Sweep Algorithms for the Capacitated Vehicle Routing Problem with Structured Time Windows

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Abstract

The capacitated Vehicle Routing Problem with structured Time Windows (cVRPsTW) is concerned with finding optimal tours for vehicles with given capacity constraints to deliver goods to customers within assigned time windows. In our problem variant these time windows have a special structure, namely they are non-overlapping and each time window holds several customers. This is a reasonable assumption for Attended Home Delivery services. Sweep algorithms are known as simple, yet effective heuristics for the classical capacitated Vehicle Routing Problem. We propose variants of the sweep algorithm that are not only able to deal with time windows, but also exploit the additional structure of the time windows in a cVRPsTW. Afterwards we suggest local improvement heuristics to decrease our objective function even further. A carefully constructed benchmark set that resembles real-world data is used to prove the efficacy of our algorithms in a computational study.

Key words. Vehicle routing; time windows; sweep algorithm; attended home delivery; transportation; logistics.

1 Introduction

Attended Home Delivery (AHD) services, e.g., online grocery shopping services, have encountered a significant growth in popularity in recent years. In a typical setup, customers choose time windows during which they want to receive their goods from a set of available time windows that is provided by the supplying company. This set is called *structured* if the number of customers is significantly larger than the number of time windows and all windows are pair-wise non-overlapping. Once all orders have been placed, the supplier aims to minimize the fulfillment costs. This involves solving a *capacitated Vehicle Routing Problem with structured Time Windows* (cVRPsTW), which is a special case of the *capacitated Vehicle Routing Problem with Time Windows* (cVRPTW). Heuristics are the method of choice in practice to produce high-quality solutions in reasonable time since the problem instances occurring are typically rather large.

So-called *cluster-first, route-second* methods have been proven to be effective for the classical *capacitated Vehicle Routing Problem* (cVRP). These methods first partition the customers into subsets that are small enough such that they can all be visited by one vehicle. In a second step, they compute a route for each vehicle. The most prominent example of this group of algorithms is the *sweep algorithm* [3]. It clusters the customers by dividing the plane into radial sectors originating from the depot's location. In this work we first generalize the sweep algorithm to the cVRPTW. Secondly, we propose two variants that exploit the additional structure of the time windows in a cVRPsTW. Due to the imposed structure the total number of time windows is quite small, which allows us to use window-dependent angles. After obtaining a first feasible solution, improvement heuristics are applied that try to decrease the objective function by slightly altering the angles obtained by our sweep algorithms.

Finally, we conduct an extensive computational study using a large variety of carefully constructed benchmark instances which show that our approach is capable of finding good initial solutions for instances containing up to

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2000 customers within a few seconds. Further, we demonstrate that the proposed improvement heuristics allow us to significantly improve the solution quality within a few minutes. We notice that the performance of the different variants is dependent on the characteristics of the considered instance, e.g., whether vehicle capacities or time windows are the stronger restriction.

For an overview of exact resp. heuristic methods for the cVRP(TW) we refer to [1, 2]. Solomon [6] proposes a sweep heuristic that takes time window constraints into account. In contrast to our approach, it does not consider the time windows of the customers when partitioning them. Instead, the time windows are only respected when computing the routes for each vehicle. While Solomon's approach utilizes an insertion heuristic to obtain the routes, we apply a Mixed-Integer Linear Program (MILP) to decide the feasibility of an assignment of customers to a vehicle, as well as to obtain the optimal solution of the occurring routing subproblems.

2 Formal Problem Definition

A cVRPsTW instance consists of a set of time windows $\mathscr{W} = \{w_1, \ldots, w_q\}$, where each $w_i \in \mathscr{W}$ is defined through its start time s_{w_i} and its end time e_{w_i} with $s_{w_i} < e_{w_i}$, a set of customers \mathscr{C} , $|\mathscr{C}| = n$, a time window assignment function $w: \mathscr{C} \to \mathscr{W}$, a depot *d* from which all vehicles depart from and return to, $\overline{\mathscr{C}} := \mathscr{C} \cup \{d\}$, a travel time function $t: \overline{\mathscr{C}} \times \overline{\mathscr{C}} \to \mathbb{R}_{\geq 0}$, a service time function $s: \mathscr{C} \to \mathbb{R}_{>0}$, a common vehicle capacity $C \in \mathbb{R}_{>0}$, and an order weight function $c: \mathscr{C} \to [0, C]$. We require the windows to be structured, i.e., $n \gg q$ and for $1 \le i < j \le q$ it holds $s_{w_j} \ge e_{w_i}$. Moreover, each element in $\overline{\mathscr{C}}$ has coordinates in the two-dimensional plane. We assume that the travel times are correlated to the geographical distances, but not purely determined by them.

A *tour* consists of a set $\mathscr{A} = \{a_1, a_2, \dots, a_k\}$ of customers with corresponding arrival times $\alpha_{a_1}, \dots, \alpha_{a_k}$ during which the vehicles are scheduled to arrive. A tour is called *capacity-feasible*, if $\sum_{i=1}^{k} c(a_i) \leq C$. Moreover, we call it *time-feasible*, if every customer is served within its assigned time window, i.e., $s_{w(a_i)} \leq \alpha_{a_i} \leq e_{w(a_i)}, i = 1, \dots, k$, and if there is sufficient time to respect the required service and travel times, i.e., $\alpha_{a_{i+1}} - \alpha_{a_i} \geq s(a_i) + t(a_i, a_{i+1}), i = 1, \dots, k - 1$. A *schedule* $\mathscr{S} = \{\mathscr{A}, \mathscr{B}, \dots\}$ is a set of tours where each customer occurs in exactly one tour. It is called *feasible*, if all tours are capacity- and time-feasible.

We consider three objectives: The first one is the *number of vehicles used*, i.e. $\lambda_1(\mathscr{S}) := |\mathscr{S}|$. Secondly, the *schedule duration* $\lambda_2(\mathscr{S})$ is defined as the sum of all *tour durations*, i.e. $\lambda_2(\mathscr{A}) := t(d,a_1) + \alpha_{a_k} - \alpha_{a_1} + s(a_k) + t(a_k,d)$. Thirdly, the *schedule travel time* $\lambda_3(\mathscr{S})$ is defined as the sum of all *tour travel times*, i.e. $\lambda_3(\mathscr{A}) := t(d,a_1) + \sum_{i=1}^{k-1} t(a_i,a_{i+1}) + t(a_k,d)$. Similar to Solomon [6], we aim to minimize these three objectives with respect to the lexicographical order $(\lambda_1, \lambda_2, \lambda_3)$, since providing a vehicle is usually the most expensive cost component, followed by the drivers' salaries, and the costs for fuel.

3 Sweep Algorithms for Structured Time Windows

In this section we describe several variants of the sweep algorithm. First, we introduce some more notation and definitions.

Tree-Feasibility: For a tour \mathscr{A} let $\mathscr{A}_w \subseteq \mathscr{A}$ be the set of customers assigned to $w \in \mathscr{W}$. We consider the complete directed graph with vertex set \mathscr{A}_w and edge weights $t(a_i, a_j) + s(a_i)$ assigned to each edge (a_i, a_j) . The existence of a spanning arborescence through \mathscr{A}_w with length of at most $e_w - s_w$ for all time windows w forms a necessary condition for the time-feasibility of a tour \mathscr{A} . In this case we call \mathscr{A} tree-feasible. Existence of such an arborescence can be checked in $\mathscr{O}(|\mathscr{A}_w|^2)$ time [7]. Hence, time-infeasibility of a potential tour can often be detected without solving a time-consuming MILP. Moreover, we apply the concept of tree-feasibility in the Corrective Sweep algorithm below.

Angles of Customers: Any sweep algorithm is based on the polar coordinate representation of the customers \mathscr{C} , where the depot *d* forms the origin of the coordinate system and $\theta(a) \in [0, 2\pi]$ denotes the angle component of a customer $a \in \mathscr{C}$. The direction of the zero angle θ_0 is a choice parameter of the algorithm as it impacts the result. We choose the zero angle such that it separates the two consecutive customers with the largest angle gap, i.e., $\max_{a,b\in\mathscr{C}} \theta(a) - \theta(b)$ is minimized. Moreover, we run all algorithms in clockwise and counterclockwise direction and in each case select the variant that gives the better result.

In the following, we denote $\mathscr{C}(\theta, \theta') = \{a \in \mathscr{C} \mid \theta(a) \in [\theta, \theta'[\}, \mathscr{C}_w = \{a \in \mathscr{C} \mid w(a) = w\}, \text{ and } \mathscr{C}_w(\theta, \theta') = \mathscr{C}(\theta, \theta') \cap \mathscr{C}_w.$

3.1 Sweep Strategies

We propose the following general strategy to obtain high-quality solutions for a given cVRPTW or cVRPsTW instance. It consists of three steps:

- 1. Use a variant of the sweep algorithm to determine a feasible clustering of \mathscr{C} .
- 2. Apply local improvement heuristics to enhance the quality of the clustering.
- 3. Compute the optimal route for each cluster.

In all three steps, the *Traveling Salesperson Problem with Time Windows* (TSPTW) respectively the *Traveling Salesperson Problem with structured Time Windows* (TSPsTW) occurs as a subproblem to check the time-feasibility or to obtain the optimal solution of a single tour. We apply two MILP formulations that have been proposed in previous work [4], a general one for the TSPTW, and a more efficient one that is tailored to the TSPsTW. Following the lexicographical order, it first minimizes λ_2 , and then λ_3 while keeping λ_2 fixed. Next let us relate our notation to the well-known sweep algorithm [3] for the cVRP.

Traditional Sweep: The clustering method proposed in [3] relates to Step 1. It finds angles θ_i such that the *i*-th cluster is given by $\mathscr{C}(\theta_{i-1}, \theta_i)$ as follows: Set $\theta_0 = 0$. For i = 1, 2, ... make θ_i as large as possible such that $\mathscr{C}(\theta_{i-1}, \theta_i)$ forms a capacity-feasible cluster. Here, and in all following algorithms, the range for *i* is chosen such that all customers are scheduled.

Next, we propose a natural generalization of the Traditional Sweep algorithm that works for instances having structured (cVRPsTW), as well as for instances having arbitrary time windows (cVRPTW).

Simple Sweep: Choose the angle θ_i as large as possible while ensuring that the resulting cluster $\mathscr{C}(\theta_{i-1}, \theta_i)$ is still small enough such that a time- and capacity-feasible tour that visits all contained customers can be found. We check the time-feasibility using the TSPTW- resp. TSPsTW-MILP.

Now we present two variants of the sweep algorithm that exploit the additional structure of the time windows of cVRPsTW instances.

Window-wise Sweep: In case of structured time windows there are quite few time windows in comparison to customers, i.e., $q \ll n$. Hence, we can define window-dependent angles θ_i^j , $i \ge 1$, j = 1, ..., q, such that the *i*-th cluster is given by $\bigcup_{j=1}^{q} \mathscr{C}_{w_j}(\theta_{i-1}^j, \theta_i^j)$. We propose to add the customers to the clusters window by window. The resulting algorithm is described as follows: While there are unclustered customers in \mathscr{C}_{w_j} , make θ_i^j as large as possible such that $\mathscr{C}_{w_j}(\theta_{i-1}^j, \theta_i^j)$ can be added to the *i*-th cluster while ensuring that the cluster stays time- and capacity-feasible. If necessary, increase the number of clusters.

Corrective Sweep: If vehicle capacity is a stronger restriction than time windows, then Window-wise Sweep creates large sectors for the first few time windows and runs out of capacity later on, causing an increased need of vehicles. Therefore, we propose another variant that prevents this from happening. We first initialize angels θ_i^0 of maximal size such that all sets $\mathscr{C}(\theta_{i-1}^0, \theta_i^0)$ are capacity- and tree-feasible. Then we start with empty clusters. For j = 1, ..., q we set $\theta_i^j = \theta_i^{j-1}$ and add the customers of \mathscr{C}_{w_j} accordingly. Since this may result in some infeasible clusters, the angles θ_i^j have to be adjusted. Starting with i = 1, we check whether the *i*-th cluster is still feasible. If not, we try to reduce the size of cluster *i* by increasing the angles $\theta_{i-1}^j, ..., \theta_1^j$, such that cluster *i* becomes feasible while the clusters 1, ..., i - 1 remain feasible. If this procedure does not succeed, we decrease θ_i^j until the *i*-th cluster becomes feasible. Then, we increment *i* and repeat this procedure. If necessary, we increase the number of clusters.

Local Improvement Heuristic: After obtaining an initial clustering using one of the heuristics described above, we aim to improve the clustering during Step 2. In case that the clustering was obtained using the Simple Sweep algorithm, we set $\theta_i^j = \theta_i$. For each angle θ_i^j we try to improve the objective function iteratively by decreasing or increasing the angle. In each step the angle is slightly altered such that one customer moves to another cluster. A change is accepted if the lexicographical objective is improved. This procedure is repeated until a local minimum is reached.

Routing: In Step 3 a tour for each cluster is obtained by solving the TSPTW- or TSPsTW-MILP with lexicographical objective (λ_2, λ_3) to optimality.

4 Computational Experiments

As to our best knowledge, none of the available cVRPTW benchmark instances comply with the considered AHD use-case, we created a new benchmark set that resembles urban settlement structures in order to provide mean-

Vehicle Capacity	200				400			
Runtime/Objectives	t	λ_1	λ_2	λ_3	t	λ_1	λ_2	λ_3
Without Improvement								
Simple Sweep	7.2	54.0	507.7	332.7	41.0	42.1	406.5	315.3
Window-wise Sweep	96.9	64.4	381.2	371.5	306.7	35.0	340.2	328.1
Corrective Sweep	7.6	54.0	507.2	332.5	34.6	41.5	402.1	314.8
With Improvement								
Simple Sweep	241.0	54.0	480.3	318.1	1324.7	41.9	368.5	300.8
Window-wise Sweep	141.9	64.4	378.5	369.3	679.1	35.0	336.2	324.8
Corrective Sweep	248.6	53.9	479.9	318.0	1353.5	41.4	366.8	300.5

Table 1: Results for instances with n = 2000. We report average values over 100 instances each. The runtime in seconds is denoted by t, while λ_1 denotes the number of vehicles used and λ_2 resp. λ_3 denote the tour duration resp. the travel time in hours.

ingful computational experiments. We placed customers on a $20 \text{ km} \times 20 \text{ km}$ square grid (roughly the size of Vienna). In order to achieve varying customer densities, only 20 % of the customer locations are sampled from a two-dimensional uniform distribution. The remaining 80 % of the customer locations have been sampled from clusters whose centers and shapes have been randomly sampled. Travel times are calculated proportional to the Euclidean distances rounded to integer seconds. As proposed by Pan et al. [5] we assume a travel speed of 20 km/h. Each customer is randomly assigned to one time window out of a set of ten consecutive time windows, where each is one hour long. We assume the service time at each customer to be 5 minutes. The order weights are sampled from a truncated normal distribution centered around 5 units. We consider configurations with n = 250,500,1000,2000 and C = 200,400. Choosing C = 200 results in the vehicle capacity being the bottleneck, while choosing C = 400 causes that the time window constraints are the most limiting factor. For each configuration we constructed 100 instances and report the average results. All instances can be downloaded from http://tinyurl.com/vrpstw.

All experiments were performed on an Ubuntu 14.04 machine equipped with an Intel Xeon E5-2630V3 @ 2.4 GHz 8 core processor and 132 GB RAM. We use Gurobi 8.0 in single thread mode to solve the MILPs. To demonstrate the effectiveness of the improvement heuristics, we compare the case where all three steps are performed against the case where Step 2 is omitted.

In Table 1 we present the results of the computational study for n = 2000. In the case that the vehicle capacity is the more limiting factor, i.e., C = 200, Simple Sweep and Corrective Sweep produced nearly identical results with respect to all three objectives. In terms of λ_1 both algorithms clearly outperform Window-wise Sweep. However, with respect to λ_2 , Window-wise Sweep produced the best results. In the case that time window constraints pose the strongest restriction, i.e., C = 400, Window-wise Sweep clearly produced the best results with respect to λ_1 and λ_2 . In general, we suggest to apply Corrective Sweep as an algorithm producing good solutions in most cases. If the time window constraints are the most limiting factor of the considered instances, or if λ_2 is more relevant than λ_1 , then Window-wise Sweep is the best choice.

As it is rather hard to remove a whole vehicle from a schedule, we notice that applying our local improvement heuristic rarely results in a reduction of the primary objective λ_1 . However, the study shows that the improvement heuristic heavily impacts the schedule duration λ_2 , while also having positive impact on the travel time λ_3 . The experiments show that our heuristics, when applied to large instances containing 2000 customers, take a few seconds to around five minutes to produce a feasible schedule, and below 25 minutes to produce an improved schedule.

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