

Founding Editors

Gerhard Goos

Karlsruhe Institute of Technology, Karlsruhe, Germany

Juris Hartmanis

Cornell University, Ithaca, NY, USA

Editorial Board Members

Elisa Bertino

Purdue University, West Lafayette, IN, USA

Wen Gao

Peking University, Beijing, China

Bernhard Steffen

TU Dortmund University, Dortmund, Germany

Gerhard Woeginger

RWTH Aachen, Aachen, Germany

Moti Yung

Columbia University, New York, NY, USA

More information about this series at <http://www.springer.com/series/7412>

Frank Nielsen · Frédéric Barbaresco (Eds.)

Geometric Science of Information

4th International Conference, GSI 2019
Toulouse, France, August 27–29, 2019
Proceedings

Editors

Frank Nielsen
Sony Computer Science Laboratories, Inc.
Tokyo, Japan

Frédéric Barbaresco
Thales
Limours, France

ISSN 0302-9743

ISSN 1611-3349 (electronic)

Lecture Notes in Computer Science

ISBN 978-3-030-26979-1

ISBN 978-3-030-26980-7 (eBook)

<https://doi.org/10.1007/978-3-030-26980-7>

LNCS Sublibrary: SL6 – Image Processing, Computer Vision, Pattern Recognition, and Graphics

© Springer Nature Switzerland AG 2019

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, expressed or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Cover page painting: Woman teaching Geometry, from French medieval edition of Euclid's Elements (14th century) © The British Library, used with granted permission.

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Foreword

On behalf of both the Organizing and the Scientific Committees, it is our great pleasure to welcome you to the proceedings of the 4th International SEE Conference on Geometric Science of Information (GSI 2019), hosted at ENAC in Toulouse, during August 27–29, 2019.

GSI 2019 benefited from the following scientific and financial sponsors: SMF, SMAI, GDR CNRS ISIS & MIA, ENAC, THALES, Ecole Polytechnique, Mines ParisTech, Sony Computer Science Laboratories Inc (Sony CSL). GSI 2019 was the opening event of the CIMI labex trimester on “Statistics with Geometry and Topology”: <https://perso.math.univ-toulouse.fr/statistics-geometry-and-topology/>.

The three-day conference was also organized in the framework of the relations set up between SEE and scientific institutions or academic laboratories: ENAC, Institut Mathématique de Bordeaux, Ecole Polytechnique, Ecole des Mines ParisTech, Inria, CentraleSupélec, Institut Mathématique de Bordeaux, Sony Computer Science Laboratories.

We would like to express all our thanks to the local organizers (ENAC, IMT and CIMI Labex) for hosting this event at the interface between geometry, probability, and information geometry.

The GSI conference cycle was initiated by the [Brillouin Seminar Team](#) as early as 2009. The GSI 2019 event was motivated in the continuity of the first initiatives launched in 2013 (<https://www.see.asso.fr/gsi2013>) at Mines ParisTech, consolidated in 2015 (<https://www.see.asso.fr/gsi2015>) at Ecole Polytechnique, and opened up to new communities in 2017 (<https://www.see.asso.fr/gsi2017>) at Mines ParisTech. We mention that in 2011, we organized an Indo-French workshop on “Matrix Information Geometry” that yielded an [edited book in 2013](#), and in 2017, we collaborated with the CIRM seminar in Luminy TGSi 2017 “Topological and Geometrical Structures of Information” (<http://forum.cs-dc.org/category/94/tgsi2017>). The last GSI 2017 proceedings were published by Springer in their *Lecture Notes in Computer Science* series (<https://www.springer.com/fr/book/9783319684444>) and the most important contributions to the GSI 2017 conference are collected in the Springer volume *Geometric Structures of Information* (<https://www.springer.com/us/book/9783030025199>).

The technical program of GSI 2019 covered all the main topics and highlights in the domain of “geometric science of information” including information geometry manifolds of structured data/information and their advanced applications. These proceedings consist solely of original research papers that were carefully peer-reviewed by two or three experts and revised before acceptance.

The GSI 2019 program included a renowned honorary speaker, one guest honorary speaker, and three keynote distinguished speakers, as well as a history session with a talk on “Fermat, Pascal, and the Geometry of Chance” and a “Tribute to Jean-Louis Koszul” (who passed away in January 2018).

As with GSI 2013, GSI 2015, and GSI 2017, GSI 2019 addressed inter-relations between different mathematical domains like shape spaces (geometric statistics on manifolds and Lie groups, deformations in shape space, etc.), probability/optimization and algorithms on manifolds (structured matrix manifold, structured data/Information etc.), relational and discrete metric spaces (graph metrics, distance geometry, relational analysis, etc.), computational and Hessian information geometry, geometric structures in thermodynamics and statistical physics, algebraic/infinite dimensional/Banach information manifolds, divergence geometry, tensor-valued morphology, optimal transport theory, manifold and topology learning, and applications such as geometries of audio-processing, inverse problems, and signal/image processing. The GSI 2019 topics were enriched with contributions on Lie group machine learning, harmonic analysis on Lie groups, geometric deep learning, geometry of Hamiltonian Monte Carlo, geometric and (poly)symplectic integrators, contact geometry and Hamiltonian control, geometric and structure preserving discretizations, probability density estimation and sampling in high dimension, geometry of graphs and networks and geometry in neuroscience and cognitive sciences.

At the turn of the century, new and fruitful interactions were discovered between several branches of science: information science (information theory, digital communications, statistical signal processing), mathematics (group theory, geometry and topology, probability, statistics, sheaves theory) and physics (geometric mechanics, thermodynamics, statistical physics, quantum mechanics). The GSI conference cycle is an attempt to discover joint mathematical structures to all these disciplines by elaboration of a “general theory of information” embracing physics, information science, and cognitive science in a global scheme.

GSI 2019 addressed the following sessions with associated chairs:

- Probability on Riemannian Manifolds – Marc Arnaudon, Ana Bela Cruzeiro
- Optimization on Manifold – Salem Said, Rodolphe Sepulchre
- Shape Space – Nicolas Charon, Pietro Gori
- Statistics on Non-linear Data – Xavier Pennec, Stefan Sommer
- Lie Group Machine Learning – Elena Celledoni, Frédéric Barbaresco
- Statistical Manifold and Hessian Information Geometry – Michel Nguiffo Boyom, Hiroshi Matsuzoe
- Monotone Embedding and Affine Immersion of Probability Models – Jun Zhang, Atsumi Ohara
- Non-parametric Information Geometry – Lorenz Schwachhöfer, John Armstrong
- Divergence Geometry – Frank Nielsen, Wolfgang Stummer
- Computational Information Geometry – Frank Nielsen, Olivier Schwander
- Wasserstein Information Geometry/Optimal Transport – Guido Montufar, Wuchen Li
- Geometric Structures in Thermodynamics and Statistical Physics – Goffredo Chirco, François Gay-Balmaz
- Geometric and Structure-Preserving Discretizations – François Gay-Balmaz, Joël Bensoam
- Geometry of Quantum States – Florio Maria Ciaglia, Giuseppe Marmo
- Geometry of Tensor-Valued Data – Jesús Angulo, Geert Verdoolaeye

- Geometric Mechanics – G ry de Saxc , Jean Lerbet
- Geometric Science of Information Libraries – Nina Miolane, Alice Le Brigant
- Poster Session – Pierre Baudot

We were also honored to have the following keynote talks:

- Invited Honorary Speaker
 - Alain Chenciner (Universit  Paris 7, Observatoire de Paris), “N-body Relative Equilibria in Higher Dimensions”
- Guest Honorary Speaker
 - Karl Friston (Wellcome Trust Centre for Neuroimaging), “Markov Blankets and Bayesian Mechanics”
- Keynote Speakers
 - Elena Celledoni (Norwegian University of Science and Technology), “Structure preserving algorithms for geometric numerical integration”
 - Gabriel Peyr  (CNRS, Ecole Normale Sup rieure), “Optimal Transport for Machine Learning”
 - Jean-Baptiste Hiriart-Urruty (Universit  de Toulouse), “Fermat, Pascal: Geometry and Chance”

August 2019

Frank Nielsen
Fr d ric Barbaresco

Organization

Program Chairs

Frédéric Barbaresco
Frank Nielsen

Thales Air Systems, France
Sony Computer Science Laboratories Inc., Japan

Patronage

SEE: Société de l'électricité, de l'électronique et des technologies de l'information et de la communication

Jean Vieille
Valérie Alidor
Marianne Emorine

SEE Webmaster, France
SEE, France
SEE, France

Scientific Committee

Alain Trouvé
Alessandro Barp
Alex Lin
Alexis Arnaudon
Ali Mohammad-Djafari
Alice Barbara
Alice Le Brigant

ENS Paris-Saclay, France
Imperial College London, UK
University of California, Los Angeles, USA
Imperial College London, UK
CNRS, France
Université Lille, France
Thales Air Systems/Institut Mathématique
de Bordeaux, France

Amor Keziou
Ana Bela Cruzeiro
Anatole Chessel
Anton Thalmaier
Antonio Mucherino
Arjan Van Der Schaft
Atsumi Ohara
Barbara Opozda
Barbara Trivellato
Benjamin Charlier
Bernhard Maschke
Casimiro Cavalcante
Christian Leonard
Cyrus Mostajeran
Daniel Luft
Darryl Holm

Université de Reims, France
Universidade de Lisboa, Portugal
École polytechnique, France
University of Luxembourg, Luxembourg
IRISA, University of Rennes 1, France
University of Groningen, The Netherlands
University of Fukui, Japan
Jagiellonian University, Kraków, Poland
Politecnico di Torino, Italy
Université de Montpellier, France
Université Lyon 1, France
Federal University of Ceará, Brazil
Université Paris Nanterre, France
University of Cambridge, UK
University Trier, Germany
Imperial College London, UK

Dominique Spehner	University of Grenoble Alpes, France
Elena Celledoni	Norwegian University of Science and Technology, Norway
Emmanuel Chevallier	Weizmann Institute, Israel
Eric Grivel	Université de Bordeaux, France
Evan Gawlik	University of Hawaii at Manoa, USA
Fabio Di Cosmo	Università degli Studi di Napoli Federico II, Italy
Fabio Mele	University of Regensburg, Germany
Fabrice Gamboa	Institut de Mathématiques de Toulouse, France
Flavien Leger	University of California, Los Angeles, USA
Florence Nicol	ENAC, Toulouse, France
Florio Maria	Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany
Frédéric Barbaresco	Thales Air Systems, France
Francesco Becattini	Università di Firenze, Italy
Francois Gay-Balmaz	Laboratoire de Météorologie Dynamique, École Normale Supérieure, France
Francois Xavier Vialard	Université Paris Est Marne La Vallée, France
Frank Nielsen	Sony Computer Science Laboratories, Tokyo, Japan
Geert Verdoolaege	Ghent University, Belgium
Gery de Saxce	Laboratoire de Mécanique de Lille, France
Giovanni Conforti	Ecole Polytechnique, Palaiseau, France
Giuseppe Marmo	University of Naples Federico II, Italy
Goffredo Chirco	Max Planck Institute for Gravitational Physics, Albert Einstein Institute, Germany
Guido Montufar	Max Planck Institute for Mathematics in the Sciences, Germany
Haomin Zhou	Georgia Institute of Technology, USA
Hiroaki Yoshimura	Waseda University, Japan
Hiroshi Matsuzoe	Nagoya Institute of Technology, Japan
Hiroyasu Satoh	Nippon Institute of Technology, Japan
Hitoshi Furuhashi	Hokkaido University, Japan
Hong van Le	Institute of Mathematics of ASCR, Czech Republic
Ishi Hideyuki	Nagoya University, Japan
Itoh Mitsuhiro	University of Tsukuba, Japan
Jésus Angulo	Centre de Morphologie Mathématique, École des Mines de Paris, France
Jan Naudts	Universiteit Antwerpen, Belgium
Jean Pierre Gazeau	Université Paris Diderot Paris 7, France
Joël Bensoam	IRCAM, France
Juhyun Par	Lancaster University, UK
Jun Zhang	University of Michigan, USA
Kathrin Welker	Helmut Schmidt University/University of the Federal Armed Forces Hamburg, Germany
Keisuke Haba	Nagoya Institute of Technology, Japan

Kinobu Shimizu	Tokyo University of Agriculture and Technology, Japan
Klas Modin	Chalmers University of Technology, Sweden
Koichi Tojo	The University of Tokyo, Japan
Leland McInnes	Tutte Institute for Mathematics and Computing, Canada
Lorenz Schwachhoefer	Technical University Dortmund, Germany
Luca Schiavone	Ostrava University, Italy
Luigi Malagò	Romanian Institute of Science and Technology, Romania
Marc Arnaudon	Université de Bordeaux, France
Marco Laudato	Università dell'Aquila, Italy
Marion Pilté	Mines ParisTech, France
Martin Bauer	Florida State University, USA
Maziar Esfahanian	University of Polytechnic of Turin, Italy
Michel Nguiffo Boyom	University Montpellier, France
Michel Broniatowski	LSTA Université Pierre et Marie Curie, Paris, France
Michele Pavon	Università di Padova, Italy
Miguel Tierz	Universidade de Lisboa, Portugal
Minh Ha Quang	RIKEN, Japan
Nicolas Brunel	ENSIIE, France
Nicolas Charon	Johns Hopkins University, USA
Nicolas Le Bihan	Gipsa-Lab, CNRS, France
Nihat Ay	Max Planck Institute for Mathematics in the Sciences, Germany
Nikolas Tapi	Norwegian University of Science and Technology, Norway
Nina Marinette Miolane	Stanford, USA
Olivier Peltre	Université Denis Diderot Paris 7, France
Olivier Schwander	LIP6 Sorbonne University, Paris, France
Paola Siri	Politecnico di Torino, Italy
Paul Marriott	University of Waterloo, Canada
Philipp Harms	University of Freiburg, Germany
Pierre Baudot	Inserm U1072, France
Pierre Roussillon	Télécom ParisTech, Paris, France
Pierre-Antoine Absil	Université Catholique de Louvain, Belgium
Pietro Gori	Telecom ParisTech, France
Qi Feng	University of Southern California, USA
Radmila Pribic	Thales Nederland BV, The Netherlands
Remco Duits	Eindhoven University of Technology, The Netherlands
Rodolphe Sepulchre	University of Cambridge, UK
Salem Said	Laboratoire IMS, Bordeaux, France
Shiwei Lan	University of Illinois Urbana-Champaign, USA
Stefan Sommer	University of Copenhagen, Denmark
Stephane Puechmorel	Ecole Nationale de l'Aviation Civile, Toulouse, France
Tomonari Sei	The University of Tokyo, Japan

Wolfgang Stummer	University of Erlangen-Nurnberg, Germany
Wuchen Li	University of California, Los Angeles, USA
Xavier Pennec	Inria, France
Yifan Chen	California Institute of Technology, USA

Local Organizing Committee

Gilles Baroin	ENAC, Toulouse, France
Alice Le Brigant	ENAC, Toulouse, France
Murat Bronz	ENAC, Toulouse, France
Georges Mykoniatis	ENAC, Toulouse, France
Simon Le Sénéchal	ENAC, Toulouse, France
Florence Nicol	ENAC, Toulouse, France
Ludovic d'Estampes	ENAC, Toulouse, France
Stéphane Puechmorel	ENAC, Toulouse, France
Thierry Klein	ENAC, Toulouse, France
Tat Dat Tô	ENAC, Toulouse, France
Simon Le Senechal	ENAC, Toulouse, France

Sponsors and Organizers

ENAC, Toulouse, France
 Institut Mathématique de Bordeaux, France
 Ecole Polytechnique, France
 Ecole des Mines ParisTech, France
 Inria, France
 Centrale Supélec, France
 Institut Mathématique de Bordeaux, France
 Sony Computer Science Laboratories, Tokyo, Japan



Contents

Shape Space

On Geometric Properties of the Textile Set and Strict Textile Set	3
<i>Tomonari Sei and Ushio Tanaka</i>	
Inexact Elastic Shape Matching in the Square Root Normal Field Framework.	13
<i>Martin Bauer, Nicolas Charon, and Philipp Harms</i>	
Signatures in Shape Analysis: An Efficient Approach to Motion Identification	21
<i>Elena Celledoni, Pål Erik Lystad, and Nikolas Tapia</i>	
Dilation Operator Approach and Square Root Velocity Transform for Time/Doppler Spectra Characterization on $SU(n)$	31
<i>Guillaume Bouleux and Frederic Barbaresco</i>	
Selective Metamorphosis for Growth Modelling with Applications to Landmarks	39
<i>Andreas Bock, Alexis Arnaudon, and Colin Cotter</i>	

Geometric Mechanics

Intrinsic Incremental Mechanics	51
<i>Jean Lerbet, Noël Challamel, François Nicot, and Félix Darve</i>	
Multi-symplectic Lie Group Thermodynamics for Covariant Field Theories . . .	55
<i>Goffredo Chirco, Marco Laudato, and Fabio M. Mele</i>	
Euler-Poincaré Equation for Lie Groups with Non Null Symplectic Cohomology. Application to the Mechanics	66
<i>Géry de Saxcé</i>	
Geometric Numerical Methods with Lie Groups	75
<i>Joël Bensoam and Pierre Carré</i>	
Souriau Exponential Map Algorithm for Machine Learning on Matrix Lie Groups.	85
<i>Frédéric Barbaresco</i>	

Geometry of Tensor-Valued Data

$\mathbb{R}$ -Complex Finsler Information Geometry Applied to Manifolds of Systems	99
<i>Christophe Corbier</i>	
Minkowski Sum of Ellipsoids and Means of Covariance Matrices	107
<i>Jesús Angulo</i>	
Hyperquaternions: An Efficient Mathematical Formalism for Geometry	116
<i>Patrick R. Girard, Patrick Clarysse, Romaric Pujol, Robert Goutte, and Philippe Delachartre</i>	
α -power Sums on Symmetric Cones	126
<i>Keiko Uohashi</i>	
Packing Bounds for Outer Products with Applications to Compressive Sensing	135
<i>Sebastian Semper and Thomas Hotz</i>	

Lie Group Machine Learning

On a Method to Construct Exponential Families by Representation Theory . . .	147
<i>Koichi Tojo and Taro Yoshino</i>	
Lie Group Machine Learning and Gibbs Density on Poincaré Unit Disk from Souriau Lie Groups Thermodynamics and $SU(1,1)$ Coadjoint Orbits . . .	157
<i>Frédéric Barbaresco</i>	
Irreversible Langevin MCMC on Lie Groups	171
<i>Alexis Arnaudon, Alessandro Barp, and So Takao</i>	
Predicting Bending Moments with Machine Learning	180
<i>Elena Celledoni, Halvor S. Gustad, Nikita Kopylov, and Henrik S. Sundklakk</i>	
The Exponential of Nilpotent Supergroups in the Theory of Harish-Chandra Representations	188
<i>Claudio Carmeli, Rita Fioresi, and V. S. Varadarajan</i>	

Geometric Structures in Thermodynamics and Statistical Physics

Dirac Structures in Open Thermodynamics	199
<i>Hiroaki Yoshimura and François Gay-Balmaz</i>	
From Variational to Bracket Formulations in Nonequilibrium Thermodynamics of Simple Systems	209
<i>François Gay-Balmaz and Hiroaki Yoshimura</i>	

A Homological Approach to Belief Propagation and Bethe Approximations . . .	218
<i>Olivier Peltre</i>	
About Some System-Theoretic Properties of Port-Thermodynamic Systems . . .	228
<i>Arjan van der Schaft and Bernhard Maschke</i>	
Expectation Variables on a Para-Contact Metric Manifold Exactly Derived from Master Equations.	239
<i>Shin-itiro Goto and Hideitsu Hino</i>	
Monotone Embedding and Affine Immersion of Probability Models	
Doubly Autoparallel Structure on Positive Definite Matrices and Its Applications.	251
<i>Atsumi Ohara</i>	
Toeplitz Hermitian Positive Definite Matrix Machine Learning Based on Fisher Metric	261
<i>Yann Cabanes, Frédéric Barbaresco, Marc Arnaudon, and Jérémie Bigot</i>	
Deformed Exponential and the Behavior of the Normalizing Function	271
<i>Francisca Leidmar Josue Vieira, Rui Facundo Vigelis, Luiza Helena Felix de Andrade, and Charles Casimiro Cavalcante</i>	
Normalization Problems for Deformed Exponential Families.	279
<i>Hiroshi Matsuzoe, Antonio M. Scarfone, and Tatsuaki Wada</i>	
New Geometry of Parametric Statistical Models	288
<i>Jun Zhang and Gabriel Khan</i>	
Divergence Geometry	
The Bregman Chord Divergence.	299
<i>Frank Nielsen and Richard Nock</i>	
Testing the Number and the Nature of the Components in a Mixture Distribution	309
<i>Michel Broniatowski, Emilie Miranda, and Wolfgang Stummer</i>	
Robust Estimation by Means of Scaled Bregman Power Distances. Part I. Non-homogeneous Data	319
<i>Birgit Roensch and Wolfgang Stummer</i>	
Robust Estimation by Means of Scaled Bregman Power Distances. Part II. Extreme Values	331
<i>Birgit Roensch and Wolfgang Stummer</i>	

Computational Information Geometry

Topological Methods for Unsupervised Learning.	343
<i>Leland McInnes</i>	
Geometry and Fixed-Rate Quantization in Riemannian Metric Spaces Induced by Separable Bregman Divergences.	351
<i>Erika Gomes-Gonçalves, Henryk Gzyl, and Frank Nielsen</i>	
The Statistical Minkowski Distances: Closed-Form Formula for Gaussian Mixture Models	359
<i>Frank Nielsen</i>	
Parameter Estimation with Generalized Empirical Localization	368
<i>Takashi Takenouchi</i>	
Properties of the Cross Entropy Between ARMA Processes	377
<i>Eric Grivel</i>	

Statistical Manifold and Hessian Information Geometry

Inequalities for Statistical Submanifolds in Hessian Manifolds of Constant Hessian Curvature	389
<i>Aliya Naaz Siddiqui, Kamran Ahmad, and Cenap Özel</i>	
B. Y. Chen Inequalities for Statistical Submanifolds in Sasakian Statistical Manifolds	398
<i>Mohd. Aquib, Michel Nguiffo Boyom, Ali H. Alkhaldi, and Mohammad Hasan Shahid</i>	
Generalized Wintgen Inequality for Legendrian Submanifolds in Sasakian Statistical Manifolds	407
<i>Michel Nguiffo Boyom, Zamrooda Jabeen, Mehraj Ahmad Lone, Mohamd Saleem Lone, and Mohammad Hasan Shahid</i>	
Logarithmic Divergences: Geometry and Interpretation of Curvature	413
<i>Ting-Kam Leonard Wong and Jiaowen Yang</i>	
Hessian Curvature and Optimal Transport	423
<i>Gabriel Khan and Jun Zhang</i>	

Non-parametric Information Geometry

Divergence Functions in Information Geometry	433
<i>Domenico Felice and Nihat Ay</i>	
Sobolev Statistical Manifolds and Exponential Models.	443
<i>Nigel J. Newton</i>	

Minimization of the Kullback-Leibler Divergence over a Log-Normal Exponential Arc	453
<i>Paola Siri and Barbara Trivellato</i>	
Riemannian Distance and Diameter of the Space of Probability Measures and the Parametrix	462
<i>Mitsuhiro Itoh and Hiroyasu Satoh</i>	
Statistics on Non-linear Data	
A Unified Formulation for the Bures-Wasserstein and Log-Euclidean/Log-Hilbert-Schmidt Distances between Positive Definite Operators.	475
<i>Hà Quang Minh</i>	
Exploration of Balanced Metrics on Symmetric Positive Definite Matrices . . .	484
<i>Yann Thanwerdas and Xavier Pennec</i>	
Affine-Invariant Midrange Statistics.	494
<i>Cyrus Mostajeran, Christian Grussler, and Rodolphe Sepulchre</i>	
Is Affine-Invariance Well Defined on SPD Matrices?	
A Principled Continuum of Metrics	502
<i>Yann Thanwerdas and Xavier Pennec</i>	
Shape Part Transfer via Semantic Latent Space Factorization	511
<i>Raphaël Groscot, Laurent Cohen, and Leonidas Guibas</i>	
Geometric and Structure Preserving Discretizations	
Variational Discretization Framework for Geophysical Flow Models	523
<i>Werner Bauer and François Gay-Balmaz</i>	
Finite Element Methods for Geometric Evolution Equations	532
<i>Evan S. Gawlik</i>	
Local Truncation Error of Low-Order Fractional Variational Integrators	541
<i>Fernando Jiménez and Sina Ober-Blöbaum</i>	
A Partitioned Finite Element Method for the Structure-Preserving Discretization of Damped Infinite-Dimensional Port-Hamiltonian Systems with Boundary Control	549
<i>Anass Serhani, Denis Matignon, and Ghislain Haine</i>	
Geometry, Energy, and Entropy Compatible (GEEC) Variational Approaches to Various Numerical Schemes for Fluid Dynamics	559
<i>Antoine Llor and Thibaud Vazquez-Gonzalez</i>	

Optimization on Manifold

Canonical Moments for Optimal Uncertainty Quantification on a Variety	571
<i>Jérôme Stenger, Fabrice Gamboa, Merlin Keller, and Bertrand Iooss</i>	
Computational Investigations of an Obstacle-Type Shape Optimization Problem in the Space of Smooth Shapes	579
<i>Daniel Luft and Kathrin Welker</i>	
Bézier Curves and C^2 Interpolation in Riemannian Symmetric Spaces	589
<i>Chafik Samir and Ines Adouani</i>	
A Formalization of the Natural Gradient Method for General Similarity Measures	599
<i>Anton Mallasto, Tom Dela Haije, and Aasa Feragen</i>	
The Frenet-Serret Framework for Aligning Geometric Curves	608
<i>Nicolas J.-B. Brunel and Juhyun Park</i>	

Geometry of Quantum States

When Geometry Meets Psycho-Physics and Quantum Mechanics: Modern Perspectives on the Space of Perceived Colors	621
<i>Michel Berthier and Edoardo Provenzi</i>	
Quantum Statistical Manifolds: The Finite-Dimensional Case	631
<i>Jan Naudts</i>	
Generalized Gibbs Ensembles in Discrete Quantum Gravity	638
<i>Goffredo Chirco and Isha Kotecha</i>	
On the Notion of Composite System	647
<i>Florio Maria Ciaglia, Alberto Ibort, and Giuseppe Marmo</i>	

Probability on Riemannian Manifolds

The Riemannian Barycentre as a Proxy for Global Optimisation	657
<i>Salem Said and Jonathan H. Manton</i>	
Hamiltonian Monte Carlo on Lie Groups and Constrained Mechanics on Homogeneous Manifolds	665
<i>Alessandro Barp</i>	
On the Fisher-Rao Information Metric in the Space of Normal Distributions	676
<i>Julianna Pinele, Sueli I. R. Costa, and João E. Strapasson</i>	

Simulation of Conditioned Diffusions on the Flat Torus	685
<i>Mathias Højgaard Jensen, Anton Mallasto, and Stefan Sommer</i>	
Towards Parametric Bi-Invariant Density Estimation on $SE(2)$	695
<i>Emmanuel Chevallier</i>	
Wasserstein Information Geometry/Optimal Transport	
Affine Natural Proximal Learning	705
<i>Wuchen Li, Alex Tong Lin, and Guido Montúfar</i>	
Parametric Fokker-Planck Equation	715
<i>Wuchen Li, Shu Liu, Hongyuan Zha, and Haomin Zhou</i>	
Multi-marginal Schrödinger Bridges	725
<i>Yongxin Chen, Giovanni Conforti, Tryphon T. Georgiou, and Luigia Ripani</i>	
Hopf–Cole Transformation and Schrödinger Problems	733
<i>Flavien Léger and Wuchen Li</i>	
Curvature of the Manifold of Fixed-Rank Positive-Semidefinite Matrices Endowed with the Bures–Wasserstein Metric	739
<i>Estelle Massart, Julien M. Hendrickx, and P.-A. Absil</i>	
Geometric Science of Information Libraries	
Second-Order Networks in PyTorch	751
<i>Daniel Brooks, Olivier Schwander, Frédéric Barbaresco, Jean-Yves Schneider, and Matthieu Cord</i>	
Symmetric Algorithmic Components for Shape Analysis with Diffeomorphisms	759
<i>Nicolas Guigui, Shuman Jia, Maxime Sermesant, and Xavier Pennec</i>	
Author Index	769