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Volume 156

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Fawaz Alsolami · Mohammad Azad ·  
Igor Chikalov · Mikhail Moshkov

# Decision and Inhibitory Trees and Rules for Decision Tables with Many-valued Decisions

Fawaz Alsolami  
Computer, Electrical and Mathematical  
Sciences and Engineering Division  
King Abdullah University of Science  
and Technology  
Thuwal, Saudi Arabia

Mohammad Azad  
Computer, Electrical and Mathematical  
Sciences and Engineering Division  
King Abdullah University of Science  
and Technology  
Thuwal, Saudi Arabia

Igor Chikalov  
Computer, Electrical and Mathematical  
Sciences and Engineering Division  
King Abdullah University of Science  
and Technology  
Thuwal, Saudi Arabia

Mikhail Moshkov  
Computer, Electrical and Mathematical  
Sciences and Engineering Division  
King Abdullah University of Science  
and Technology  
Thuwal, Saudi Arabia

ISSN 1868-4394

ISSN 1868-4408 (electronic)

Intelligent Systems Reference Library

ISBN 978-3-030-12853-1

ISBN 978-3-030-12854-8 (eBook)

<https://doi.org/10.1007/978-3-030-12854-8>

Library of Congress Control Number: 2019930960

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The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

*To our families*

# Preface

This book is devoted to the study of decision and inhibitory trees and rules for decision tables with many-valued decisions. In conventional decision tables, a single decision is associated with each row. However, situations in which a set of decisions is associated with each row are often possible. For some decision tables, inhibitory trees and rules can represent more information than decision trees and rules.

We consider various examples of problems and decision tables with many-valued decisions and discuss the difference between decision and inhibitory trees and rules for decision tables with many-valued decisions. We mention without proofs some relatively simple results obtained earlier for decision trees, tests, rules, and rule systems for binary decision tables with many-valued decisions. We generalize these results to the inhibitory trees, tests, rules, and rule systems.

We extend the multi-stage and bi-criteria optimization approaches to the case of decision trees and rules for decision tables with many-valued decisions and then generalize them to the case of inhibitory trees and rules. The applications of these techniques include the study of totally optimal (optimal relative to a number of criteria simultaneously) decision and inhibitory trees and rules, the comparison of greedy heuristics for tree and rule construction as single-criterion and bi-criteria optimization algorithms, the development of the restricted multi-pruning approach used in classification and knowledge representation, etc.

We also study the time complexity of decision and inhibitory trees and rule systems over arbitrary sets of attributes represented by information systems.

The results presented in this book can be useful for researchers in data mining, knowledge discovery, and machine learning, especially those who work in rough set theory, test theory, and logical analysis of data. The book can be used for the creation of courses for graduate students.

Thuwal, Saudi Arabia  
November 2018

Fawaz Alsolami  
Mohammad Azad  
Igor Chikalov  
Mikhail Moshkov

# Acknowledgements

We are greatly indebted to King Abdullah University of Science and Technology for the immense support.

We are grateful to our coauthors in papers devoted to the creation of extensions of dynamic programming for decision and inhibitory trees, rules, and rule systems: Hassan AbouEisha, Mohammed Al Farhan, Abdulaziz Alkhalid, Saad Alrawaf, Talha Amin, Monther Busbait, Shahid Hussain, and Beata Zielosko.

We are thankful to Prof. Andrzej Skowron for stimulating discussions.

We extend an expression of gratitude to Prof. Janusz Kacprzyk, to Dr. Thomas Ditzinger, and to the Series Intelligent Systems Reference Library staff at Springer for their support in making this book possible.

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