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An Introduction to Analytical Fuzzy Plane Geometry

Springer

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To our Gurus,
who teach us how to deal with
the uncertainties in real life…

Preface

Understanding any optimization model involves a sequential process of obtaining a reasonable set of solutions. Instead of just achieving the final solution from a black-box algorithm, we wanted to look into the solution space of an optimization model which is defined in a fuzzy environment. Solution space of a fuzzy optimization model is hazy in nature and optimality lies on the edge of it. Thus our quest started by traversing the boundary of the fuzzy feasible region.

Fuzzy Geometry is a study of shape, size, and nature of a region, surrounded by a hazy line or curve. In this book, the shape analysis has been done for obtaining the mathematical equations of hazy or fuzzy objects. The fuzzy objects have been visualized geometrically, and the construction procedure has been suggested to visualize it in the Euclidean space. Any entity in Euclidean space is the collection of points, restricted with some predefined constraints. The basic defining element in fuzzy geometry is a fuzzy point. This book starts with the concept of fuzzy point as a collection of different fuzzy numbers in the space. Though a fuzzy number is defined on the real line but to represent a fuzzy point in Euclidean space, there was a need to extend the concept of fuzzy number on any real line, excluding the real axis.

The book mainly focuses on two-dimensional analytical fuzzy geometry. But the ideas may be extended to higher dimensions as well. The aim of the book is to synergize the different concepts of geometry and the algebra of real numbers in a methodical way. The joining of two fuzzy points as a fuzzy line segment and extending it bi-infinitely as fuzzy line are defined. The concepts of fuzzy triangle, fuzzy circle, and fuzzy parabola are illustrated. Proper care has been taken so that every concept coincides with the conventional definitions in classical geometry with zero uncertainty. The ideas on fuzzy geometry are applied to fuzzy multi-objective optimization problems. It is also shown that the proposed geometry can highly reduce the computational cost in obtaining the fuzzy decision feasible region.

This book is suggested as a basic research monograph on fuzzy geometry. There is ample scope to extend the ideas further.

Keywords Fuzzy point • Same and inverse point • Fuzzy distance • Fuzzy line • Different forms of fuzzy line • Fuzzy angle • Fuzzy triangle • Fuzzy circle • Fuzzy parabola • Fuzzy multi-objective optimization problem • Fuzzy non-dominance • Construction of fuzzy feasible space

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