A SAT-based System for Consistent Query Answering

Akhil A. Dixit¹ and Phokion G. Kolaitis^{1,2}

University of California Santa Cruz
² IBM Research - Almaden

Abstract. An inconsistent database is a database that violates one or more integrity constraints, such as functional dependencies. Consistent Query Answering is a rigorous and principled approach to the semantics of queries posed against inconsistent databases. The consistent answers to a query on an inconsistent database is the intersection of the answers to the query on every repair, i.e., on every consistent database that differs from the given inconsistent one in a minimal way. Computing the consistent answers of a fixed conjunctive query on a given inconsistent database can be a coNP-hard problem, even though every fixed conjunctive query is efficiently computable on a given consistent database.

We designed, implemented, and evaluated CAvSAT, a SAT-based system for consistent query answering. CAvSAT leverages a set of natural reductions from the complement of consistent query answering to SAT and to Weighted MaxSAT. The system is capable of handling unions of conjunctive queries and arbitrary denial constraints, which include functional dependencies as a special case. We report results from experiments evaluating CAvSAT on both synthetic and real-world databases. These results provide evidence that a SAT-based approach can give rise to a comprehensive and scalable system for consistent query answering.

1 Introduction

Managing inconsistencies in databases is a challenge that arises in several different contexts. Data cleaning is the main approach towards managing inconsistent databases (see the survey [16]). In data cleaning, clustering techniques and/or domain knowledge are used to resolve violations of integrity constraints in a given inconsistent database, thus producing a single consistent database. This approach, however, is often *ad hoc*; for example, if a person has two different social security numbers in a database, which of the two should be kept?

The framework of database repairs and consistent query answering, introduced by Arenas, Bertossi, and Chomicki [3], is an alternative, and arguably more principled, approach to data cleaning. In contrast to data cleaning, the inconsistent database is left as is; instead, inconsistencies are handled at query time by considering all possible repairs of the inconsistent database, where a *repair* of an inconsistent database I is a consistent database I that differs from I in a "minimal" way. The main algorithmic problem in this framework is to compute the *consistent answers* to a query q on a given database I, that is, the tuples that lie in the intersection of the results of q applied on each repair of I (see the monograph [6]). Computing the consistent answers to a query q on I can be computationally harder than evaluating q on I, because an

inconsistent database may have exponentially many repairs. By now there is an extensive literature on the computational complexity of the consistent answers for different classes of constraints and queries [7,17,21,28,29]. For key constraints (the most common constraints) and for conjunctive queries (the most frequently asked queries), the consistent answers appear to exhibit an intriguing trichotomy, namely, the consistent answers of every fixed conjunctive query under key constraints are either first-order rewritable (hence, polynomial-time computable), or are polynomial-time computable but not first-order rewritable, or are coNP-complete. So far, this trichotomy has been proved for self-join free conjunctive queries by Koutris and Wijsen [20,21]. Moreover, Koutris and Wijsen designed a quadratic algorithm that, given such a conjunctive query and a set of key constraints, determines the side of the trichotomy in which the consistent answers to the query fall. Prior to this work, Fuxman and Miller identified a class of conjunctive queries, called C_{forest} , whose consistent answers are FO-rewritable [11,13]. Membership in C_{forest} , however, is sufficient but not necessary condition for the FO-rewritability of the consistent answers.

Several academic prototype systems for consistent query answering have been developed [4,5,9,11,12,14,18,23,24]. In particular, the ConQuer system [11,12] is tailored to queries in the class C_{forest} . Other systems use logic programming [5,14], compact representations of repairs [8], or reductions to solvers. Specifically, the system in [23] uses reductions to answer set programming, while the EQUIP system in [18] uses reductions to binary integer programming and the subsequent deployment of CPLEX. It is fair to say, however, no comprehensive and scalable system for consistent query answering exists at present; this state of affairs has impeded the broader adoption of the framework of repairs and consistent answers as a principled alternative to data cleaning.

In this paper, we report on a SAT-based system for consistent query answering, which we call CAvSAT (Consistent Answers via SAT). The CAvSAT system leverages natural reductions from the complement of consistent query answering to SAT and to WEIGHTED MAXSAT. As such, it can handle the consistent answers to unions of conjunctive queries under *denial* constraints, a broad class of integrity constraints that include functional dependencies (hence also key constraints) as a special case. CAvSAT is the first SAT-based system for consistent query answering. We carried out a preliminary stand-alone evaluation of CAvSAT on both synthetic and real-world databases. The first set of experiments involved the consistent answers of conjunctive queries under key constraints on synthetic databases in which each relation has up to one million tuples. One of the a priori unexpected findings is that, for conjunctive queries whose consistent answers are first-order rewritable, CAvSAT had comparable or even better performance to evaluating the first-order rewritings using a database engine, such as PostgreSQL. The second set of experiments involved the consistent answers of (unions of) conjunctive queries under functional dependencies on restaurant inspection records in Chicago and New York with some of the relations exceeding 200000 tuples. The CAvSAT source code is available at the GitHub repository https://github.com/uccross/cavsat via a BSD-style license.

While much more work remains to be done, the experimental finding reported here provide evidence that a SAT-based approach can indeed give rise to a comprehensive and scalable system for consistent query answering.

2 Basic Notions and Background

Databases, Constraints, and Queries A *relational database schema* \mathcal{R} is a finite collection of relation symbols, each with a fixed positive integer as its arity. The attributes of a relation symbol are names for its columns; attributes can also be identified by their positions, thus $Attr(R) = \{1,...,n\}$ denotes the set of attributes of R. An \mathcal{R} -database instance or, simply, an \mathcal{R} -instance is a collection I of finite relations R^I , one for each relation symbol R in \mathcal{R} . An expression of the form $R^I(a_1,...,a_n)$ is a fact of the instance I if $(a_1,...,a_n) \in R^I$. Every \mathcal{R} -instance can be identified with the (finite) set of its facts. The active domain of I is the set of all values occurring in facts of I.

Relational database schemas are often accompanied by a set of integrity constraints that impose semantic restrictions on the allowable instances. A functional dependency (FD) $x \to y$ on a relation symbol R is an integrity constraint asserting that if two facts agree on the attributes in x, then they must also agree on the attributes in y. A key is a minimal subset x of Attr(R) such that the FD $x \to Attr(R)$ holds. In this case, the attributes in x are called key attributes of R and they are denoted by underlining their corresponding positions; thus, $R(\underline{A},\underline{B},C)$ denotes that the attributes A and B form a key of B. Every functional dependency is expressible in first-order logic. For example, the key constraint A, $B \to C$ in R(A,B,C) is expressed by the first-order formula

$$\forall x, y, z, z' (R(x, y, z) \land R(x, y, z') \rightarrow z = z')$$

Functional dependencies are an important special case of *denial constraints* (DCs), which are expressible by first-order formulas of the form

$$\forall x_1, ..., x_n \neg (\varphi(x_1, ..., x_n) \land \psi(x_1, ..., x_n)),$$

or, equivalently,

$$\forall x_1, ..., x_n (\varphi(x_1, ..., x_n) \to \neg \psi(x_1, ..., x_n)),$$

where $\varphi(x_1,...,x_n)$ is a conjunction of atomic formulas and $\psi(x_1,...,x_n)$ is a conjunction of expressions of the form $(x_i \text{ op } x_j)$ with each op a built-in predicate, such as $=, \neq, <, >, \leq, \geq$. In words, a denial constraint prohibits a set of tuples that satisfy certain conditions from appearing together in a database instance.

Let k be a positive integer. A k-ary query on a relational database schema \mathcal{R} is a function q that takes an \mathcal{R} -instance I as argument and returns a k-relation q(I) on the active domain of I as value. A boolean query on \mathcal{R} is a function that takes an \mathcal{R} -instance I as argument and returns true or false as value. As is well known, first-order logic has been successfully used as a query language. In fact, it forms the core of SQL, the main commercial database query language.

A *conjunctive query* is a first-order formula built using the relational symbols, conjunctions, and existential quantifiers. Thus, each conjunctive query is expressible by a first-order formula of the form

$$q(\boldsymbol{z}) := \exists \boldsymbol{w} \ (R_1(\boldsymbol{x_1}) \wedge ... \wedge R_m(\boldsymbol{x_m})),$$

where each x_i is a tuple consisting of variables and constants, z and w are tuples of variables, and the variables in $x_1, ..., x_m$ appear in exactly one of z and w. Clearly, a conjunctive query with k free variables z is a k-ary query, while a conjunctive query with no free variables (i.e., all variables are existentially quantified) is a boolean query.

Conjunctive queries are also known as $select\text{-}project\text{-}join}$ (SPJ) queries and are among the most frequently asked queries in databases. For example, the binary conjunctive query $q(s,t) := \exists c(\operatorname{Enrolls}(s,c) \land \operatorname{Teaches}(t,c))$ returns the set of all pairs (s,t) such that student s is enrolled in a course taught by teacher t, while the boolean conjunctive query $q() := \exists x, y, z(E(x,y) \land E(y,z) \land E(z,x))$ tests whether or not a graph with an edge relation E contains a triangle.

Repairs and Consistent Answers Let \mathcal{R} be a database schema and let Σ be a set of integrity constraints on \mathcal{R} . An \mathcal{R} -instance I is *consistent* if $I \models \Sigma$, that is, I satisfies every constraint in Σ ; otherwise, I is *inconsistent*. A *repair* of an inconsistent instance I w.r.t. Σ is a consistent instance J that differs from I in a "minimal" way. Different notions of minimality give rise to different types of repairs (see [6] for a comprehensive survey). Here, we focus on *subset repairs*, the most extensively studied type of repairs. An instance J is a *subset repair* of an instance I if $J \subseteq I$ (where I and J are viewed as sets of facts), $J \models \Sigma$, and there exists no instance J' such that $J' \models \Sigma$ and $J \subset J' \subset I$. From now on, by *repair* we mean a subset repair. Arenas, Bertossi, and Chomicki [3] used repairs to give rigorous semantics to query answering on inconsistent databases. Specifically, assume that g is a query, g is an g-instance, and g is a tuple of values. We say that g is a *consistent answer* (also referred as a *certain answer*) to g on g w.r.t. g if g

$$\operatorname{CONS}(q,I,\varSigma) = \bigcap \{q(J): J \text{ is a repair of } I \text{ w.r.t. } \varSigma\}.$$

If Σ is a fixed set of integrity constraints and q is a fixed query, then the main computational problem associated with the consistent answers is: given an instance I, compute $\mathrm{Cons}(q,\,I,\,\Sigma)$. If q is a boolean query, then computing the certain answers becomes the decision problem $\mathrm{Certainty}(q,\Sigma)$: given an instance I, is q true on every repair I of I w.r.t. Σ ? When the constraints Σ are understood from the context, we will write $\mathrm{Cons}(q,I)$ and $\mathrm{Certainty}(q)$, instead of $\mathrm{Cons}(q,I,\Sigma)$ and $\mathrm{Certainty}(q,\Sigma)$.

Computational Complexity of Consistent Answers If Σ is a fixed finite set of denial constraints and q is a k-ary conjunctive query, where $k \geq 1$, then the following problem is in coNP: given an instance I and a tuple t, is t a certain answer to q on I w.r.t. Σ ? This is so because to check that t is not a certain answer to q on I w.r.t. Σ , we guess a repair J of I and verify that $t \notin q(J)$ (note that J is a subset of I, evaluating a fixed conjunctive query on a given database is a polynomial-time task, and testing if J is a repair of I w.r.t. denial constraints is a polynomial-time task as well). Similarly, if q is a boolean conjunctive query, then the decision problem CERTAINTY (q, Σ) is in coNP.

Even for key constraints and boolean conjunctive queries, Certainty (q, Σ) exhibits a variety of behaviors within coNP. Indeed, consider the queries

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1. Path() := \exists x, y, z \ R(\underline{x}, y) \land S(\underline{y}, z);
2. Cycle() := \exists x, y \ R(\underline{x}, y) \land S(\underline{y}, x);
3. Sink() := \exists x, y, z \ R(\underline{x}, z) \land S(\overline{y}, z).
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Fuxman and Miller [13] showed that CERTAINTY(PATH) is FO-rewritable, i.e., there is a first-order definable boolean query q' such that CONS(PATH, $\Sigma, I) = q'(I)$, for every instance I. In fact, q' is $\exists x, y, z \ R(\underline{x}, y) \land S(\underline{y}, z) \land \forall y'(R(\underline{x}, y') \rightarrow \exists z' S(\underline{y'}, z'))$.

Wijsen [27] showed that CERTAINTY(CYCLE) is in P, but it is not FO-rewritable, while Fuxman and Miller [13] showed that CERTAINTY(SINK) is coNP-complete via a reduction from the complement of MONOTONE 3-SAT.

The preceding state of affairs sparked a series of investigations aiming to obtain classification results concerning the computational complexity of the consistent answers (e.g., see [15,17,22,26,27]). The most definitive result to date is a *trichotomy* theorem, established by Koutris and Wijsen [19,20,21], for boolean self-join free conjunctive queries, where a conjunctive query is *self-join free* if no relation symbol occurs more than once in the query. This trichotomy theorem asserts that if q is a self-join free conjunctive query with one key per relation symbol, then CERTAINTY(q) is FO-rewritable, or in P but not FO-rewritable, or coNP-complete. Moreover, there is a quadratic algorithm to decide, given such a query, which of the three cases of the trichotomy holds. It remains an open problem whether or not this trichotomy extends to arbitrary boolean conjunctive queries and to arbitrary functional dependencies or denial constraints.

3 Consistent Query Answering for Key Constraints

In this section, we assume that \mathcal{R} is a database schema and Σ is a finite set of primary key constraints on \mathcal{R} , i.e., there is one key constraint per each relation of \mathcal{R} . We first consider boolean conjunctive queries and, for each fixed boolean conjunctive query q, we give a natural polynomial-time reduction from CERTAINTY(q) to UNSAT. We then extend this reduction to non-boolean conjunctive queries, so that for every fixed non-boolean conjunctive query q, the consistent answers to q can be computed by iteratively solving WEIGHTED MAXSAT instances. In what follows, we heavily use the notions of key-equal groups of facts and minimal witnesses to a conjunctive query.

Definition 1. Key-Equal Group. Let I be an \mathcal{R} -instance. We say that two facts of a relation R of I are key-equal, if they agree on the key attributes of R. A set S of facts of I is called a key-equal group of facts if every two facts in S are key-equal, and no fact in S is key-equal to some fact in $I \setminus S$.

Definition 2. *Minimal Witness.* Let I be an \mathcal{R} -instance and let S be a sub-instance of I. We say that S is a *minimal witness* to a conjunctive query q on I, if $S \models q$, and for every proper subset S' of S, we have that $S' \not\models q$.

For each relation R of I, the key-equal groups of R are computed by an SQL query that involves grouping the key attributes of R. Similarly, the set of minimal witnesses to a fixed conjunctive query q on I are computed efficiently as follows. A unique integer factID is attached to each fact, by adding an attribute FactID to each relation in I that appears in q. Thus, a new instance I' is built, where each relation $R'(FactID, \underline{A}, \underline{B})$ in I' is obtained from a relation $R(\underline{A}, \underline{B})$ in I. A new non-boolean query q' is constructed, such that each atom $R'(factID_R, \underline{x}, \underline{y})$ in q' is constructed from an atom $R(\underline{x}, \underline{y})$ of q. The variables of q' that correspond to the FactID attributes are not existentially quantified. It is easy to see that each tuple (without duplicate factIDs) in q'(I') is in 1-1 correspondence with a minimal witness to q on I.

Boolean Conjunctive Queries Let q be a fixed boolean conjunctive query over \mathcal{R} .

Reduction 1 Given an R-instance I, we construct a CNF-formula ϕ as follows.

For each fact f_i of I, introduce a boolean variable x_i , $1 \le i \le n$. Let \mathcal{G} be the set of key-equal groups of facts of I, and let \mathcal{W} be the set of minimal witnesses to g on I.

- For each $G_j \in \mathcal{G}$, construct the clause $\alpha_j = \bigvee_{f_i \in G_j} x_i$.
- For each $W_j \in \mathcal{W}$, construct the clause $\beta_j = \bigvee_{f_i \in W_j} \neg x_i$.
- Construct the boolean formula $\phi = \begin{pmatrix} |\mathcal{G}| & \alpha_i \\ \wedge & \alpha_i \end{pmatrix} \wedge \begin{pmatrix} |\mathcal{W}| & \beta_j \\ \wedge & j=1 \end{pmatrix}$.

Proposition 1. Let ϕ be the CNF-formula constructed using Reduction 1.

- The size of ϕ is polynomial in the size of I.
- The formula ϕ is satisfiable if and only if CERTAINTY(q, Σ) is false on I.

The proofs of all propositions are given in the Appendix.

Non-boolean Conjunctive Queries Let q be a fixed non-boolean query on \mathcal{R} , i.e., q has one or more free variables. We extend Reduction 1 to Reduction 2, so that one can reason about the certain answers to q on an \mathcal{R} -instance I using the satisfying assignments of the CNF-formula ϕ constructed via Reduction 2.

We use the term *potential answers* to refer to the answers to q on I. If a_l is such a potential answer, we write $q[a_l]$ to denote the boolean conjunctive query obtained from q by replacing the free variables in the body of q by corresponding constants from a_l .

Reduction 2 Given an R-instance I, we construct a CNF-formula ϕ as follows.

For each fact f_i of I, introduce a boolean variable x_i , $1 \le i \le n$, Let \mathcal{G} be the set of key-equal groups of facts of I and let \mathcal{A} be the set of potential answer to q on I. For each $\mathbf{a}_l \in \mathcal{A}$, let \mathcal{W}^l denote the set of minimal witnesses to the boolean query $q[\mathbf{a}_l]$ on I. For each $\mathbf{a}_l \in \mathcal{A}$, introduce a boolean variable p_1 , $1 \le l \le ..., |\mathcal{A}|$.

- For each $G_j \in \mathcal{G}$, construct the clause $\alpha_j = \bigvee_{f_i \in G_j} x_i$.
- For each $\mathbf{a}_l \in \mathcal{A}$ and for each $W_j^l \in \mathcal{W}^l$, construct the clause $\beta_j^l = \begin{pmatrix} \vee \\ f_i \in W_j^l \end{pmatrix} \vee \neg p_l$.
- Construct the boolean formula $\phi = \begin{pmatrix} |\mathcal{G}| & \alpha_i \\ \wedge & \alpha_i \end{pmatrix} \wedge \begin{pmatrix} |\mathcal{A}| & \langle |\mathcal{W}^l| & \beta_j \\ \wedge & & \gamma_j & \beta_j \end{pmatrix}$.

Proposition 2. Let ϕ be the CNF-formula constructed using Reduction 2.

- The size of ϕ is polynomial in the size I.
- There exists a satisfying assignment to ϕ in which a variable p_l is set to 1 if and only if $\mathbf{a}_l \notin \text{CONS}(q, I)$.

Example 1. Consider the flights information database in Table 1. The database schema has three relations, namely, Airlines, Tickets, and Flights; the key attributes of each relation are underlined. This database is inconsistent, as the sets $\{f_1, f_3\}$ and $\{f_8, f_9\}$ of facts violate the key constraints of the relations Airlines and Flights, respectively.

Suppose we want to find out the codes of the flights that belong to an airline from Canada and fly to the airport OAK. This can be expressed by the unary conjunctive query $q(x) := Flights(x, y, z, p, 'OAK', q, r) \land Airlines(z, 'Canada')$.

Table 1: Flight information records.

	Airlines						
Fact	AIRLINE	COUNTRY					
f_1	Southwest	United States					
f_2	Jazz Air	Canada					
f_3	Southwest	Canada					

Tickets					
Fact	<u>PNR</u>	CODE	CLASS	FARE	
f_4	MJ9C8R	SWA 1568	Economy	430 USD	
f_5	KLF88V	MI 471	First	914 USD	
f_6	NJ5RT3	SWA 1568	First	112 USD	

Flights							
Fact	CODE	DATE	AIRLINE	FROM	TO	DEPARTURE	ARRIVAL
f_7	JZA 8329	01/29/19	Jazz Air	GEG	OAK	16:12 PST	18:00 PST
f_8	SWA 1568	01/29/19	Silkair	YYZ	YAM	18:55 EST	18:44 EST
f_9	SWA 1568	01/29/19	Southwest	LAX	OAK	16:18 PST	17:25 PST

There are two potential answers to q, namely, 'JZA 8329' and 'SWA 1568', so we introduce their corresponding variables p_1 and p_2 . Since the facts f_1 and f_3 form a keyequal group, we construct an α -clause $(x_1 \vee x_3)$. Similarly, since the set $\{f_2, f_7\}$ of facts is a minimal witness to q ['JZA 8329'], we construct the β -clause $(\neg x_2 \vee \neg x_7 \vee \neg p_1)$. By continuing this way, we obtain the following CNF-formula ϕ :

 $(x_1 \lor x_3) \land x_2 \land x_4 \land x_5 \land x_6 \land x_7 \land (x_8 \lor x_9) \land (\neg x_2 \lor \neg x_7 \lor \neg p_1) \land (\neg x_3 \lor \neg x_9 \lor \neg p_2).$ Clauses x_2 , x_7 , and $(\neg x_2 \lor \neg x_7 \lor \neg p_1)$ force p_1 to take value 0 in each satisfying assignment of ϕ , because the facts f_2 and f_7 appear in every repair of I, thus making 'JZA 8329' a consistent answer to q. In contrast, there is a satisfying assignment of ϕ in which p_2 is set to 1, which implies that 'SWA 1568' is not a consistent answer to q.

Optimizing the reductions In real-life applications, a large part of the inconsistent database is consistent. For a boolean query q, if a minimal witness to q is present in the consistent part of the database instance, then we can immediately conclude that CERTAINTY(q, I, Σ) is true. This can be checked with simple SQL queries that involve grouping on the key attributes of each relation. Similarly, for non-boolean queries, the consistent answers coming from the witnesses that belong to the consistent part of the database can be computed efficiently using SQL queries. All additional consistent answers can then be found using the preceding reduction. In this case, we need to introduce boolean variables corresponding to only those facts that contribute to the additional potential answers. This significantly reduces the size of the CNF-formulas

produced by Reductions 1 or 2. This optimization has been used earlier in [18], where $Cons(q, I, \Sigma)$ was reduced to an instance of binary integer programming.

4 Consistent Query Answering Beyond Key Constraints

In this section, we consider the broader class of denial constraints and the more expressive class of unions of conjunctive queries. Note that computing the consistent answers of unions of conjunctive queries under denial constraints is still in coNP, but the consistent answers of a union $Q := q_1 \cup \ldots \cup q_k$ of conjunctive queries q_1, \ldots, q_k is not, in general, equal to the union of the consistent answers of q_1, \ldots, q_k .

We give a polynomial-time reduction from $\mathrm{Cons}(Q,I,\Sigma)$ to UnSAT , where Σ is a fixed finite set of denial constraints and Q is a fixed union of non-boolean conjunctive queries. The potential answers to Q are treated in the same way as the potential answers to the conjunctive query q in Reduction 2; to this effect, we introduce a boolean variable for each potential answer. The reduction we give here relies on the notions of *minimal violations* and *near-violations* to the set of denial constraints that we introduce next.

Definition 3. *Minimal violation.* Assume that Σ is a set of denial constraints, I is an \mathcal{R} -instance, and S is a sub-instance of I. We say that S is a *minimal violation* to Σ , if $S \not\models \Sigma$ and for every set $S' \subset S$, we have that $S' \models \Sigma$.

Definition 4. Near-violation. Assume that Σ is a set of denial constraints, I is an \mathcal{R} -instance, S is a sub-instance of I, and f is a fact of I. We say that S is a near-violation w.r.t. Σ and f, if $S \models \Sigma$ and $S \cup \{f\}$ is a minimal violation to Σ . As a special case, if $\{f\}$ itself is a minimal violation to Σ , then we say that there is exactly one near-violation w.r.t. f, and it is the singleton $\{f_{true}\}$, where f_{true} is an auxiliary fact.

For a fixed finite set Σ of denial constraints, the set of minimal violations to Σ on a given database instance I are computed as follows. The body of a denial constraint $d \in \Sigma$ is treated as a boolean conjunctive query q_d , possibly containing atomic formulas from d that use built-in predicates such as =, \neq , <, >, \leq , and \geq , in addition to the relation symbols. The set of minimal witnesses to q_d on I is computed as described in Section 3, which is also, precisely, the set of minimal violations to d. The union of the sets of minimal violations over all denial constraints in Σ gives us the set of minimal violations to Σ . For each fact $f \in I$, the set of near-violations to Σ w.r.t. f can be obtained by removing f from every minimal violation to Σ that contains f.

Let \mathcal{R} be a database schema, let Σ be a fixed finite set of denial constraints on \mathcal{R} , and let $Q := q_1 \cup \ldots \cup q_k$ be a union of conjunctive queries q_1, \ldots, q_k . Let I be an \mathcal{R} -instance, and let Q be the fixed union of k non-boolean conjunctive queries q_1, \ldots, q_k .

Reduction 3 Given an R-instance I, we construct a boolean formula ϕ' as follows. Compute the following sets:

- V: the set of minimal violations to Σ on I.
- \mathcal{N}^i : the set of near-violations to Σ , on I, w.r.t. each fact $f_i \in I$.
- A: the set of potential answers to Q on I.
- W^l : the set of all minimal witnesses to $Q[a_l]$ on I, for each $a_l \in A$.

For each fact f_i of I, introduce a boolean variable x_i , $1 \le i \le n$. For the auxiliary fact f_{true} , introduce a constant $x_{true} = true$. For each $N_i^i \in \mathcal{N}^i$, introduce a boolean variable y_i^i , and for each $a_l \in A$, introduce a boolean variable p_l .

- 1. For each $V_j \in \mathcal{V}$, construct a clause $\alpha_j = \bigvee_{f_i \in V_j} \neg x_i$. 2. For each $\mathbf{a}_l \in \mathcal{A}$ and for each $W_j^l \in \mathcal{W}^l$, construct a clause $\beta_j^l = \left(\bigvee_{f_i \in W^l} \neg x_i \right) \lor \neg p_l.$
- 3. For each $f_i \in I$, construct a clause $\gamma_i = x_i \vee \left(\bigvee_{N^i \in \mathcal{N}^i} y^i_j\right)$.
- 4. For each variable y_j^i , construct an expression $\theta_j^i = y_j^i \leftrightarrow \left(\bigwedge_{f_d \in N_i^i} x_d \right)$.
- 5. Construct the following boolean formula ϕ :

$$\phi' = \begin{pmatrix} |\mathcal{V}| \\ \wedge \\ \alpha_i \end{pmatrix} \wedge \begin{pmatrix} |\mathcal{A}| \\ \wedge \\ l=1 \end{pmatrix} \begin{pmatrix} |\mathcal{W}^l| \\ \wedge \\ j=1 \end{pmatrix} \beta_j^l \end{pmatrix} \wedge \begin{pmatrix} |I| \\ \wedge \\ i=1 \end{pmatrix} \begin{pmatrix} |\mathcal{N}^i| \\ \wedge \\ j=1 \end{pmatrix} \theta_j^i \wedge \gamma_i \end{pmatrix}$$

Proposition 3. Let ϕ' be the boolean formula constructed using Reduction 3.

- The formula ϕ' can be transformed to an equivalent CNF-formula ϕ whose size is polynomial in the size of I.
- There exists a satisfying assignment to ϕ' in which a variable p_l is set to 1 if and only if $a_l \notin Cons(Q, I, \Sigma)$.

Example 2. Consider the database instance from Table 1. In addition to the three key constraints from Example 1, suppose the schema now has two additional integrity constraints: (a) if a flight departs from YYZ, then its airline must be Jazz Air; and (b) for Southwest airlines, if two tickets have the same code, then the ticket with an economy class must have lower fare than the one with the first class. These can be expressed as the following denial constraints:

(a)
$$\forall x, y, z, w, p, q \neg (Flights(x, y, z, 'YYZ', w, p, q) \land z \neq 'Jazz Air')$$

(b) $\forall x, y, z, w, p, q \neg (Flights(x, y, 'Southwest', z, w, p, q) \land Tickets(x, x, y, z, w, p, q, z, w, p, z, w, p, z, w, z,$

(b)
$$\forall x, y, z, w, p, q \neg (Flights(x, y, `Southwest', z, w, p, q) \land Tickets(r, x, `First', t) \land Tickets(r', x, `Economy', t') \land t \leq t')$$

Let us say that we want to find the PNR numbers of the tickets booked with first class, or with Silkair airlines. This can be expressed as the union $Q := q_1 \cup q_2$ of two unary conjunctive queries, where

$$q_1(x) := \exists x, y, z \; \textit{Tickets}(x, y, '\textit{First'}, z)$$

 $q_2(x) := \exists x, y, z, w, p, q, r, s, t \; \textit{Tickets}(x, y, z, w) \land \textit{Flights}(y, '\textit{Silkair'}, p, q, r, s, t)$

The minimal witnesses to Q, the minimal violations to Σ , and the near-violations to Σ w.r.t. each fact of the database are shown in Figure 1. With these, we construct the α -, β -, γ -clauses, and the θ -expressions of ϕ , as shown in Figure 2. Even though, for simplicity, it is not mentioned in Reduction 3, we do the following optimization in practice: if $|N_i^i| = 1$, we do not introduce a variable y_i^i , but, we use the x-variable corresponding to the only fact in N_i^i . In each satisfying assignment to ϕ , the variable p_1

Minimal violations to Σ :

Fig. 1: Minimal violations, minimal witnesses, and near-violations in Example 2.

Near-violations to Σ :

Fig. 2: The α -, β -, γ -clauses, and the θ -expressions in Example 2.

```
 \begin{array}{l} \alpha\text{-clauses: } (\neg x_1 \vee \neg x_3), (\neg x_8), (\neg x_4 \vee \neg x_6 \vee \neg x_9) \\ \beta\text{-clauses: } (\neg x_5 \vee \neg p_1), (\neg x_6 \vee \neg p_2), (\neg x_4 \vee \neg x_8 \vee \neg p_3) \\ \gamma\text{-clauses: } (x_1 \vee y_1^1), (x_2), (x_3 \vee y_1^3), (x_4 \vee y_1^4), (x_5), (x_6 \vee y_1^6), (x_7), (x_8 \vee y_1^8), (x_9 \vee y_1^9) \\ \theta\text{-expressions: } (y_1^1 \leftrightarrow x_3), (y_1^3 \leftrightarrow x_1), (y_1^4 \leftrightarrow (x_6 \wedge x_9)), (y_1^6 \leftrightarrow (x_4 \wedge x_9)), (y_1^8 \leftrightarrow x_{true}), \\ (y_1^9 \leftrightarrow (x_4 \wedge x_6)) \end{array}
```

must take the value 0. In contrast, this is not the case for p_2 and p_3 . By Proposition 3, 'KLF88V' is a consistent answer to Q, but 'MJ9C8R' and 'NJ5RT3' are not.

5 Computing Consistent Answers via WEIGHTED MAXSAT

By Proposition 1, the consistent answer to a boolean conjunctive query over a schema \mathcal{R} with primary key constraints can be computed by solving the UNSAT instance constructed in Reduction 1. For non-boolean queries, however, in a CNF-formula ϕ constructed using Reduction 2 or 3, one needs to identify each variable p_l such that there exists at least one satisfying assignment to ϕ in which p_l gets set to 1. By Proposition 3, the corresponding potential answers can then be discarded for being inconsistent. One way to do this is as follows. Add a clause $(p_1 \vee ... \vee p_{|A|})$ to ϕ , and solve ϕ using a SAT solver. For each p_l that gets set to 1 in the solution of ϕ , remove the literal p_l from ϕ and then solve ϕ again. Repeat this process until ϕ is no longer satisfiable. At the end of this iterative process, the potential answers corresponding to the p-variables that still occur positively in ϕ are precisely the consistent answers to Q on I. This approach, however, requires many SAT instances to be solved when the number of potential answers is large. For this reason, we developed and tested a different method that uses solving WEIGHTED MAXSAT instances. The construction of these WEIGHTED MAXSAT instances is described in Reduction 4.

Reduction 4 *Let the setup be the same as that of Reduction 2 (or Reduction 3).*

- 1. Construct a CNF-formula ϕ using Reduction 2 (or Reduction 3).
- 2. Make all clauses in ϕ hard.
- 3. For each $\mathbf{a}_l \in \mathcal{A}$, construct a unit ϵ -clause $\epsilon_l = (p_l)$.
- 4. Make all ϵ -clauses soft, and of equal weights.
- 5. Construct the WCNF-formula $\psi = \phi \land \begin{pmatrix} |A| \\ \land \\ l=1 \end{pmatrix}$.

Algorithm 1 Eliminating Inconsistent Potential Answers

```
1: procedure ELIMINATEWITHMAXSAT(\psi, \mathcal{A})
 2:
        let ANS = bool array[|A|]
 3:
        for l=1 to |\mathcal{A}| do
 4:
            Ans[l] \leftarrow true
 5:
        let bool moreAnswers \leftarrow true
 6:
        while moreAnswers do
 7:
            moreAnswers \leftarrow false
            let opt \leftarrow MAXSAT(\psi)
 8:
                                                                9:
            for l=1 to |\mathcal{A}| do
10:
                if opt[p_l] = 1 then
11:
                    moreAnswers \leftarrow true
12:
                    ANS[l] \leftarrow false
13:
                    Remove the unit clause (p_l) from \psi
14:
                    Remove all clauses containing the literal \neg p_l from \psi
                    Add a new unit hard clause (\neg p_l) to \psi
15:
16:
        return ANS
```

The preceding Algorithm 1 computes the consistent answers by iteratively solving WEIGHTED MAXSAT instances. It takes as inputs the instance ψ constructed using Reduction 4 and the set $\mathcal A$ of potential answers. The idea is to eliminate, in each iteration, as many inconsistent answers from $\mathcal A$ as possible by solving ψ . After each iteration, ψ is modified in such a way that additional inconsistent answers, if any, can be eliminated in subsequent iterations. In Section 6, we carried out experiments in which it turned out that the number of iterations taken by Algorithm 1 is less than 4, even when there is a large number of potential answers.

Proposition 4. Algorithm 1 returns an array ANS such that $a_l \in Cons(Q, I)$ if and only if the entry ANS[l] is true.

6 Preliminary Experimental Results

We evaluated the performance of CAvSAT using two different scenarios. First, we experimented with large synthetically generated databases having primary key constraints. We implemented Reduction 2 without the optimization mentioned in Section 3. We also implement Reduction 4 and Algorithm 1. We found out that for seven non-boolean FO-rewritable queries that were also used in [18], CAvSAT significantly outperformed the database evaluation of the FO-rewritings obtained using the algorithm from [21]. We also implemented Reduction 2 with the optimization, and evaluated its performance on fourteen additional conjunctive queries whose consistent answers are coNP-complete or are in P but are not FO-rewritable. In the second scenario, we evaluated the performance of CAvSAT using Reduction 3 on a real-world database with functional dependencies. The definitions of the queries used in the experiments, as well as the FO-rewritings of the first seven queries, can be found in the Appendix.

Experimental Setup All experiments were carried out on a machine running on Intel Core i7 2.7 GHz, 64 bit Ubuntu 16.04, with 8GB of RAM. We used PostgreSQL

10.1 as an underlying DBMS, and MaxHS v3.0 solver [10] for solving the WEIGHTED MAXSAT instances. Our system is implemented in Java 9.04.

6.1 Synthetic Data Generation

The synthetic data were generated in two phases: (a) generation of consistent data; (b) injection of inconsistency into consistent data. The parameters used to generate the data were the number of tuples per relation (*rSize*), degree of inconsistency (*inDeg*), and the size of each key-equal group (*kSize*).

Generating consistent data Each relation in the consistent database was generated with the same number of tuples, so that injecting inconsistency with specified kSize and inDeg will make the total number of tuples in the relation equal to rSize. For each query used in the experiment, the data was generated so the evaluation of the query on the consistent database results in a relation that has the size 15% to 20% of rSize. The values of the third attribute in all of the ternary relations, were chosen from a uniform distribution in the range [1, rSize/10]. This was done to simulate a reasonably large number of potential answers. The remaining attributes take values from randomly generated alphanumeric strings of length 10.

Injecting inconsistency In each relation, the inconsistency was injected by inserting new tuples to the consistent data, that share the values of the key attributes with some already existing tuples from the consistent data. The parameter *inDeg* denotes in percentage the number of tuples per relation, that participate in a key violation. We conducted experiments with the varying values for *inDeg*, ranging from 5% to 15%. The values of *kSize* were uniformly distributed between 2 to 5. The non-key attributes of the newly injected tuples were uniform random alphanumeric strings of length 10.

6.2 Experimental Results

CAvSAT on FO-rewritable Queries In this set of experiments, we compare the performance of CAvSAT against the FO-rewritings of seven queries over the database with primary key constraints. For queries q_1, \ldots, q_7 , we computed the FO-rewritings using the algorithm of Koutris and Wijsen in [21]. We refer to these FO-rewritings as KW-FO-rewritings. Since the queries q_1, \ldots, q_4 happen to be in the class C_{forest} , we computed additional FO-rewritings for them using the algorithm implemented in the consistent query answering system ConQuer [11]; we refer to these rewritings as ConQuer-FO-rewritings. Each FO-rewriting was translated into SQL, and fed to PostgreSQL.

Table 2 shows the size of the WCNF-formulas produced by Reduction 4 without optimization (where Reduction 2 is used inside Reduction 4), on these queries over the databases having one million tuples per relation. Figure 3 shows the evaluation time of CAvSAT with these formulas using MaxHS v3.0 solver [10]. The letter E denotes the time required for encoding the problem into a WEIGHTED MAXSAT instance, and the letter S denotes the time taken by Algorithm 1. The percentage adjacent to the letters S and E denotes the degree of inconsistency. Figure 4 (left) shows the significant gain in performance due to the optimization, for databases of size one million tuples per relation and with 10% inconsistency. This is not surprising; since 90% of the data were consistent, it is expected that most of the consistent answers lie in the consistent part of

the database. Table 3 shows the size of the WCNF-formulas produced by Reduction 4, with the optimization in place.

Figure 4 (right) shows that for the queries q_1, \ldots, q_4 in the class C_{forest} , the performance of CAvSAT is slightly worse, but comparable, to their ConQuer-FO-rewritings. For all seven queries q_1, \ldots, q_7 , however, CAvSAT significantly outperformed their KW-FO-rewritings, as PostgreSQL hit the two hours timeout while evaluating each KW-FO-rewriting. In fact, this timeout was hit by all seven queries even for databases of size as small as 100K tuples per relation. For q_1, \ldots, q_7 , the average number of iterations taken by Algorithm 1 to eliminate all inconsistent potential answers was 2.85.

Table 2: Size of CNF-formula

Query	Variables	Clauses
q_1	2.08M	2.18M
q_2	2.07M	2.12M
q_3	3.15M	3.07M
q_4	3.23M	3.07M
q_5	2.07M	2.12M
q_6	3.3M	3.06M
q_7	3.25M	3.06M

Fig. 3: Evaluation time of CAvSAT without optimization, for 1M tuples/relation.

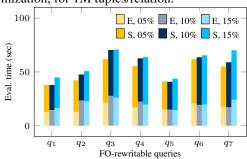
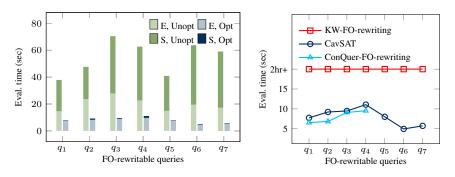


Fig. 4: Evaluation time of CAvSAT with and without optimization (left). Evaluation time of CAvSAT with optimization, in comparison with the KW-FO-rewriting and the ConQuer-FO-rewriting, for 1M tuples/relation with 10% inconsistency (right).



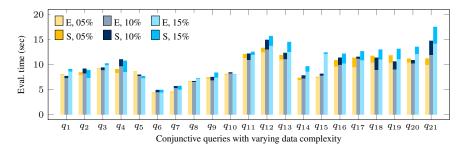
CAvSAT on Harder Queries In this set of experiments, we considered fourteen additional non-boolean conjunctive queries whose consistent answers are coNP-complete or in P but not FO-rewritable (Figure 7 in the Appendix). Figure 5 shows that the time required for the optimizing and then constructing the WEIGHTED MAXSAT instance using Reduction 4, dominates over the time taken by Algorithm 1. The solver takes comparatively more time for the queries that have more free variables or more atoms.

Table 3 shows the size of the CNF-formulas constructed by Reduction 4 (where Reduction 2 is used inside Reduction 4) in this experiment. The average number of iterations taken by Algorithm 1 to eliminate all inconsistent potential answers to a query was 3.2.

Table 3: The size of the CNF-formulas with optimization, for 1M tuples per relat	Table 3: The six	ze of the CNF-form	ulas with optimization	. for 1M tuples	per relation.
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Query	Variables	Clauses	Query	Variables	Clauses	Query	Variables	Clauses
q_1	16.5K	20.9K	q_8	16.6K	16.8K	q_{15}	14.9K	15K
q_2	68.6K	76.0K	q_9	58K	57.7K	q_{16}	58.8K	58.4K
q_3	31.9K	36.8K	q_{10}	31.3K	36.6K	q_{17}	40.1K	41.4K
q_4	117.2K	123.7K	q_{11}	105K	118.1K	q_{18}	107.5K	121.4K
q_5	16.3K	20.6K	q_{12}	116.8K	123.4K	q_{19}	114.4K	120.7K
q_6	32.8K	33.2K	q_{13}	63.2K	65.7K	q_{20}	53.4K	63.7K
q_7	32.5K	33.8K	q_{14}	53.9K	59.2K	q_{21}	170K	199K

Fig. 5: Evaluation time of CAvSAT for conjunctive queries with varying data complexity, with optimization, over the databases of size 1M tuples/relation.



Results on the Real-World Databases In this set of experiments, we evaluated the performance of CAvSAT using Reduction 3 on real-world data having key constraints on each relation, along with one functional dependency. The data used are about inspections of food establishments in New York and Chicago, and are taken from [2] and [1]. Part of this data have been previously used for evaluating data cleaning systems, such as HoloClean [25]. Since the structure of the schema or the constraints on the database were not specified by the source, we decomposed the data into four relations, and assumed reasonable key constraints for all relations and also one additional functional dependency, as shown in Table 4. We evaluated the performance of Reduction 3 on six queries depicted in Figure 10 in the Appendix. For example, query Q_3 returns the names of the restaurants, such that they are present in both New York and Chicago, and they were inspected on the same day. Figure 6 shows that the solver took the most amount of time to compute answers to this query. Not surprisingly, the evaluation time increases as the number of atoms or the number of free variables in the query grow. Table 5 shows the size of the CNF-formulas produced by Reduction 4 (where Reduction 3 is used inside Reduction 4). No optimization was implemented in this set of experiments.

Relation # Tuples

NY_Insp (LicenseNo, Risk, InspDate, InspType, Result) 229K

NY_Rest (Name, LicenseNo, Cuisine, Address, Zip) 26.5K

CH_Insp (LicenseNo, Risk, InspDate, InspType, Result) 167K

CH_Rest (Name, LicenseNo, Facility, Address, Zip) 31.1K

Table 4: The schema and the constraints of the real-world database.

Constraint	Type	Violations
$\boxed{\text{NY_Insp (LicenseNo, InspDate, InspType} \rightarrow \text{Risk, Result)}}$	Key	25.6%
NY_Rest (LicenseNo → Name, Cuisine, Address, Zip)	Key	0%
$\hline \text{CH_Insp (LicenseNo, InspDate, InspType} \rightarrow \text{Risk, Result)}$	Key	0.07%
CH_Rest (LicenseNo → Name, Cuisine, Address, Zip)	Key	5.86%
$CH_Rest (Name \rightarrow Zip)$	FD	9.73%

Fig. 6: Evaluation time of CAvSAT on real data.

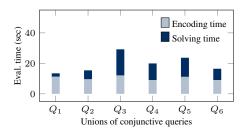


Table 5: Size of the CNF-formula.

Query	Variables	Clauses
Q_1	455.1K	793.7K
Q_2	456.5K	794K
Q_3	455.1K	671.5K
Q_4	476K	861.5K
Q_5	486.7K	836.2K
Q_6	455.5K	1.12M

7 Concluding Remarks

We designed and implemented CAvSAT, the first SAT-based system for consistent query answering. Our preliminary stand-alone evaluation shows that a SAT-based approach can give rise to a scalable system for consistent query answering. We note that, on queries with first-order rewritable consistent answers, CAvSAT had comparable or even better performance to evaluating the first-order rewritings using a database engine. This finding suggests a potential difference between theory and practice, since the study of first-order rewritability of the consistent answers was motivated from having an efficient evaluation of consistent answers using the database engine alone.

The next step in this investigation is to carry out an extensive comparative evaluation of CAvSAT with other systems for consistent query answering and, in particular, with systems that use reduction-based methods [18,23].

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Appendix

Proof of Proposition 1

Proposition 1. Let ϕ be the CNF-formula constructed using Reduction 1.

- The size of ϕ is polynomial in the size of I.
- The formula ϕ is satisfiable if and only if CERTAINTY (q, Σ) is false on I.

Proof. Let n be the number of facts in I. There are exactly n boolean variables used in ϕ . Clearly, $|\mathcal{G}| \leq n$, therefore the number of α -clauses is bounded above by n. Similarly, for each $G_j \in \mathcal{G}$, we have that $|G_j| \leq n$. Hence, the length of each α -clause is also at most n. If d is the number of atoms in q, then we have that $|\mathcal{W}| \leq n^d$; moreover, for every $W_j \in \mathcal{W}$, we have that $|W_j| \leq d$. Hence, the number of β -clauses in ϕ is at most n^d , and the length of each β -clause is bounded above by d. Since the query q is not part of the input to the problem CERTAINTY(q), we have that d is a fixed constant.

To prove the second part of the proposition, assume first that CERTAINTY(q) is false on I. Hence, there exists a repair R of I that falsifies q. Construct an assignment \hat{a} to the variables in ϕ by setting $\hat{a}(x_i)=1$ if and only if $f_i\in R$. Since exactly one fact from each key-equal group of I is present in R, exactly one variable from each α -clause is set to 1 in \hat{a} . Also, since $R\not\models q$, no minimal witness to q is in R. Therefore, at least one variable from each β -clause is set to 0 in \hat{a} . Hence, \hat{a} satisfies ϕ . For the other direction, let \hat{a} be a satisfying assignment to ϕ . Since no two α -clauses share a variable, we can construct a set X of variables by arbitrarily choosing exactly one x_i from each α -clause, such that $\hat{a}(x_i)=1$. Construct a set R of facts of I, such that $f_i\in R$ if and only if $x_i\in X$. It is easy to see that R contains exactly one fact from each key-equal group of I, and no minimal witness to q on I is present in R. Hence, R is a repair of I that falsifies q.

Proof of Proposition 2

Proposition 2. Let ϕ be the CNF-formula constructed using Reduction 2.

- The size of ϕ is polynomial in the size I.
- There exists a satisfying assignment to ϕ in which a variable p_l is set to 1 if and only if $a_l \notin CONS(q, I)$.

Proof. Let n be the number of facts in I. Let m be the arity of the query q and let d and the number of atoms of q. Since an answer to q is a set of m facts, we have that $|\mathcal{A}| \leq n^m$. For each l, the number of witnesses in \mathcal{W}^l is bounded by n^d . Therefore, there are at most n^{m+d} β -clauses in ϕ , each of length at most d. Since the query q is not part of the input to $\mathrm{CONS}(q)$, the quantities m and d can be treated are fixed constants. It follows directly from Proposition 1 that both the number of α -clauses and the length of each α -clause in ϕ are bounded above by n.

To prove the second part of the proposition, assume first that $a_l \notin \text{Cons}(q)$. Hence, there exists a repair R of I, such that no minimal witness to $q[a_l]$ is in R. Construct an assignment \hat{a} to the variables in ϕ as follows. Set $\hat{a}(x_i) = 1$ if and only if $f_i \in R$. Set $\hat{a}(p_l) = 1$, and set $\hat{a}(p_j) = 0$ for all $j \neq l$. Since exactly one fact from each key-equal

group of I is in R, the assignment sets to 1 exactly one variable from each α -clause. Since no minimal witness to $q[a_l]$ is in R, at least one variable from each β^l -clause is set to 0 in \hat{a} , thus satisfying all β^l -clauses, even when p_l is set to 1. All other β -clauses are satisfied trivially because of the assignment $\hat{a}(p_j)=0$, for all $j\neq l$. In the other direction, let \hat{a} be the satisfying assignment to ϕ , such that $\hat{a}(p_l)=1$. Since no two α -clauses share a variable, we can construct a set X of variables by arbitrarily choosing exactly one x_i from each α -clause, such that $\hat{a}(x_i)=1$. Construct a set R of facts of I, such that $f_i\in R$ if and only if $x_i\in X$. It is easy to see that exactly one fact from each key-equal group of I is present in R. Since $\hat{a}(p_l)=1$ and since all β^l -clauses are satisfied by \hat{a} , at least one fact from each minimal witness to $q[a_l]$ is missing in R. Hence, R must be a repair of I such that $R \not\models q[a_l]$.

Proof of Proposition 3

Proposition 3. Let ϕ' be the boolean formula constructed using Reduction 3.

- The formula ϕ' can be transformed to an equivalent CNF-formula ϕ whose size is polynomial in the size of I.
- There exists a satisfying assignment to ϕ' in which a variable p_l is set to 1 if and only if $a_l \notin CONS(Q, I, \Sigma)$.

Proof. Let n be the number of facts of I. Let d_1 be the smallest number such that there exists no denial constraint in Σ whose number of database atoms is bigger than d_1 . Also, let d_2 be the smallest number such that there exists no conjunctive query in Q whose number of database atoms is bigger than d_2 . Since Σ and Q are not part of the input to CONS(Q), the quantities d_1 and d_2 are fixed constants. We also have that $|\mathcal{V}| \leq n^{d_1}$, $|\mathcal{N}^i| \leq n^{d_1}$ for $1 \leq i \leq n$, $|\mathcal{A}| \leq n^{d_2}$, and $|\mathcal{W}^l| \leq n^{d_2}$ for $1 \leq l \leq |\mathcal{A}|$. The number of x-, y-, and p-variables in ϕ' is therefore bounded by n, n^{d_1+1} , and n^{d_2} , respectively. The formula ϕ' contains as many α -clauses as $|\mathcal{V}|$, and none of the α -clause's length exceeds n. Similarly, there are at most n^{d_2} β -clauses, and none of their lengths exceeds $d_2 + 1$. The number of γ -clauses is precisely n, and each γ -clause is at most $n^{d_1+1}+1$ literals long. There are as many θ -expressions as there are y-variables. Every θ -expression is of the form $y \leftrightarrow (x_1 \land ... \land x_d)$, where d is a constant obtained from the number of facts in the corresponding near-violation. Each θ -expression can be equivalently written in a constant number of CNF-clauses as $((\neg y \lor x_1) \land ... \land (\neg y \lor x_d)) \land (\neg x_1 \lor ... \lor \neg x_d \lor y)$, in which the length each clause is constant. This makes it possible to transform ϕ' into an equivalent CNF-formula ϕ , whose size is polynomial in the size of I.

To prove the second part of the proposition, assume first that \hat{a} is a satisfying assignment to the variables in ϕ' such that $\hat{a}(p_l)=1$. Construct a database instance R such that $f_i\in R$ if and only if $\hat{a}(x_i)=1$. The α -clauses make sure that no minimal violation to Σ is present in R, meaning that R is a consistent subset of I. The γ -clauses and the θ -expressions encode the condition that, for every fact $f\in I$, either $f\in R$ or at least one near-violation w.r.t. Σ and f is in R. This condition makes sure that R is indeed a repair of I. Since $\hat{a}(p_l)=1$, the β^l -clauses ensure that at least one fact from each minimal witness to $Q[a_l]$ is missing from R, meaning that $a_l \notin Q(R)$.

In the other direction, given a repair R that falsifies $Q[a_l]$, build an assignment \hat{a} as follows. Set $\hat{a}(x_i) = 1$ if and only if $f_i \in r$. Set $\hat{a}(p_l) = 1$, and set $\hat{a}(p_{l'}) = 0$ for

all $l' \neq l$. Since $R \models \Sigma$, no minimal violation to Σ is a subset of R, meaning that \hat{a} satisfies all α -clauses in ϕ' . Also, for every fact $f \in I$, it must be the case that either $f \in R$ or at least one near-violation w.r.t. Σ and f is in R (otherwise R would not have been a repair of I). Therefore, all γ -clauses and θ -expressions are also satisfied by the assignment \hat{a} . Since $R \not\models Q[a_l]$, at least one fact from each minimal witness to $Q[a_l]$ must be missing from R, meaning that there is at least one variable x_i in each β^l -clause such that $\hat{a}(x_i) = 0$. Hence, all β^l -clauses are satisfied by \hat{a} , even when $\hat{a}(p_l) = 1$. All other β -clauses are satisfied trivially, since $\hat{a}(p_{l'}) = 0$, for all $l' \neq l$.

Proof of Proposition 4

We first state and prove Lemma 1, that reasons about the satisfying assignments to the WCNF-formula ψ , constructed using Reduction 4. This lemma is used in proving Proposition 4.

Lemma 1. Let ϕ' be the CNF-formula constructed in Step 1 of Reduction 4, and ψ_i be the WEIGHTED MAXSAT instance at the beginning of i^{th} iteration of Algorithm 1. For all i, every optimal solution of ψ_i satisfies all clauses in ϕ' .

Proof. We prove Lemma 1 by induction on i. The CNF-formula ϕ' constructed in Step 1 of Reduction 4 can always be satisfied by setting all x-variables to 1, and all p-variables to 0. The clauses in ϕ being hard, ϵ being soft ensures that every optimal solution of ψ_0 satisfies all clauses in ϕ . Assume that for some $i \geq 0$, every optimal solution to ψ_i satisfies all clauses in ϕ . At the end of iteration i+1, if moreAnswers is true, the formula ψ_{i+1} is constructed from ψ_i by adding to ψ_i the unit hard clauses $(\neg p_l)$. This forces every optimal solution of ψ_{i+1} to satisfy all of these added clauses. Since no p_l variable occurs positively in ϕ , we have that, for all i, every optimal solution to ψ_{i+1} still satisfies ϕ .

Proposition 4. Algorithm 1 returns an array ANS such that $a_l \in Cons(Q, I)$ if and only if the entry ANS[l] is true.

Proof. In one direction, for every l, if $\mathbf{a}_l \in \mathrm{CONS}(Q, I)$, then, by Proposition 3, the variable p_l takes value 0 in every assignment that satisfies ϕ . By Lemma 1, for every i, the optimal solution of ψ_i also assigns value 0 to the variable p_l . As a result, Line 12 never gets executed, and the entry $\mathrm{ANS}[l]$ remains true.

For the other direction, we first prove that Algorithm 1 always terminates. Observe that at the end of the i^{th} iteration, for every l, a unit clause (p_l) is present in ψ_i if and only if ANS[l] is true. Hence, at the end of i^{th} iteration, if moreAnswers is true, then the optimal solution to ψ_i must have assigned value 1 to at least one variable p_l such that ANS[l] was previously true. Therefore, at the end of i^{th} iteration, at least i entries in ANS are false. It follows that the algorithm terminates after at most $|\mathcal{A}|$ iterations.

Since no clause in ϕ contains a positive literal p_l , the addition of a unit hard clause $(\neg p_l)$ to ψ_i does not suppress any satisfying assignments to ϕ while finding the optimal solution to ψ_{i+1} . Therefore, in every iteration, the optimal solution of ψ guarantees to satisfy the maximum number of p_l variables for which the unit clause (p_l) is still in ψ . As a result, Algorithm 1 does not terminate until it marks the entries ANS[l] false, for

all l, for which there exists a satisfying assignment to ϕ in which p_l gets assigned to 1. In other words, by Proposition 3, for every inconsistent answer a_l , the entry ANS gets marked as false.

Fig. 7: Queries used in the experiments with synthetic data.

FO-rewritable consistent answers:

```
\begin{aligned} q_{1}(z) &:= \exists x, y, v, w \; (R_{1}(\underline{x}, y, z) \land R_{2}(\underline{y}, v, w)) \\ q_{2}(z, w) &:= \exists x, y, v \; (R_{1}(\underline{x}, y, z) \land R_{2}(\underline{y}, v, w)) \\ q_{3}(z) &:= \exists x, y, v, u, d \; (R_{1}(\underline{x}, y, z) \land R_{3}(\underline{y}, v) \land R_{2}(\underline{v}, u, d)) \\ q_{4}(z, d) &:= \exists x, y, v, u \; (R_{1}(\underline{x}, y, z) \land R_{3}(\underline{y}, v) \land R_{2}(\underline{v}, u, d)) \\ q_{5}(z) &:= \exists x, y, v, w \; (R_{1}(\underline{x}, y, z) \land R_{4}(\underline{y}, v, w)) \\ q_{6}(z) &:= \exists x, y, x', w, d \; (R_{1}(\underline{x}, y, z) \land R_{2}(\underline{x'}, y, w) \land R_{5}(\underline{x}, y, d)) \\ q_{7}(z) &:= \exists x, y, w, d \; (R_{1}(\underline{x}, y, z) \land R_{2}(y, x, w) \land R_{5}(\underline{x}, y, d)) \end{aligned}
```

In P, but not FO-rewritable, consistent answers:

```
\begin{array}{l} q_{8}(z,w) := \; \exists x,y \; (R_{1}(\underline{x},y,z) \land R_{2}(\underline{y},x,w)) \\ q_{9}(z) := \; \exists x,y,w,u,d \; (R_{1}(\underline{x},y,z) \land R_{2}(\underline{y},x,w) \land R_{4}(\underline{y},u,d)) \\ q_{10}(z,w,d) := \; \exists x,y,u \; (R_{1}(\underline{x},y,z) \land R_{2}(\underline{y},x,w) \land R_{4}(\underline{y},u,d)) \\ q_{11}(z) := \; \exists x,y,w \; (R_{1}(\underline{x},y,z) \land R_{2}(\underline{y},x,w)) \\ q_{12}(v,d) := \; \exists x,y,z,u \; (R_{3}(\underline{x},y) \land R_{6}(\underline{y},z) \land R_{1}(\underline{z},x,d) \land R_{4}(\underline{x},u,v)) \\ q_{13}(v) := \; \exists x,y,z,u \; (R_{3}(\underline{x},y) \land R_{6}(\underline{y},z) \land R_{7}(\underline{z},x) \land R_{4}(\underline{x},u,v)) \\ q_{14}(d) := \; \exists x,y,z,u \; (R_{3}(\underline{x},y) \land R_{6}(\underline{y},z) \land R_{1}(\underline{z},x,d) \land R_{7}(\underline{x},u)) \end{array}
```

coNP-complete consistent answers:

```
\begin{array}{l} q_{15}(z) := \; \exists x,y,x',w \; (R_1(\underline{x},y,z) \land R_2(\underline{x'},y,w)) \\ q_{16}(z,w) := \; \exists x,y,x' \; (R_1(\underline{x},y,z) \land R_2(\underline{x'},y,w)) \\ q_{17}(z) := \; \exists x,y,x',w,u,d \; (R_1(\underline{x},y,z) \land R_2(\underline{x'},y,w) \land R_4(\underline{y},u,d)) \\ q_{18}(z,w) := \; \exists x,y,x',u,d \; (R_1(\underline{x},y,z) \land R_2(\underline{x'},y,w) \land R_4(\underline{y},u,d)) \\ q_{19}(z,w,d) := \; \exists x,y,x',u \; (R_1(\underline{x},y,z) \land R_2(\underline{x'},y,w) \land R_4(\underline{y},u,d)) \\ q_{20}(z) := \; \exists x,y,x',w,u,d,v \; (R_1(\underline{x},y,z) \land R_2(\underline{x'},y,w) \land R_4(\underline{y},u,d) \land R_3(\underline{u},v)) \\ q_{21}(z,w) := \; \exists x,y,x',u,d,v \; (R_1(\underline{x},y,z) \land R_2(\underline{x'},y,w) \land R_4(\underline{y},u,d) \land R_3(\underline{u},v)) \end{array}
```

Fig. 8: The KW-FO-rewritings of the consistent answers to queries q_1 to q_7 .

```
q_1(f^{(1)}) := \exists s_1 \in R_1(\forall r_1 \in R_1(\exists s_2 \in R_2(\forall r_2 \in R_2)))
                                              (s_1.1 \neq r_1.1 \vee s_2.1 \neq r_2.1 \vee (r_1.2 = r_2.1 \wedge r_1.3 = f.1)))))
q_2(f^{(2)}) := \exists s_1 \in R_1 (\forall r_1 \in R_1 (\exists s_2 \in R_2 (\forall r_2 \in R_2)))
                                              (s_1.1 \neq r_1.1 \vee s_2.1 \neq r_2.1 \vee (r_2.3 = f.1 \wedge r_1.2 = r_2.1 \wedge r_1.3 = f.2)))))
q_3(f^{(1)}) := \exists s_1 \in R_1(\forall r_1 \in R_1(\exists s_2 \in R_3(\forall r_2 \in R_3(\exists s_3 \in R_2(\forall r_3 \in R_2)))))
                                               ((s_1.1 \neq r_1.1 \vee s_2.1 \neq r_2.1 \vee s_3.1 \neq r_3.1 \vee (r_2.2 = r_3.1 \wedge r_1.2 = r_2.1 \wedge r_1.3 = f.1))))))))
q_4(f^{(2)}) := \exists s_1 \in R_1(\forall r_1 \in R_1(\exists s_2 \in R_3(\forall r_2 \in R_3(\exists s_3 \in R_2(\forall r_3 \in R_2)))))
                                               (s_1.1 \neq r_1.1 \vee s_2.1 \neq r_2.1 \vee s_3.1 \neq r_3.1
                                                \forall (r_3.3 = f.1 \land r_2.2 = r_3.1 \land r_1.2 = r_2.1 \land r_1.3 = f.2)))))))
q_5(f^{(1)}) := \exists s_1 \in R_1 (\forall r_1 \in R_1 (\exists s_2 \in R_4 (\forall r_2 \in R_4))) \in R_1 (\exists s_2 \in R_4 (\forall r_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\forall r_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\forall r_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\forall r_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\forall r_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\forall r_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\forall r_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\forall r_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\forall r_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\forall r_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\forall r_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\forall r_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\forall r_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\forall r_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\forall r_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\forall r_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\forall r_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\forall r_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\forall r_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\forall r_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\forall r_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\forall r_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\exists s_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\exists s_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\exists s_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\exists s_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\exists s_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\exists s_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\exists s_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\exists s_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\exists s_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\exists s_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\exists s_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\exists s_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\exists s_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\exists s_2 \in R_4)) \in R_2 (\exists s_2 \in R_4)) \in R_2 (\exists s_2 \in R_4 (\exists s_2 \in R_4)) \in R_2 (\exists s_2 \in R_4))
                                               (s_1.1 \neq r_1.1 \vee s_2.1 \neq r_2.1 \vee s_2.2 \neq r_2.2 \vee (r_1.2 = r_2.1 \wedge r_1.3 = f.1)))))
q_6(f^{(1)}) := \exists s_1 \in R_2(\forall r_1 \in R_2(\exists s_2 \in R_5(\forall r_2 \in R_5(\exists s_3 \in R_1(\forall r_3 \in R_1)))))
                                               (s_1.1 \neq r_1.1 \vee s_2.1 \neq r_2.1 \vee s_3.1 \neq r_3.1)
                                                \lor (r_2.1 = r_3.1 \land r_1.2 = r_2.2 \land r_1.2 = r_3.2 \land r_3.3 = f.1)))))))
q_7(f^{(1)}) := \exists s_1 \in R_2(\forall r_1 \in R_2(\exists s_2 \in R_5(\forall r_2 \in R_5(\exists s_3 \in R_1(\forall r_3 \in R_1)))))
                                              (s_1.1 \neq r_1.1 \vee s_2.1 \neq r_2.1 \vee s_3.1 \neq r_3.1
                                                \forall (r_1.2 = r_2.1 \land r_1.2 = r_3.1 \land r_1.1 = r_2.2 \land r_1.1 = r_3.2 \land r_3.3 = f.1)))))))
```

All variables range over tuples in relations; $f^{(i)}$ denotes a tuple with i elements, i = 1, 2.

Fig. 9: The SQL translations of the ConQuer-FO-rewritings of the consistent answers to queries q_1 to q_4 .

```
q_2(R1_3, R2_3):
q_1(R1_{-3}):
Candidates as (
                                                   Candidates as (
select distinct R1.3 as R1_3,R1.1 as R1_1
                                                   select distinct R1.3 as R1_3, R2.3 as R2_3,
from R1, R2
                                                   R1.1 as R1_1 from R1, R2
where (R1.2 = R2.1)
                                                   where (R1.2 = R2.1)
Filter as (select R1_1
                                                   Filter as (select R1_1
from Candidates C join R1 on C.R1_1 =
                                                   from Candidates C join R1 on C.R1_1 =
R1.1
                                                   R1.1
left outer join R2 on R1.2 = R2.1
                                                   left outer join R2 on R1.2 = R2.1
where (R2.1 is null)
                                                   where (R2.1 is null)
union all select C.R1_1 from Candidates C
                                                   union all select C.R1_1 from Candidates C
group by C.R1_1
                                                   group by C.R1_1
having count(*) > 1)
                                                   having count(*) > 1)
select R1_3
                                                   select R1_3, R2_3
                                                   from Candidates C
from Candidates C
where not exists
                                                   where not exists
(select * from Filter F where C.R1_1 =
                                                   (select * from Filter F where C.R1_1 =
F.R1_1)
                                                   F.R1_{-1}
q_3(R1_-3):
                                                   q_4(R1_3, R2_3):
Candidates as (
                                                   Candidates as (
select distinct R1.3 as R1_3, R1.1 as R1_1
                                                   select distinct R1.3 as R1_3, R2.3 as
from R1, R3, R2
                                                   R2_3,R1.1 as R1_1
where (R3.2 = R2.1) and (R1.2 = R3.1))
                                                   from R1, R3, R2
                                                   where (R3.2 = R2.1) and (R1.2 = R3.1)
Filter as (
select R1_1
                                                   Filter as (select R1_1
from Candidates C join R1 on C.R1_1 =
                                                   from Candidates C join R1 on C.R1_1 =
left outer join R3 on R1.2 = R3.1 left outer
                                                   left outer join R3 on R1.2 = R3.1 left outer
join R2 on R3.2 = R2.1
                                                   join R2 on R3.2 = R2.1
where (R3.1 is null OR R2.1 is null)
                                                   where (R3.1 is null or R2.1 is null)
union all select C.R1_1 from Candidates C
                                                   union all select C.R1_1 from candidates C
group by C.R1_1
                                                   group by C.R1_1
having count(*) > 1)
                                                   having count(*) > 1)
select R1_3
                                                   select R1_3, R2_3
from Candidates C
                                                   from Candidates C
where not exists
                                                   where not exists
(select * from Filter F where C.R1_1 =
                                                   (select * from Filter F where C.R1_1 =
F.R1_1)
                                                   F.R1_1)
```

Fig. 10: Queries used on the real-world database.

```
\begin{aligned} Q_1() &:= \exists x,y,z,w,v,y',z',w',v' \text{ (NY_Rest}(x,y,z,w,v) \land \text{CH_Rest}(x,y',z',w',v'))} \\ Q_2(x) &:= \exists y,z,w,v,y',z',w',v' \text{ (NY_Rest}(x,y,z,w,v) \land \text{CH_Rest}(x,y',z',w',v'))} \\ Q_3(x) &:= \exists y,z,w,v,y',z',w',v',q,r,s,t,q',s',t' \text{ (NY_Rest}(x,y,z,w,v) \land \text{CH_Rest}(x,y',z',w',v')} \\ & \land \text{NY_Insp}(y,q,r,s,t) \land \text{CH_Insp}(y',q',r,s',t')) \\ Q_4(x,y) &:= \exists z,w,v,q,r,s \text{ (CH_Rest}(x,y,z,w,v) \land \text{CH_Insp}(y,q,r,s,\text{`Pass'})) \\ Q_5(x) &:= \exists y,z,w,v,q,r,s \text{ (CH_Rest}(x,y,z,w,v) \land \text{CH_Insp}(y,q,r,s,\text{`Fail'})) \cup \\ & \exists y,z,w,v,q,r,s \text{ (NY_Rest}(x,y,z,w,v) \land \text{NY_Insp}(y,q,r,s,\text{`Fail'})) \\ Q_6(x,v) &:= \exists y,z,w,y',z',w',v',q,r,s \text{ (CH_Rest}(x,y,z,w,v) \land \text{NY_Rest}(x,y',z',w',v')} \\ & \land \text{NY_Insp}(y',\text{`Not Critical'},q,r,s)) \end{aligned}
```