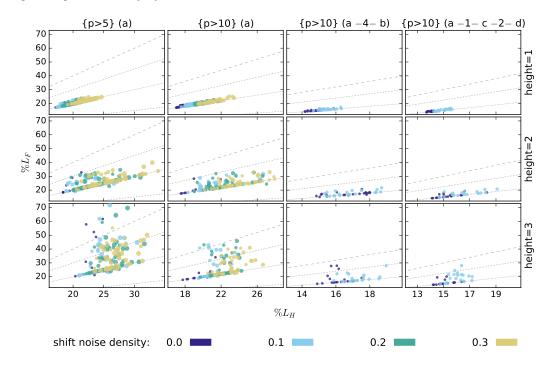


Fig. A.14. Compression ratios for planted and extracted pattern collections ($\% L_H$ and $\% L_F$, respectively) on synthetic sequences perturbed only by shift noise.

Fig. A.15. Compression ratios for planted and extracted pattern collections ($\%L_H$ and $\%L_F$, respectively) on synthetic sequences perturbed by additive noise (a, 0.1).

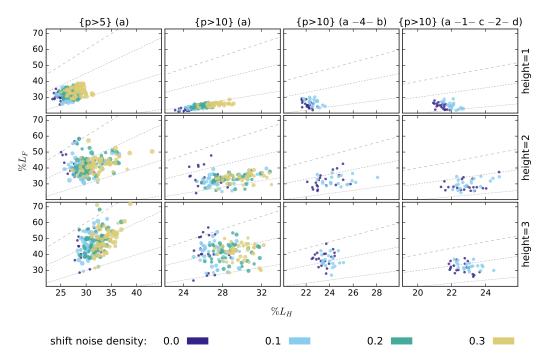


Fig. A.16. Compression ratios for planted and extracted pattern collections ($\% L_H$ and $\% L_F$, respectively) on synthetic sequences perturbed by additive noise (a, 0.5).

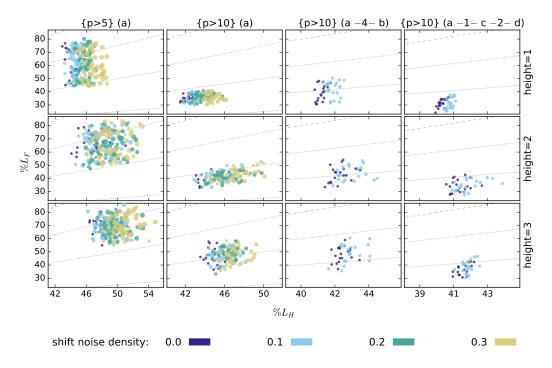
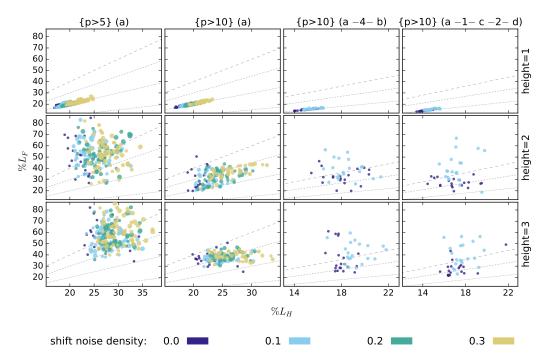


Fig. A.17. Compression ratios for planted and extracted pattern collections ($\% L_H$ and $\% L_F$, respectively) on synthetic sequences containing interleaving.



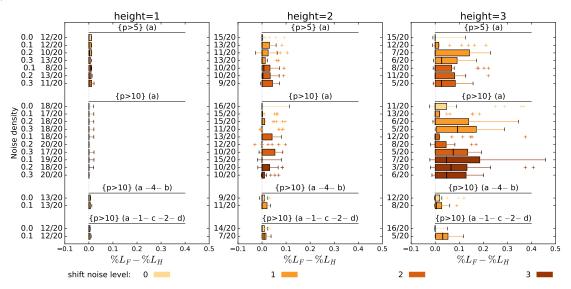
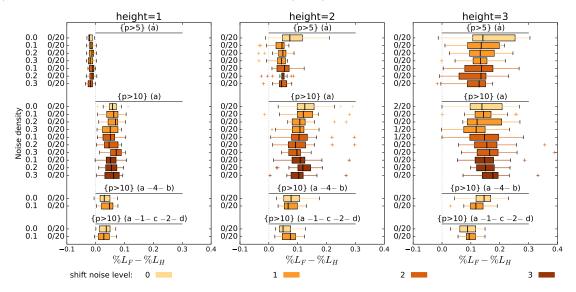


Fig. A.18. Differences in compression ratios for planted and extracted pattern collections ($\%L_H$ and $\%L_F$, respectively) on synthetic sequences perturbed only by shift noise.

Fig. A.19. Differences in compression ratios for planted and extracted pattern collections ($\%L_H$ and $\%L_F$, respectively) on synthetic sequences perturbed by additive noise (a, 0.1).



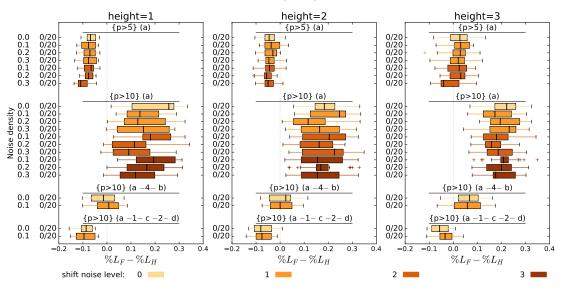
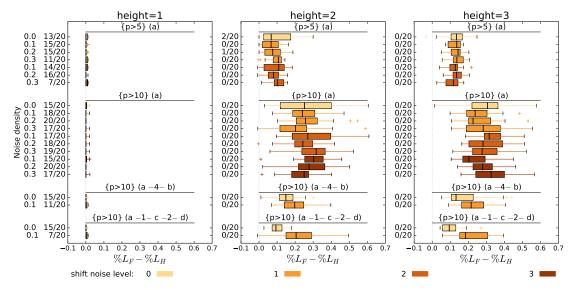


Fig. A.20. Differences in compression ratios for planted and extracted pattern collections ($\%L_H$ and $\%L_F$, respectively) on synthetic sequences perturbed by additive noise (a, 0.5).

Fig. A.21. Differences in compression ratios for planted and extracted pattern collections ($\%L_H$ and $\%L_F$, respectively) on synthetic sequences containing interleaving.



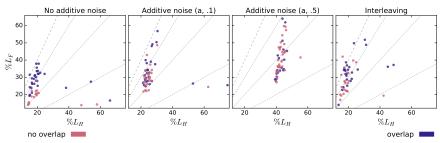
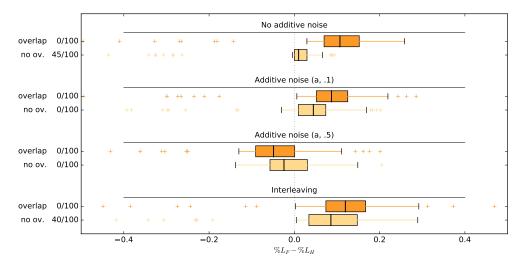


Fig. A.22. Compression ratios for planted and extracted pattern collections ($\% L_H$ and $\% L_F$, respectively) on synthetic sequences with multiple planted patterns.

Fig. A.23. Differences in compression ratios for planted and extracted pattern collections ($\%L_H$ and $\%L_F$, respectively) on synthetic sequences with multiple planted patterns.



	S	$\Delta(S)$	$ \Omega $	S	$S^{(\alpha)}$	$L(\emptyset, S)$	RI	r (s)
				med	max		cycles	overall
3zap-0	181644	181643	443	22	36697	4154277	2094	35048
3zap-1	129532	129531	214	57	29376	2849285	1697	32125
bugzilla-0	16775	16774	91	6	3332	303352	112	522
bugzilla-1	15418	15417	61	24	3551	276298	116	504
samba	28751	7461	119	44	2905	520443	214	2787

 Table A.7. Statistics for application log trace sequences.

 Table A.8. Statistics for sacha sequences.

	S	$\Delta(S)$	$ \Omega $	$S^{(\alpha)}$	$L(\emptyset,S)$	RТ	' (s)
				med max		cycles	overall
sacha-abs-G1	72516	3321680	94	523 5531	1987678	700	8734
sacha-abs-G15	65977	221445	141	$231 \ \ 4389$	1573140	2963	14377
sacha-abs-G30	58447	110722	141	$254 \ \ 3033$	1343757	980	9125
sacha-abs-G60	47880	55361	141	$154 \ 4270$	1045284	598	5310
sacha-abs-G720	26380	4613	69	$174 \ 3547$	450453	212	2287
sacha-abs-G1440	22261	2306	55	$306 \ 2287$	359005	153	1533
sacha-rel	36258	36257	47	523 5531	721270	373	22252

	S	$\Delta(S)$	$ \Omega $	S	$S^{(\alpha)}$	$L(\emptyset, S)$	R	Г (s)
				med	max		cycles	overa
25-F	413	11391	10	23	211	6599	1	
10-M	1290	21116	17	28	194	23110	14	1
9-M	1483	29499	25	26	365	27362	6	1
21-F	5506	92897	85	38	479	121869	33	5
23-F	8262	7389217	50	63	699	227374	68	15
14-F	9682	100684	49	47	1818	199781	76	15
1-M	10529	42774	89	32	1215	214629	92	24
30-F	11567	42653	69	67	1136	234214	98	31
13-F	13328	78490	73	42	2183	279248	122	33
20-M	15449	98993	118	39	2159	339895	168	42
29-F	16460	88050	80	41	1093	352049	152	102
8-M	17648	83771	59	52	1719	365481	251	99
6-M	17652	87591	132	26	1425	382647	156	60
15-F	19782	44587	80	38	1344	400786	170	71
2-F	19842	546993	107	54	1079	485405	172	81
26-F	23859	68872	61	48	2240	486633	276	86
12-M	24138	79753	103	80	1153	528938	202	135
17-F	25024	80935	79	58	2351	524069	510	175
27-F	25034	79156	156	48	919	560681	232	196
5-F	26215	17900307	100	65	2629	769981	254	146
16-F	28809	90751	75	52	2131	611065	439	278
11-F	35579	92393	86	94	2240	766086	511	329
31-F	40564	1596375	92	49	3975	1027350	355	138
28-F	42832	103996	111	79	4967	934544	423	354
7-F	43657	81890	87	83	4707	916811	521	267
35-F	57443	184647	122	129	4389	1309032	638	608
22-M	59374	70461	121	58	2547	1281232	1564	1826
24-F	73921	72563	136	41	4567	1566364	1372	939
33-F	83954	94870	160	68	4047	1845429	2297	1673
19-F	113885	164231	175	59	5361	2572126	2098	1698
18-F	167863	90623	241	68	6101	3733349	1812	2897

 Table A.9. Statistics for ubiqLog-abs sequences.

	S	$\Delta(S)$	$ \Omega $	2	$S^{(\alpha)}$	$L(\emptyset,S)$	RI	Г (s)
				med	max		cycles	overall
25-F	372	371	6	33	211	3896	1	3
10-M	905	904	7	101	334	10899	7	15
9-M	973	972	18	25	365	12693	5	8
21-F	4234	4233	51	51	806	71351	29	49
23-F	5274	5273	23	83	991	83287	36	160
14-F	6676	6675	29	29	2670	104860	48	103
1-M	8686	8685	56	34	1635	148938	56	417
30-F	8983	8982	40	65	1759	151815	57	937
20-M	10891	10890	64	58	3234	193871	73	597
13-F	11029	11028	46	69	2742	190835	84	856
6-M	11425	11424	62	45	1797	201859	84	339
29-F	11974	11973	43	31	1655	206150	90	665
8-M	12463	12462	31	65	3375	207949	98	1918
15-F	13680	13679	46	36	2163	235866	97	1273
2-F	13907	13906	49	80	1891	249331	113	1102
26-F	13995	13994	27	68	2975	235133	99	1954
17-F	15790	15789	38	78	3415	273659	114	3171
27-F	18406	18405	109	56	1289	361470	143	493
12-M	18807	18806	69	73	1600	356307	148	402
5-F	21185	21184	67	83	4781	400094	207	638
16-F	21417	21416	49	34	3610	382984	156	1990
31-F	25032	25031	57	45	5332	457534	183	2795
11-F	26512	26511	49	108	4160	496154	223	1055
35-F	38794	38793	52	129	6842	743513	294	15822
28-F	39162	39161	79	99	5002	782479	373	1714
7-F	39216	39215	54	136	5551	752947	381	1388
22-M	44533	44532	61	58	3872	870531	364	11686
24-F	51636	51635	67	48	7069	998823	447	14480
33-F	62824	62823	95	80	6620	1266835	545	20740
19-F	74421	74420	101	59	8513	1500824	675	36644
18-F	103681	103680	142	55	8640	2203689	739	16320

 Table A.10. Statistics for ubiqLog-rel sequences.

	%L	$L(\mathcal{C},S)$	$L:\mathcal{R}$	$ \mathcal{R} $	$ \mathcal{C} $	s	/	v	/	h	/	m	$c_{>3}$	c^{M}	c^+
					3zap-	0									
\mathcal{C}_S	56.32	2339741	0.41	37048	11852	11852	2 /	0	/	0	/	0	0.94	5	2325
$\widetilde{\mathcal{C}_V}$	55.14	2290523	0.40		11162						1	0	0.93	5	2325
\mathcal{C}_H	47.84	1987311	0.35	26773	8371	3459	/	0	/ 4	4912	$2^{\prime}/$	0	0.97	8	2325
\mathcal{C}_{V+H}	47.40	1969139	0.34	26261	8220	3499	14	419	/ 4	1302	2/	0	0.97	8	2325
\mathcal{C}_F	46.99	1952299	0.34	25982	8012	3499	/	91	/ 4	4154	1/1	268	0.96	8	2325
					3zap-	1									
\mathcal{C}_S	54.21	1544589	0.40	25280	8604	8604	/	0	/	0	/	0	0.96	5	4653
\mathcal{C}_V	53.21	1516077	0.41	24984	7927	7471	/ 4	456	/	0	/	0	0.95	5	4653
\mathcal{C}_H	48.41	1379470	0.35	19980	6326	3492							0.97	8	4653
\mathcal{C}_{V+H}	48.10	1370402	0.36	19969	6118	3286	/:	329	12	250;	3/	0	0.98	8	4653
\mathcal{C}_F	47.49	1353263	0.36	19662	5856	3181	/	83	/2	2368	3/2	224	0.97	7	4653
				יל	ıgzill	a-0									
\mathcal{C}_S	48.58	147374	0.12	773	262	262	/			0	/	0	0.98	$\overline{7}$	1652
\mathcal{C}_V	48.56	147321	0.12	773	260	259	/	1	/	0	/	0	0.98	7	1652
\mathcal{C}_H	42.43	128712	0.12	722	203	133	/	0	/	70	/	0	0.98	9	1652
\mathcal{C}_{V+H}	42.39	128599	0.12	711	203	130	/		/	72	/	0	0.98	9	1652
\mathcal{C}_F	42.41	128656	0.13	734	197	124	/	1	/	70	/	2	0.98	9	1652
				ול	ıgzill	a-1									
\mathcal{C}_S	46.05	127230	0.16	1005	411	411	/	0	/	0	/	0	0.97	6	869
\mathcal{C}_V	45.68	126202	0.16	989	385	361	/	24	/	0		0	0.96	6	869
\mathcal{C}_H	43.56	120362	0.15	889	331	208	/	0	/	123		0	0.99	8	869
\mathcal{C}_{V+H}	43.48	120143	0.15	868	336	216	/	8	/	112	/	0	0.99	8	869
\mathcal{C}_F	43.32	119698	0.15	863	327	213	/	3	/	99	/	12	0.98	8	869
					samba	1									
\mathcal{C}_S	28.42	147889	0.14	956	429	429	/	0	/	0	/	0	0.94	10	2657
\mathcal{C}_V	28.42	147889	0.14	956	429	429	/	0	/	0	/	0	0.94	10	2657
\mathcal{C}_H	28.37	147638	0.13	937	426	409	/	0	/	17	/	0	0.95		2657
\mathcal{C}_{V+H}	28.37	147638	0.13	937	426	409	/	0	/	17	/	0	0.95	10	2657
\mathcal{C}_F	28.37	147638	0.13	937	426	409	/	0	/	17	/	0	0.95	10	2657

 Table A.11. Detailed results for application log trace sequences.

	%L	$L(\mathcal{C},S)$	$L : \mathcal{R}$	$ \mathcal{R} $	$ \mathcal{C} $	s	/	v	/ h /	m	$c_{>3}$	c^{M}	c^+
				sa	cha-ab	s-G1							
\mathcal{C}_S	86.24	1714241	0.44	27295	13134	13134	/	0	/ 0 /	0	0.37	3	10
\mathcal{C}_V	86.24	1714241	0.44	27295	13134	13134	/	0	/ 0 /	0	0.37	3	10
${\mathcal C}_H$	84.19	1673502	0.44	26754	11327	9321	/	0	/ 2006 /	0	0.47	3	28
\mathcal{C}_{V+H}	84.19	1673502	0.44	26754	11327	9321	/	0	/ 2006 /	0	0.47	3	28
\mathcal{C}_F	84.19	1673502	0.44	26754	11327	9321	/	0	/ 2006 /	0	0.47	3	28
				sac	ha-ab	s-G15							
\mathcal{C}_S	74.34	1169517	0.37	17586		9602				0	0.71	4	304
\mathcal{C}_V	74.34	1169511	0.37	17583	9602				/ 0 /		0.71	4	304
\mathcal{C}_H	68.64	1079861	0.35	15605	6953	3957	/	0	/ 2996 /	0	0.82	6	582
\mathcal{C}_{V+H}	68.64	1079861	0.35	15605	6953	3957	/	0	/ 2996 /	0	0.82	6	582
\mathcal{C}_F	68.64	1079861	0.35	15605	6953	3957	/	0	/ 2996 /	0	0.82	6	582
				sac	ha-ab	s-G30							
\mathcal{C}_S	70.42	946325	0.32	12647	7969	7969				0	0.72	4	328
\mathcal{C}_V	70.42	946328	0.32	12638	7971	7969				0	0.72	4	328
\mathcal{C}_H	64.22	862899	0.30	11085					/2561/		0.84	6	468
\mathcal{C}_{V+H}	64.22	862899	0.30	11085	5513	2952	/	0	/ 2561 /	0	0.84	6	468
\mathcal{C}_F	64.22	862899	0.30	11085	5513	2952	/	0	/2561/	0	0.84	6	468
				sac	ha-ab:	s-G60							
\mathcal{C}_S	64.61	675374	0.34	9977	5100	5100	/	0	/ 0 /	0	0.75	4	1150
\mathcal{C}_V	64.62	675488	0.34	9971	5101	5098	/	3	/ 0 /	0	0.75	4	1150
\mathcal{C}_H	60.03	627462	0.31	8477	3746	2197	/	0	/ 1549 /	0	0.88	6	1150
\mathcal{C}_{V+H}	60.03	627460	0.31	8460	3754	2207	/	3	/1544/	0	0.88	6	1150
\mathcal{C}_F	60.11	628321	0.31	8600	3746	2206	/	2	/ 1537 /	1	0.88	6	1150
				sacl	ha-abs	-G720							
\mathcal{C}_S	30.45	137162	0.14	958	384	384	/	0	/ 0 /	0	0.99	14	3540
\mathcal{C}_V	30.45	137162	0.14	958	384	384		0	/ 0 /	0	0.99	14	3540
\mathcal{C}_{H}	30.23	136169	0.12	863	382	351		0	/ 31 /	0	1.00	15	3540
\mathcal{C}_{V+H}	30.23	136169	0.12	863	382	351	/	0	/ 31 /	0	1.00	15	3540
\mathcal{C}_F	30.23	136169	0.12	863	382	351	/	0	/ 31 /	0	1.00	15	3540
				sach	ia-abs-	-G1440							
\mathcal{C}_S	24.87	89270	0.07	343	208	208	/	0	/ 0 /	0	1.00	24	2260
\mathcal{C}_V	24.87	89270	0.07	343	208	208	·			0	1.00	24	2260
${\cal C}_H$	24.85		0.07	332	209				/ 4 /		1.00		
\mathcal{C}_{V+H}	24.85			332	209	205				0	1.00		
\mathcal{C}_F	24.85			332	209				/ 4 /				2260
				s	acha-1	rel							
\mathcal{C}_S	56.31	406137	0.29	5624	2446	2446	/	0	/ 0 /	0	0.93	5	4629
$\widetilde{\mathcal{C}_V}$	56.28			5602		2442					0.93		4629
\mathcal{C}_H	55.94			5611					/ 280 /		0.93		4629
\mathcal{C}_{V+H}	55.91			5613					/ 273 /		0.93		4629
\mathcal{C}_F	55.91	403261		5616			/		/ 271 /		0.93		4629

Table A.12. Detailed results for sacha.

	07. I	$L(\mathcal{C},S)$	I.D	$ \mathcal{R} $	$ \mathcal{C} $	s	/ 01	/ h / m	c>3 c	M c
	/01/	$L(\mathbf{C}, D)$	<i>L</i> . <i>R</i>	$ \mathcal{N} $		3	/ 0	/ 11 / 111	t>3 t	L L
					25-F					
\mathcal{C}_S	85.52	5643	0.57	193	41	41	/ 0		0.88	5 1'
\mathcal{C}_V	85.52	5643	0.57	193	41		/ 0	, ,	0.88	5 1'
\mathcal{C}_H	84.33	5564	0.58	194	36		/ 0	/ /	0.86	5 1'
V+H	84.33	5564		194	36		/ 0		0.86	5 1'
\mathcal{C}_F	84.33	5564	0.58	194	36	31 ,	/ 0	/ 5 / 0	0.86	5 1
					10-M					
\mathcal{C}_S	73.17	16908	0.33	297	147	147 ,	/ 0	/ 0 / 0	0.73	5 2
\mathcal{C}_V	73.17	16908	0.33	297	147	147 ,			0.73	$5 \ 2$
\mathcal{C}_H	68.92	15927	0.37	315	118	66 ,			0.85	6 3
V+H	68.92	15927	0.37	315	118		/ 0		0.85	6 3
\mathcal{C}_F	68.92	15927	0.37	315	118	66 ,	/ 0	/ 52 / 0	0.85	6 3
					9-M					
\mathcal{C}_S	55.01	15052	0.60	445	84	84	/ 0	/ 0 / 0	0.68	5 6
\mathcal{C}_V	55.01	15052	0.60	445	84	84 ,	/ 0	/ 0 / 0	0.68	5 6
\mathcal{C}_H	52.50	14366	0.60	425	65	38 ,	/ 0		0.68	$5 \ 12$
V+H	52.50	14366	0.60	425	65	38	/ 0	/ 27 / 0	0.68	$5 \ 12$
\mathcal{C}_F	52.50	14366	0.60	425	65	38	/ 0	/ 27 / 0	0.68	$5 \ 12$
					21-F					
\mathcal{C}_S	80.87	98556	0.55	2394	707	707	/ 0	/ 0 / 0	0.43	3 8
\mathcal{C}_V	80.87	98556		2394	707	707		, ,	0.43	3 8
\mathcal{C}_H	76.80	93601	0.56	2289	545	363		, ,	0.56	4 8
V+H	76.80	93601	0.56	2289	545	363			0.56	4 8
\mathcal{C}_F	76.80	93601	0.56	2289	545			/ 182 / 0	0.56	4 8
U _F	10.00	00001	0.00	2200	23-F	000 /	, 0	/ 102 / 0	0.00	1 0
\mathcal{C}_S	55.01	127122	0.91	1957		797	/ 0	/ 0 / 0	0.67	4 8
\mathcal{C}_S \mathcal{C}_V		127122 127122		$1357 \\ 1357$	737 737	$737 \\ 737 $			0.67	4 8
									0.67	
\mathcal{C}_H	38.77	88153		981 081	$393 \\ 393$	147		, ,	0.85	9 51
V+H	38.77	88153		981 081		147			0.85	9 51
\mathcal{C}_F	38.77	88153	0.32	981	393	147	/ 0	/246/0	0.85	9 51
0	F1 00	100940	0.40	1055	14-F	001	/ 0	/ 0 / 0	0.79	4 10
\mathcal{C}_S		102348		1857	821	821		, ,	0.73	4 12
\mathcal{C}_V		102348		1857	821				0.73	4 12
\mathcal{C}_H	49.05	97984		1766				/ 175 / 0	0.75	5 24
V+H	49.05			1766				/175/0	0.75	5 24
\mathcal{C}_F	49.09	98070	0.39	1766		507	/ 0	/174/1	0.75	5 24
-					1-M		,			
\mathcal{C}_S		161808				1397,		· . · .	0.76	4 3
\mathcal{C}_V		161808						/ 0 / 0	0.76	4 3
\mathcal{C}_H		131647		2520		379 ,			0.80	8 10
V+H		131647		2520		379 ,			0.80	8 10
\mathcal{C}_F	61.34	131647	0.42	2520	820	370	/ 0	/441/0	0.80	8 10

Table A.13. Detailed results for ubiqLog-abs sequences (1/5).

	%L	$L(\mathcal{C},S)$	$L:\mathcal{R}$	$ \mathcal{R} $	$ \mathcal{C} $	s /	v	/ h / r	n $c_{>3}$	c^{M}	c^+
					30-F						
\mathcal{C}_S	73.52	172191	0.40	3305	1497	1497 /	0	/ 0 / 0	0 0.75	4	54
\mathcal{C}_V		172185						· · · · ·	0 0.75	4	54
\mathcal{C}_H		128612		2101				· . · .	0 0.79		
\mathcal{C}_{V+H}		128612		2101				· . · .	0 0.79	12	
\mathcal{C}_{F}		128612		2101				/ 535 / 0			
U _F	01.01	120012	0.00	2101	13-F	,	Ŭ	/ 000 / 1	0.10		100
	70.00	010001	0.44	4907	-		0		0 0 70	4	
\mathcal{C}_S		213301						/ /	0 0.72	4	59
\mathcal{C}_V		213301						/ /	0 0.72	4	59
\mathcal{C}_H		199390						/ /	0 0.80	6	59
\mathcal{C}_{V+H}		199390						/ /	0 0.80	6	
\mathcal{C}_F	71.43	199456	0.42	3847	1319	812 /	0	/ 506 /	1 0.81	6	59
					20-M						
\mathcal{C}_S	70.67	240215	0.44					/ /	0 0.75	4	69
\mathcal{C}_V	70.65	240145	0.44	4585	1771	1768/	3	/ 0 / 0	0 0.75	4	69
${\mathcal C}_H$	64.62	219625	0.44						0 0.80	6	128
\mathcal{C}_{V+H}	64.62	219625	0.44	4195	1332	824 /	0	/ 508 / 0	0.80	6	128
${\mathcal C}_F$	64.60	219562	0.44	4186	1334	826 /	0	/ 507 /	1 0.80	6	128
					29-F	,		, ,			
\mathcal{C}_S	74.64	262784	0.36	4200	2280	2380 /	0	/ 0 / 0	0 0.73	4	24
\mathcal{C}_{V}		262784 262784								4	
						,		, ,			
\mathcal{C}_H		178132		2919				/ /	0 0.81	12	
\mathcal{C}_{V+H}		178132		2919				/ 612 / 0			120
\mathcal{C}_F	50.64	178260	0.37	2932		349 /	0	/ 614 /	0 0.82	12	120
					8-M						
\mathcal{C}_S	73.77	269623	0.35					/ 0 / 0	0 0.83	4	43
\mathcal{C}_V	73.81	269774	0.35	4445	2402	2398/	4	/ 0 / 0	0 0.83	4	43
\mathcal{C}_H	57.18	208993	0.33	3142	1318	398 /	0	/ 920 /	0 0.88	8	120
\mathcal{C}_{V+H}	57.18	208993	0.33	3142	1318	398 /	0	/ 920 /	0 0.88	8	120
\mathcal{C}_F	57.18	208993	0.33	3142	1318	398 /	0	/ 920 /	0 0.88	8	120
					6-M						
\mathcal{C}_S	70.27	268896	0.39	4553	2223	2223 /	0	/ 0 / 0	0 0.71	4	46
$\widetilde{\mathcal{C}_V}$		269015						/ 0 / 0		4	46
\mathcal{C}_H		207147				,		/ 731 /			222
\mathcal{C}_{V+H}		207147						/ 731 /			222
\mathcal{C}_F		207261						/ 731 /			222
					15-F						
\mathcal{C}_S	64.73	259438	0.29	3497	1919	1919 /	0	/ 0 / 0	0 0.89	5	68
\mathcal{C}_V		259751						/ 0 / 0			
\mathcal{C}_{H}		178482						/ 537 / 0			
\mathcal{C}_{V+H}		178482						/ 537 / 0			
\mathcal{C}_{V+H} \mathcal{C}_{F}		178552		2390 2376				/ 537 / 1			
$\mathbf{v}_{F'}$	11.00	110002	0.43	2010	543	тт /	0	/ 001/	· 0.00	14	040

Table A.14. Detailed results for ubiqLog-abs sequences (2/5).

	%L	$L(\mathcal{C},S)$	$L:\mathcal{R}$	$ \mathcal{R} $	$ \mathcal{C} $	s /	v	/	h	/	m	<i>C</i> >3	c^{M}	c^+
					2-F									
\mathcal{C}_S	66.44	322518	0.32	4061	2325	2325 /	0	/	0	/	0	0.80	5	60
\mathcal{C}_V	66.40	322299	0.32			2321 /				1		0.80	5	60
\mathcal{C}_H		225305				458 /				1	0	0.81	8	315
V+H		225305				458 /						0.81		315
\mathcal{C}_F		225345				457 /						0.81		315
					26-I	,		,		'				
\mathcal{C}_S	63.59	309454	0.27	3871	2206	2206 /	0	/	0	/	0	0.87	5	73
$\tilde{\mathcal{C}_V}$		309454				2206 /						0.87	5	73
\mathcal{C}_H		197816				296 /						0.87		336
, V+H		197816				296 /						0.87		336
\mathcal{C}_F		197816				296 /						0.87		336
U _F	10.00	101010	0.20	2011	12-1		Ŭ	/	000	/	Ŭ	0.01	10	000
\mathcal{C}_S	70.66	373721	0.34	5604		- 3068 /	0	/	0	/	0	0.71	4	58
\mathcal{C}_V		373711				3067 /				/		0.71	4	
\mathcal{C}_{H}		273712				702 /				/		0.78		215
		273753				705 /						0.78		213
\mathcal{C}_{F}		273725				705 /						0.78		$210 \\ 215$
C_F	51.75	210120	0.30	4224	17-E	,	T	/	000	/	T	0.78	1	210
C	69 54	250199	0.24	5561			0	/	0	/	0	0.80	5	40
\mathcal{C}_S		359182				3114 /						0.80		4(
\mathcal{C}_V		359324				3112 /		'		'		0.80	5	
\mathcal{C}_H		259470				470 /							12	
V+H		259470				470 /							12	
\mathcal{C}_F	49.52	259531	0.30	3450		477 /	0	/	1122	2 /	3	0.82	12	240
					27-I									
\mathcal{C}_S		438767				4194 /				/		0.61		29
\mathcal{C}_V		437645				4134/						0.61	4	29
\mathcal{C}_H	58.13	325913	0.42			731 /						0.73	6	174
V+H	58.13	325926	0.42			726 /						0.73	6	174
\mathcal{C}_F	58.10	325774	0.42	5845	1789	732 /	0	/	1053	3/	4	0.73	6	17_{-}
					5-F									
\mathcal{C}_S	62.19	478816	0.32	5016	3069	3069 /	0	/	0	/	0	0.74	4	88
\mathcal{C}_V	62.19	478816	0.32	5016	3069	3069 /	0	/	0	/	0	0.74	4	88
\mathcal{C}_H	56.86	437799	0.32	4608	2307	1265 /	0	/	1042	2/	0	0.86	6	176
V+H	56.86	437799	0.32			1265 /						0.86	6	176
\mathcal{C}_F		437799				1265 /						0.86		176
					16-H	7								
		410010	0.30	5687	3739	3739 /	0	/	0	/	0	0.88	5	38
\mathcal{C}_S	67.74	413913	0.50											~ ~
$\mathcal{C}_S \mathcal{C}_V$		413913 413913				3739 /	0	/	0	/	0	0.88	5	- 38
	67.74		0.30	5687	3739	3739 / 770 /				'	$\begin{array}{c} 0 \\ 0 \end{array}$	$\begin{array}{c} 0.88\\ 0.86 \end{array}$		
\mathcal{C}_V	$67.74 \\ 47.69$	413913	$\begin{array}{c} 0.30\\ 0.29 \end{array}$	$5687 \\ 3654$	$\begin{array}{c} 3739\\1798\end{array}$		0	/	1028	3/	0		10	38 168 168

Table A.15. Detailed results for ubiqLog-abs sequences (3/5).

	%L	$L(\mathcal{C}, S)$	$L:\mathcal{R}$	$ \mathcal{R} $	$ \mathcal{C} $	<i>s</i> /	' v	/	h	/	\overline{m}	C>3	c^{M}	<i>c</i> ⁺
					11-F	,								
\mathcal{C}_S	66.37	508461	0.31	7047	4382	4382 /	′ 0	/	0	/	0	0.82	4	96
\mathcal{C}_V		508739				4373 /				1		0.82	4	
${\cal C}_H$		394797				1279/				/	0	0.83	6	294
\mathcal{C}_{V+H}		395014				1270 /						0.83	6	294
\mathcal{C}_{F}		395154				1282 /						0.82	6	294
- 1			0.20		31-F			/ -		- /		0.01	, in the second s	
\mathcal{C}_S	40.18	412820	0.26	3943	2402	2402 /	′ 0	/	0	/	0	0.81	5	388
\mathcal{C}_V		412719				2392 /				'	0	0.81	5	
\mathcal{C}_H		327613				555 /					0	0.87		2328
\mathcal{C}_{V+H}		327630				551 /						0.87		2328
\mathcal{C}_F		327552				551 /						0.87		2328
- 1					28-F			/		/			Ũ	
\mathcal{C}_S	58.81	549595	0.34	8104	4261	4261 /	0	/	0	/	0	0.74	4	177
\mathcal{C}_V		549623				4250 /				'		0.74	4	
\mathcal{C}_H		528707				2609 /						0.81	6	
\mathcal{C}_{V+H}		528715				2612 /						0.81	6	177
\mathcal{C}_{F}		528684				2607 /						0.81	6	177
U _F	00.01	020001	0.01	1001	7-F	2001 /	0	/	001	/	1	0.01	0	111
\mathcal{C}_S	60.62	555786	0.28	6842	3538	3538 /	′ 0	/	0	/	0	0.79	5	149
\mathcal{C}_V		555782				3526 /						0.79	5	149
\mathcal{C}_{H}		424663				1118 /						0.15	8	447
\mathcal{C}_{V+H}		424663				1118/						0.85 0.85	8	447
\mathcal{C}_{V+H} \mathcal{C}_{F}		424689				1116/						0.85	8	447
\mathcal{O}_F	10.02	121005	0.20	0210	35-F		0	/ -	1011	• /	1	0.01	0	111
\mathcal{C}_S	63 97	837346	0.32	11397			<u> </u>	/	0	/	0	0.79	5	97
\mathcal{C}_V		838065		11482						1		0.79	5	97
${\cal C}_H$		664994				1521 /				/		0.87	8	356
\mathcal{C}_{V+H}		664905				1526 /						0.87	8	356
\mathcal{C}_{V+H} \mathcal{C}_{F}		664865				1542 /						0.87	8	356
C_F	00.15	004000	0.00	5200	22-M	,	2	/ 4	2200	, ,	11	0.01	0	000
\mathcal{C}_S	65 78	842835	0.30	11427			<u> </u>	/	0	/	0	0.93	5	66
\mathcal{C}_V		842855 843367		11427						/		0.93	5	66
\mathcal{C}_V \mathcal{C}_H		574219				1561 /				'		0.93 0.91	_	
~													8	
\mathcal{C}_{V+H}		574166 573759				1558 / 1569 /						0.91	8	390 300
\mathcal{C}_F	44.70	010109	0.29	7450	24-F		1	/	1010)/	2	0.91	0	390
	E9.00	941059	0.92	000-			0	/	0	/	0	0.90	٣	1.40
\mathcal{C}_S		841052				5313/			0	΄,	0	0.89	5	142
\mathcal{C}_V		841242				5306 /				'	0	0.89	5	142
\mathcal{C}_H		558998				1308 /						0.88	9	546
\mathcal{C}_{V+H}		558998				1308/						0.88	9	546
\mathcal{C}_F	35.73	559639	0.24	5825	2647	1309 /	0	/1	1338	5/	U	0.88	9	546

Table A.16. Detailed results for ubiqLog-abs sequences (4/5).

	%L 1	$L(\mathcal{C},S)$	$L : \mathcal{R}$	$ \mathcal{R} $	$ \mathcal{C} $	s ,	v v	/	h	/	m	C>3	c^{M}	c^+
					33-F									
\mathcal{C}_S	59.41 10)96442	0.27	12832	7861	7861	/ 0	/	0	/	0	0.85	5	94
\mathcal{C}_V	$59.41 \ 10$	96403	0.27	12832	7862	7861	/ 1	/	0	/	0	0.85	5	94
\mathcal{C}_H	37.54	592736	0.32	9270	3213	1683	0 /	/1	1530)/	0	0.81	6	581
\mathcal{C}_{V+H}	37.54 (592736	0.32	9270	3213	1683	0 /	/1	1530)/	0	0.81	6	581
\mathcal{C}_F	37.54 (592736	0.32	9270	3213	1683	0 /	/1	1530)/	0	0.81	6	581
					19-F									
\mathcal{C}_S	47.34 12	217567	0.23	11314	6620	6620	/ 0	/	0	/	0	0.85	6	150
\mathcal{C}_V	47.33 12	217493	0.23	11314	6617	6615	2	/	0	/	0	0.85	6	150
\mathcal{C}_H	30.76	791271	0.26	7995	3211	1644	0 /	/1	1567	7 /	0	0.83	8	1043
\mathcal{C}_{V+H}	30.76	791230	0.26	7995	3210	1643	/ 1	/1	1566	3/	0	0.83	8	1043
\mathcal{C}_F	30.76	791263	0.26	7997	3208	1641	/ 1	/1	1565	5/	1	0.83	8	1043
					18-F									
\mathcal{C}_S	41.62 1	553796	0.23	14769	9468	9468	/ 0	/	0	/	0	0.85	5	180
\mathcal{C}_V	41.62 15	553727	0.23	14767	9457	9445	/ 12	./	0	/	0	0.85	5	180
\mathcal{C}_H	30.08 11	122920	0.26	11580	5086	3113	/ 0	/ 1	1973	3/	0	0.85	6	1260
\mathcal{C}_{V+H}	30.08 11	122886	0.26	11586	5082	3107	2	/ 1	1973	3/	0	0.85	6	1260
\mathcal{C}_F	30.06 11	122253	0.26	11589	5073	3102	0 /	/1	1967	7 /	4	0.85	6	1260

Table A.17. Detailed results for ubiqLog-abs sequences (5/5).

	%L	$L(\mathcal{C},S)$	$L : \mathcal{R}$	$ \mathcal{R} $	$ \mathcal{C} $	s	/	v	/	h	/	m	$c_{>3}$	c^{M}	c^+
					25-I	7									
\mathcal{C}_S	46.22	1800	0.24	34	9	9	/	0	/	0	/	0	1.00	12	211
\mathcal{C}_V	46.22		0.24	34	9		/	0	/		1		1.00		
\mathcal{C}_H	46.22		0.24	34	9		/	0	1		1	0	1.00		
\mathcal{C}_{V+H}	46.22		0.24	34	9		/	0	1		1	0	1.00		
\mathcal{C}_F	46.22		0.24	34	9		·	0		0			1.00		
-					10-1		/		,		/				
\mathcal{C}_S	39.04	4255	0.12	34	15	15	/	0	/	0	/	0	1.00	8	334
\mathcal{C}_V	39.04		0.12	34	15	15			/		1	0	1.00		
\mathcal{C}_H	39.04	4255		34	15	15			/		/		1.00		
\mathcal{C}_{V+H}	39.04	4255		34	15	15			1		1		1.00		
\mathcal{C}_F	39.04	4255		34	15	15					1		1.00		
-1					9-M		/		/	Ũ	/	Ŭ			
\mathcal{C}_S	48.02	6095	0.40	161	42		/	0	/	0	/	0	0.98	7	178
\mathcal{C}_V	48.02		0.40	161	42				/		/		0.98		
${\cal C}_{H}$	47.91		0.40	159	41	39	·		/		1	0	0.98		
\mathcal{C}_{V+H}	47.91		0.40	159	41	39	·		/			0	0.98		
\mathcal{C}_{F}	47.91		0.40	159	41								0.98		
U _F	11101	0001	0.10		21-I		/	Ŭ	/	-	/	Ū	0.00	. 0	110
\mathcal{C}_S	64.94	46336	0.45	1149			/	0	/	0	/	0	0.93	5	158
\mathcal{C}_V	64.94	46336		1149			·			0		0	0.93		
\mathcal{C}_V \mathcal{C}_H	63.48	40330 45295		1031			·			57			0.93		
\mathcal{C}_{V+H}	63.48	45295		1031 1031									0.94		
\mathcal{C}_{V+H} \mathcal{C}_{F}	63.48	45295		1031 1031									0.94		
\mathcal{O}_F	00.40	10200	0.11		021 23-I		/	0	/	01	/	0	0.01	. 0	100
	20 47	220.4.4	0.90				/	0	/	0	,	0	0.02	- C	000
\mathcal{C}_S	38.47	32044				143							0.93		
\mathcal{C}_V	38.47	32044				143			/		/		0.93		
\mathcal{C}_H	36.21	$30155 \\ 30155$				127				12			$0.95 \\ 0.95$		$2940 \\ 2940$
$\mathcal{C}_{V+H} \ \mathcal{C}_{F}$	$\begin{array}{c} 36.21\\ 36.21 \end{array}$	30155 30155				$127 \\ 127$							0.95		2940 2940
c_F	30.21	30133	0.29		139 14-I		/	0	/	12	/	0	0.95	0	2940
	10 74	40510	0.01				,		,		,	0	0.00		504
\mathcal{C}_S	40.74					180					/	0	0.99		
\mathcal{C}_V	40.74	42719				180	ſ.,		ŕ.,		/		0.99		
\mathcal{C}_{H}	40.49	42460				164							1.00		594 504
\mathcal{C}_{V+H}	40.49					$\begin{array}{c} 164 \\ 164 \end{array}$							1.00		
\mathcal{C}_F	40.49	42460	0.30	077			/	0	/	10	/	0	1.00	8	594
					1-M										
\mathcal{C}_S	44.45	66206		1329									0.91		1620
\mathcal{C}_V	44.43	66166		1330									0.92		1620
\mathcal{C}_H	43.56	64871		1274									0.91		3240
\mathcal{C}_{V+H}	43.52			1271									0.91		3240
\mathcal{C}_F	43.52	64824	0.40	1271	260	228	/	1	/	31	/	0	0.91	5	3240

Table A.18. Detailed results for ubiqLog-rel sequences (1/5).

	%L	$L(\mathcal{C},S)$	$L:\mathcal{R}$	$ \mathcal{R} $ $ \mathcal{C} $ s / v / h / m $c_{>3}$ c^{M} c
				30-F
\mathcal{C}_S	45.39	68909	0.37	1274 332 332 / 0 / 0 / 0 0.92 5 175
\mathcal{C}_V	45.31	68792		1264 332 331 / 1 / 0 / 0 0.92 5 17
\mathcal{C}_H	44.48	67520	0.35	1189 305 238 / 0 / 67 / 0 0.93 6 17
\mathcal{C}_{V+H}	44.45	67483	0.35	1194 302 237 / 1 / 64 / 0 0.93 6 17
\mathcal{C}_F	44.45	67483	0.35	1194 302 237 / 1 / 64 / 0 0.93 6 173
				20-M
\mathcal{C}_S	49.55	96071	0.41	1934 456 456 / 0 / 0 / 0 0.99 6 234
\mathcal{C}_V	49.58	96112	0.41	1937 455 454 / 1 / 0 / 0 0.99 6 234
\mathcal{C}_H	49.06	95108	0.39	1836 450 403 / 0 / 47 / 0 1.00 6 234
\mathcal{C}_{V+H}	49.08	95149	0.39	1839 449 401 / 1 / 47 / 0 1.00 6 23
\mathcal{C}_F	49.08	95152	0.39	1841 448 400 / 0 / 47 / 1 1.00 6 23
				13-F
\mathcal{C}_S	52.75	100663	0.39	1981 531 531 / 0 / 0 / 0 0.98 5 22
\mathcal{C}_V	52.75	100663	0.39	1981 531 531 / 0 / 0 / 0 0.98 5 22
\mathcal{C}_H	52.09	99398	0.37	1849 499 416 / 0 / 83 / 0 0.99 6 22
\mathcal{C}_{V+H}	52.09	99398	0.37	1849 499 416 / 0 / 83 / 0 0.99 6 22
\mathcal{C}_F	52.09	99398	0.37	1849 499 416 / 0 / 83 / 0 0.99 6 22
				6-M
\mathcal{C}_S	45.93	92705	0.32	1463 514 514 / 0 / 0 / 0 0.83 5 11
\mathcal{C}_V	45.93	92705		1463 514 514 / 0 / 0 / 0 0.83 5 11
\mathcal{C}_H	43.29	87375		1392 475 409 / 0 / 66 / 0 0.87 5 22
\mathcal{C}_{V+H}	43.29	87375		1392 475 409 / 0 / 66 / 0 0.87 5 22
\mathcal{C}_F	43.29	87375		1392 475 409 / 0 / 66 / 0 0.87 5 22
				29-F
\mathcal{C}_S	35.04	72233	0.28	956 287 287 / 0 / 0 / 0 0.80 5 16
\mathcal{C}_V	35.04	72233		956 287 287 / 0 / 0 / 0 0.80 5 16
\mathcal{C}_H	31.22	64368		921 282 262 / 0 / 20 / 0 0.83 5 82
\mathcal{C}_{V+H}	31.22	64368		921 282 262 / 0 / 20 / 0 0.83 5 82
\mathcal{C}_F	31.22	64368	0.30	921 282 262 / 0 / 20 / 0 0.83 5 82
				8-M
\mathcal{C}_S	38.16	79350	0.31	1226 344 344 / 0 / 0 / 0 0.99 5 33
\mathcal{C}_V	38.16	79350		$1226 \ 344 \ 344 \ / \ 0 \ / \ 0 \ / \ 0 \ 0.99 \ 5 \ 33'$
\mathcal{C}_H	37.74	78475		$\begin{array}{cccccccccccccccccccccccccccccccccccc$
\mathcal{C}_{V+H}	37.74	78475		1161 329 287 / 0 / 42 / 0 0.99 6 33'
\mathcal{C}_F	37.74	78475		1161 329 287 / 0 / 42 / 0 0.99 6 33
				15-F
\mathcal{C}_S	33.23	78369	0.29	1079 249 249 / 0 / 0 / 0 0.90 5 210
\mathcal{C}_V	33.23	78369		$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
\mathcal{C}_H	31.84	75091		$\begin{array}{cccccccccccccccccccccccccccccccccccc$
\mathcal{C}_{V+H}	31.84	75091		$1024 \ 244 \ 214 / \ 0 \ / \ 30 \ / \ 0 \ 0.95 \ 6 \ 433$
\mathcal{C}_F	31.84	75091		$1024\ 244\ 214\ /\ 0\ /\ 30\ /\ 0$ 0.95 6 43
- 1'			0.40	

Table A.19. Detailed results for ubiqLog-rel sequences (2/5).

	%L	$L(\mathcal{C},S)$	$L:\mathcal{R}$	$ \mathcal{R} $	$ \mathcal{C} $	s /	/ v	/	h /	m	$c_{>3}$	c^{M}	c^+
					2-F	7							
\mathcal{C}_S	44.31	110476	0.33	1749	584	584	/ 0	/	0 /	0	0.76	5	1891
$\tilde{\mathcal{C}_V}$		110476		1749		584			'		0.76	5	1891
${\cal C}_H$		103753		1616	552	,					0.82	5	5673
\mathcal{C}_{V+H}		103753		1616	552	,					0.82	5	5673
\mathcal{C}_F		103753		1616		469					0.82	5	5673
					26-	F							
\mathcal{C}_S	26.05	61256	0.26	761	171	171	/ 0	/	0 /	0	0.91	5	2975
$\widetilde{\mathcal{C}_V}$	26.05	61256	0.26	761	171	171					0.91	5	2975
\mathcal{C}_H	25.91	60913		730	167	154					0.93	6	2975
\mathcal{C}_{V+H}	25.91	60913		730	167	154					0.93	6	2975
\mathcal{C}_F	25.91	60913		730	167	154					0.93	6	2975
					17-	F		,					
\mathcal{C}_S	37.23	101886	0.32	1564	424	424	/ 0	/	0 /	0	0.96	5	3415
\mathcal{C}_V		101886		1564		424					0.96	5	3415
\mathcal{C}_{H}		100223		1442		343				0	0.98	6	3415
\mathcal{C}_{V+H}		100223		1442	410	'		'			0.98		3415
\mathcal{C}_F		100223		1442		343					0.98	6	3415
01	00102	100-20	0.00		27-	,		/	o. /	Ŭ	0.000	Ŭ	0110
\mathcal{C}_S	63 51	229552	0.34	3700		1781	/ 0	/	0 /	0	0.74	4	253
\mathcal{C}_V		229352 229499				1778		'			0.74	4	253 253
									'				
\mathcal{C}_H		203920				1077					0.79	5	1030
\mathcal{C}_{V+H}		203901				1085					0.79	5	1030
\mathcal{C}_F	30.41	203901	0.58	3079	1420	1085 /	/ 1	/ 3	oo4 /	0	0.79	5	1030
					12-	М							
\mathcal{C}_S	53.55	190797	0.27	2417	1139	1139/	/ 0	/	0 /	0	0.78	5	312
\mathcal{C}_V	53.55	190797	0.27	2417	1139	1139/	/ 0	/	0 /	0	0.78	5	312
\mathcal{C}_H	47.85	170502	0.29	2376	991	817	/ 0	/ 1	74/	0	0.83	5	1560
\mathcal{C}_{V+H}	47.85	170502	0.29	2376	991	817	/ 0	/ 1	74/	0	0.83	5	1560
\mathcal{C}_F	47.85	170502	0.29	2376	991	817	/ 0	/ 1	74/	0	0.83	5	1560
					5-F	7							
\mathcal{C}_S	57.05	228234	0.41	4499	1343	1343	/ 0	/	0 /	0	0.96	5	1232
$\widetilde{\mathcal{C}_V}$	57.05	228234	0.41			1343				0	0.96	5	1232
${\cal C}_H$	56.34	225400	0.39			1070		'	'	0	0.98	6	1232
\mathcal{C}_{V+H}		225400				1070			'		0.98	6	1232
\mathcal{C}_F		225400				1070					0.98		1232
					16-			,	,				
\mathcal{C}_S	33.19	127122	0.28	1657	507	507	/ 0	/	0 /	0	0.90	6	3610
\mathcal{C}_V		127244		1669		505					0.89	6	3610
\mathcal{C}_{H}		118994		1598		480					0.05		10830
\mathcal{C}_{V+H}		110 <i>3</i> 54 119116		1610		478					0.91		10830
\mathcal{C}_{V+H} \mathcal{C}_{F}		119116		1610		478					0.91		10830
	01.10	11/110	0.20	1010	004	-10 /	, 2	/	/	0	0.01	0	10000

Table A.20. Detailed results for ubiqLog-rel sequences (3/5).

	~ -					,			,		м	
	%L	$L(\mathcal{C},S)$	$L:\mathcal{R}$	$ \mathcal{R} $	$ \mathcal{C} $	s /	v	/ h	/ m	$c_{>3}$	c^{M}	c^+
					31-I	F						
\mathcal{C}_S	28.01	128149	0.29	1661	439	439 /	0	/ 0	/ 0	0.98	7	5196
\mathcal{C}_V	28.01	128149	0.29			439 /			/ 0		7	5196
\mathcal{C}_H	27.52	125919	0.29	1597	433	396 /	0	/ 37	/ 0	0.98	8	10392
\mathcal{C}_{V+H}	27.52	125919	0.29	1597	433	396 /	0	/ 37	/ 0	0.98	8	10392
\mathcal{C}_F	27.52	125919	0.29	1597	433	396 /	0	/ 37	/ 0	0.98	8	10392
					11-I	F						
\mathcal{C}_S	45.81	227300	0.28	3001 1	186	1186/	0	/ 0	/ 0	0.84	5	1409
\mathcal{C}_V	45.81	227300	0.28	$3001 \ 1$	186	1186/	0	/ 0	/ 0	0.84	5	1409
\mathcal{C}_H	44.05	218542	0.28	2924 1	074	915 /	0	/159	/ 0	0.89	6	2816
\mathcal{C}_{V+H}	44.05	218542	0.28	2924 1	074	915 /	0	/159	/ 0	0.89	6	2816
\mathcal{C}_F	44.05	218542	0.28	2924 1	074	915 /	0	/159	/ 0	0.89	6	2816
					35-I	F						
\mathcal{C}_S	38.44	285802	0.24	$3039\ 1$	310	1310 /	0	/ 0	/ 0	0.92	6	6388
\mathcal{C}_V	38.44	285831	0.24	$3046\ 1$	307	1305/	2	/ 0	/ 0	0.92	6	6388
\mathcal{C}_H	37.45	278445	0.23	$2892\ 1$	190	1022/	0	/168	/ 0	0.93	6	6388
\mathcal{C}_{V+H}	37.45	278449	0.23	$2894\ 1$	188	1018/	2	/168	/ 0	0.93	6	6388
\mathcal{C}_F	37.45	278449	0.23	$2894\ 1$	188	1018/	2	/168	/ 0	0.93	6	6388
					28-I	F						
\mathcal{C}_S	54.35	425270	0.30	5810 2	537	2537 /	0	/ 0	/ 0	0.88	5	998
\mathcal{C}_V	54.35	425270	0.30			2537 /				0.88	5	998
\mathcal{C}_{H}	53.92	421935	0.30	$5689\ 2$	320	2012 /	0	/ 308	/ 0	0.90	5	998
\mathcal{C}_{V+H}	53.92	421935	0.30	$5689\ 2$	320	2012 /	0	/ 308	/ 0	0.90	5	998
${\mathcal C}_F$	53.92	421935	0.30	$5689\ 2$	320	2012 /	0	/ 308	/ 0	0.90	5	998
					7-F							
\mathcal{C}_S	36.23	272826	0.25	3002 1	266	1266 /	0	/ 0	/ 0	0.88	5	1511
\mathcal{C}_V	36.23	272826	0.25			1266 /			/ 0	0.88	5	1511
\mathcal{C}_{H}	35.27	265532	0.24			1084 /			/ 0			2738
\mathcal{C}_{V+H}	35.27	265532	0.24			1084 /					6	2738
\mathcal{C}_F	35.27	265532	0.24			1084 /					6	2738
					22-1	М						
\mathcal{C}_S	26.77	233065	0.22	2225	926	926 /	0	/ 0	/ 0	0.76	5	3860
\mathcal{C}_V	26.77	233065	0.22			926 /			/ 0		5	3860
${\mathcal C}_H$	26.15	227630	0.22			766 /			/ 0	0.81	5	7720
\mathcal{C}_{V+H}		227630				766 /						
\mathcal{C}_F		227630				766 /						
					24-1	F						
\mathcal{C}_S	31.22	311878	0.21	2781 1	113	1113/	0	/ 0	/ 0	0.86	5	7060
$\widetilde{\mathcal{C}_V}$		311812				1117/		1.	́/ 0			
${\cal C}_H$		284786				1021/			/			35300
		284736		2688 1								35300
\mathcal{C}_{V+H}	20.01	204100	0.22	2000 1	000	1021/	-	/ 00	1 0	0.00		00000

Table A.21. Detailed results for ubiqLog-rel sequences (4/5).

Table A.22. Detailed results for ubiqLog-rel sequences (5/5).

	$\%L \ L(\mathcal{C},S) \ L$	$\mathcal{L}:\mathcal{R}$ $ \mathcal{R} $ $ \mathcal{C} $	s / v	/ h / m	$c_{>3} c^{\mathrm{M}} c^{+}$
		33-	F		
\mathcal{C}_S	33.75 427496 (0.28 4911 1801	1801/0	/ 0 / 0	0.72 4 6620
\mathcal{C}_V	$33.74 \ 427439$ (0.72 4 6620
\mathcal{C}_H	31.40 397794 ($0.29 4720 \ 1685$	1504 / 0	/ 181 / 0	0.78 5 26476
\mathcal{C}_{V+H}	31.39 397687 ($0.29 4715 \ 1683$	1499/1	/ 183 / 0	0.78 5 26476
${\mathcal C}_F$	31.39 397687 ($0.29 4715 \ 1683$	1499 / 1	/ 183 / 0	0.78 5 26476
		19-	F		
\mathcal{C}_S	33.32 500108 ($0.25 5101 \ 1726$	1726 / 0	/ 0 / 0	0.72 4 8500
\mathcal{C}_V	33.32 500108 ($0.25 5101 \ 1726$	1726 / 0	/ 0 / 0	0.72 4 8500
\mathcal{C}_H	30.41 456446 ($0.26 4864 \ 1564$	1313 / 0	/ 251 / 0	0.79 5 34000
\mathcal{C}_{V+H}	30.41 456446 ($0.26 4864 \ 1564$	1313 / 0	/ 251 / 0	0.79 5 34000
\mathcal{C}_F	30.41 456446 ($0.26 4864 \ 1564$	1313 / 0	/251/0	0.79 5 34000
		18-	F		
\mathcal{C}_S	30.75 677622 (0.26 6767 2567	2567 / 0	/ 0 / 0	0.70 4 4417
$\tilde{\mathcal{C}_V}$	30.75 677622 (0.70 4 4417
\mathcal{C}_H	28.60 630330 ('		0.75 5 16292
\mathcal{C}_{V+H}	28.60 630330 (,	, ,	0.75 5 16292
\mathcal{C}_F	28.60 630330 ($0.27 6659 \ 2365$	2083 / 0	/ 282 / 0	0.75 5 16292

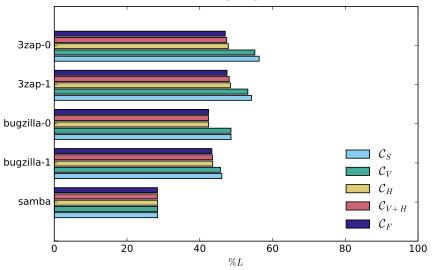
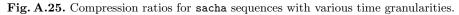
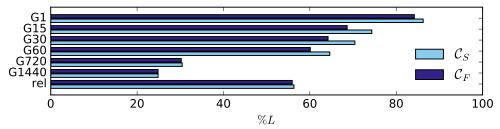


Fig. A.24. Compression ratios for 3zap, bugzilla and samba sequences.





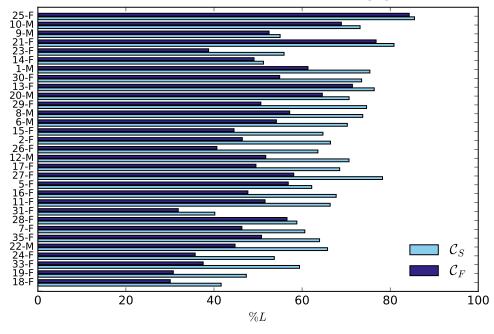
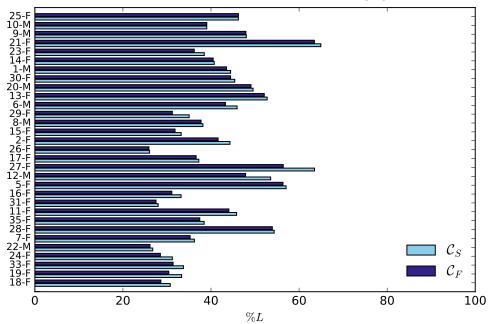


Fig. A.26. Compression ratios for the sequences from the ubiqLog-abs dataset.

Fig. A.27. Compression ratios for the sequences from the ubiqLog-rel dataset.



	τ	T	$\sum E $	occs
		sacha-abs-G1		
a)	2017-09-10 12:09	$\{r = 7, p = 1 d\}$ ([Sleep $-6 h 36 - Sleep]$ -0 min - [Childcare -17 h 24 - Childcare])	28	28
b)	2011-12-12 17:07	$\{r = 3, p = 1 d 2 \min\} (Work] - 0 \min - [Walk - 6 \min - Walk] - 0 \min - [Subway - 17 \min - Subway] - 0 \min - [Walk - 11 \min - Walk] $	13	21
c)	2012-03-06 07:40	$\{r = 4, p = 1 d\}$ ([Subway $-0 \min$ Routines] $-50 \min$ [Consulting-E)	7	12
d)		$\{r = 3, p = 23 \text{ h} 51\} ([Walk - 9 \min - [Subway - 17 \min - Subway] - 0 \min - [Walk - 5 \min - Walk] - 0 \min - [Work]$	22	18
e)	2012-05-28 16:10	{ $r=3, p=1 d 3 \min$ } (Consulting-E] $-0 \min$ [Bike $-15 \min$ Bike] $-0 \min$ [Consulting)	14	12
		sacha-abs-G15		
f)	2015-01-08 08:45	$\{r = 14, p = 7 d\}$ ([Subway -45 min - Subway] -0 min - [Consulting-E]	26	42
g)	2016-01-18 17:45	$\{r = 17, p = 1 d\}$ ([Dinner - 30 min - Dinner])	54	34
h)	2014-12-18 00:15	$\{r = 76, p = 1 d\}$ ([Sleep $-8h 30-$ Sleep])	517	152
i)	2012-03-29 16:45	$\{r = 7, p = 217 \text{ d}\}$ (Consulting-E] $-0 \min - [\text{Subway})$	12	14
		sacha-abs-G60		
j)	2011-11-27 21:30	$\{r = 968, p = 1 d\}$ ([Sleep)	2157	968
k)	2011-11-28 08:30	$\{r = 4, p = 11 \text{ h}\} (\{r = 4, p = 7 \text{ d}\} (Walk))$	8	16
l)	2015-10-24 23:30	$\{r = 22, p = 1 d\}$ ([VideoGame-B2 - 1 h - VideoGame-B2])	137	44
		sacha-rel		
m)	23200	$\{r = 3460, p = 3\}$ (Childcare)	5879	3460
n)	862	$\{r = 237, p = 12\}$ (Sleep)	775	237
o)	33140	$\{r=3, p=155\}$ ($\{r=4, p=1\}$ (Consulting-E))	1	12
p)	7091	$\{r=3, p=14207\}$ (Emacs $-445-\{r=5, p=2\}$ (Coding))	29	18

Table A.23. Example patterns from sacha sequences with different time granularities.

Table A.24. Example patterns from the 3zap-0 sequences.

	τ	T	$\sum E $	occs
a)	36060	$\{r = 110, p = 2\}$ (1561:X -1 - 1561:E)	80	220
b)	33415	$\{r = 20, p = 8\}$ (1561:I -1 - 1561:i -1 - 1561:Ix -1 - 1561:C -1 - 53:C)	63	100
c)	11680	$\{r=3, p=5116\}$ ($\{r=8, p=1\}$ (2429:U -3 - 2429:u))	40	48
d)	7908	$\{r=3, p=17729\}$ $(\{r=5, p=2\}$ $(2400:E - 1 - 2400:X)$	74	60
		$-91-\{r=5, p=2\}$ (2400:E $-1-$ 2400:X))		
e)	84347	$\{r=3, p=10563\}$ (2399:U -1- $\{r=4, p=2\}$ (2399:C -1- 2427:C))	3	27
f)	85889	$\{r = 7, p = 248\}$ $\{r = 4, p = 2\}$ $(2400:X) - 7 - 2400:C)$	48	35
g)		$\{r=3, p=17790\}$ ($\{r=5, p=6\}$ (2445:C) $-3-\{r=4, p=8\}$ (2447:C))	15	27
		$\{r = 5, p = 253\}$ (2426:C - 3 - 18:C - 3 - 2445:U - 1 - 2445:U - 1 - 2445:C	15	35
		-3-2447:C -21-2447:C)		
i)	151772	$\{r = 4, p = 221\}$ (6:C -2- $\{r = 4, p = 2\}$ (2395:X))	15	20
j)	12071	$\{r=3, p=2235\}$ $(\{r=4, p=2\}$ $(2395: X - 1 - 2395: E) -7 - \{r=4, p=6\}$ $(2395: C))$	76	36

List of Symbols

Ω	event alphabet	p. 4
α	an event	p. 4
S	an event sequence	p. 4
$S^{(lpha)}$	projection of sequence S on event α	p. 4
$\widetilde{ S }$	length of sequence S, number of timestamp-event pairs in S	p. 4
$t_{\text{start}}(S)$	smallest timestamp in S	p. 4
$t_{\rm end}(S)$	largest timestamp in S	p. 4
$\Delta(S)$	duration of sequence S , time spanned by S	p. 4
C	an event cycle	p. 5
α	cycle event	р. 5
r	cycle length	р. 5
p	cycle period	p. 5
au	cycle starting point	p. 5
E	cycle shift corrections	p. 5
$\Delta(C)$	duration of cycle C , time spanned by C	p. 5
$\sigma(E)$	sum of the shift corrections in E	p. 5
cover(C)	<i>cover</i> of cycle C , set of timestamp–event pairs reconstructed from C	p. 5
C	a collection of cycles	p. 6
$residual(\mathcal{C}, S)$	set of <i>residuals</i> , timestamp–event pairs of sequence S not covered by any cycle	p. 6
	in the collection of cycles \mathcal{C}	-
L	<i>cost</i> , code length	p. 6
P	a periodic pattern	p.10
T	pattern tree	p.10
B_X	a bock in a periodic pattern	p.10
$\Gamma(B_X)$	ordered list of children of block B_X	p.10
d_{Xi}	inter-block distance, time separating occurences of blocks $B_{X(i-1)}$ and B_{Xi}	p.10
$\gamma_{\mathbf{L}}(X)$	left-most leaf descendant of block/node X	p.11
$shift(S, t_s)$	function that shifts sequence S forward by t_s	p.13
$occs^*(P)$	list of timestamp–event pairs reconstructed from the pattern tree of P prior	p.13
	to correction, a.k.a. perfect occurences	
occs(P)	list of timestamp–event pairs reconstructed from the pattern tree of P after	p.13
	correction, a.k.a. corrected occurences	
$\epsilon(o)$	the cumulated time correction to be applied to timestamp–event pair \boldsymbol{o}	p.14
A	the string representing the event sequence of a block/node	p.14
$\Delta^*(B_X)$	time spanned by the entire cycle of block B_X	p.16
$\delta^*(B_X)$	time spanned by a single repetition of block B_X	p.16
$\Delta^*_{\max}(B_X)$	maximum time span of the entire cycle of block B_X	p.17
$\delta^*_{\max}(B_X)$	maximum time span of a repetition of block B_X	p.17
D	collection of all the periods (except of the top block) and inter-block distances	p.18
	in the pattern tree, as well as δ^*_{\max} of top block, if necessary	
%L	compression ratio, ratio of the sequence code length using the considered col-	p.30
	lection of patterns vs. using an empty collection of patterns	
\mathcal{R}	set of residuals	p.34
$L : \mathcal{R}$	fraction of the code length spent on residuals	p.34