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Interval-Valued Methods in Classifications and Decisions

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*To my Family and Friends,
especially to my Parents*

Preface

I saw the need, but I did not know how to satisfy it. I posed the problem to my best friends, Herbert Robins and Richard Bellman, because as mathematicians they were better qualified than I was to come up with a theory which was needed. Both were too busy with their own problems. I was left on my own.

Lotfi A. Zadeh [1]

Since the seminal paper on fuzzy sets was published [2], plenty of books and papers devoted to the topic of fuzzy sets theory, its extensions and applications appeared. According to the Web of Science, there are over 198,000 works with the *fuzzy* as a topic. Among them, there are also works of the best friends mentioned in the quotation by Lotfi Zadeh (e.g., [3]). The author of this monograph also would like to contribute to the subject of fuzzy sets, especially interval-valued fuzzy sets, which are one of the most important and developing generalizations of the fuzzy sets theory. However, the presented results may be also advantageous to the whole community, not only fuzzy, but more generally involved in research under uncertainty or imperfect information.

Fuzzy sets theory and its extensions are interesting not only from a theoretical point of view, but also they have applications in many disciplines such as computer science and technology. Fuzzy sets turned out to be effective tools for many practical applications in all areas, where we deal with natural language and perceptions. Fuzzy sets and fuzzy logic contributed to the development of the artificial intelligence and its applications. Fuzzy sets theory and its diverse extensions are still one of the most important approaches for dealing with uncertain, incomplete, imprecise, or vague information. The aspect of data uncertainty is studied intensively in many contexts and scientific disciplines. Many different forms of uncertainty in data have been recognized. Some come from conflicting or incomplete information, as well as from multiple interpretations of some phenomenon. Other arise from lack of well-defined distinctions or from imprecise boundaries. It is impossible to eliminate completely uncertainty and ignorance from everyday experience of scientists, specialists in various fields, and also the life of an average man. According to Lotfi A. Zadeh *As complexity rises, precise statements lose*

meaning and meaningful statements lose precision. This is why there is a need to develop effective algorithms and decision support systems that would be able to capture the arising problems.

The main aim of this monograph is to consider interval-valued fuzzy methods that improve the classification results and decision processes under incomplete or imprecise information. The presented results may be useful not only for the community working on fuzzy sets and their extensions, but also for researches and practitioners dealing with the problems of uncertain or imperfect information. The key part of the monograph is the description of the original classification algorithms based on interval-valued fuzzy methods. The described algorithms may be applied in decision support systems, for example, in medicine or other disciplines where the incomplete or imprecise information may appear (cf. Chap. 4), or for data sets with a very large number of objects or attributes (cf. Chap. 5). The presented solutions may cope with the challenges arising from the growth of data and information in our society since they enter the field of large-scale computing. As a result, they may enable efficient data processing. The presented applications are based on theoretical results connected with the family of comparability relations defined for intervals and other related notions. We show the origin, interpretation, and properties of the considered concepts deriving from the epistemic interpretation of intervals. Namely, the epistemic uncertainty represents the idea of partial or incomplete information. It may be described by means of a set of possible values of some quantity of interest, one of which is the right one [4]. Since the subject is wide, we mainly concentrate on theory and applications of new concepts of aggregation functions in interval-valued fuzzy settings. The theory of aggregation functions became an established area of research in the past 30 years [5]. Apart from theoretical results, there are many applications in decision sciences, artificial intelligence, fuzzy systems, or image processing. One of the challenges is to propose implementable aggregation methods (cf. [6]) to improve the usability of the proposed ideas. Such methods provide a heuristic which may be conveniently implemented and easily understood by practitioners. Moreover, another challenge is related to the ability of including in the proposed solutions human-specific features like intuition, sentiment, judgment, affect, etc. These features are expressed in natural language which is the only fully natural means of articulation and communication of the human beings. This idea led to considering aggregations inspired by the Zadeh idea *computing with words* [7]. Computing with words (CWW) (cf. [8]) has a very high application potential by its remarkable ability to represent and handle all kinds of descriptions of values, relations, handling imprecision. There are many aggregation methods that try, with success, to resolve the challenges of nowadays problems (cf. [9–16]). In this book, we examine the so-called possible and necessary aggregation functions defined for interval-valued fuzzy settings. One of the reasons to consider these types of aggregation operators is connected with the fact that these notions of aggregation functions were recently introduced [17] and they have not been widely examined before.

The book consists of two parts. In the first part, theoretical background is presented and next in the second part application results are analyzed. In theoretical part, in Chap. 1 elements of fuzzy sets theory and its extensions are provided. There are presented the notions of interval-valued fuzzy calculus. Diverse orders applicable for interval-valued comparing, including interval-valued fuzzy settings, are discussed. Furthermore, in Chap. 2 aggregation functions defined on the unit interval $[0, 1]$ are recalled and useful notions and properties are provided. Construction methods of interval-valued aggregation functions derive from the real-line settings and interval-valued aggregation functions often inherit the properties of their component functions defined on the unit interval $[0, 1]$. All these issues will be presented in Chap. 2.

Part II covers two major topics: decision-making and classification problems. Chapter 3 is devoted to decision-making problems with interval-valued fuzzy methods involved. It is pointed out the usage of new concepts with possible and necessary interpretation involved. Next, the classification problems are discussed. When classifiers are used there is a problem of lowering its performance due to the large number of objects or attributes and in the case of missing values in attribute data. In this book, it is shown that in such situations interval-valued fuzzy methods help to retrieve the information and to improve the quality of classification. These issues are discussed in Chaps. 4 and 5. In Chap. 4, there are proposed methods of optimization problem of k -NN classifiers that may be useful in diverse computer support systems facing the problem of missing values in data sets. Missing values appear very often in data sets of computer support systems designed for the medical diagnosis, where the lack of data may be due to financial reasons or the lack of a specific medical equipment in a given medical center. Chapter 5 presents methods of dealing with large-scale problems such as large number of objects or attributes in data sets. Specifically, there is presented a method of optimization problem of k -NN classifiers in DNA microarray methods for identification of marker genes, where typically there is faced the problem of huge number of attributes. Finally, in Chap. 6, there is presented the performance of the new types of aggregation functions for interval-valued fuzzy settings in the computer support system OvaExpert [18]. The book ends with a brief description of the future research plans in the area of presented problems, both in the theoretical and practical aspects.

The book is aimed at practitioners working in the areas of classification and decision-making under uncertainty, especially in medical diagnosis. It can serve as a brief introduction into the theory of aggregation functions for interval-valued fuzzy settings and application in decision-making and classification problems. It can also be used as supplementary reading for the students of mathematics and computer science. Moreover, the results on aggregation functions may be interesting for computer scientists, system architects, knowledge engineers, programmers, who face a problem of combining various inputs into a single output. The classification algorithms considered in this book (in Chaps. 4 and 5), along with other supplementary materials are available at [19], where there are provided suitable files to download and run the experiments.

I would like to thank Prof. Józef Drewniak for introducing me to the subject of fuzzy sets theory. Moreover, I would like to thank other Professors that helped me in better understanding the nuances of fuzzy sets theory, its extensions, and applications. Namely, these are the following persons (listed in the alphabetical order): Jan G. Bazan, Humberto Bustince, Bernard De Baets, Przemysław Grzegorzewski, Janusz Kacprzyk, Radko Mesiar, Vilém Novák, and Eulalia Szmidt. I am also grateful to my colleagues from Poland and abroad with whom I cooperated working on scientific problems or whom I met during scientific conferences. Especially, I would like to thank my colleagues from the University of Rzeszów with whom we spent many hours on seminars discussing scientific problems.

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Rzeszów, Poland
October 2018

Urszula Bentkowska

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