

Studies in Fuzziness and Soft Computing

Volume 385

Series Editor

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Fuzzy Transportation and Transshipment Problems

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