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System Theory of Continuous Time Finite Dimensional Dynamical Systems

The Memories of Tsuyoshi Matsuo
and R. E. Kalman

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Preface

This monograph deals with realization theory and control laws of continuous time finite dimensional dynamical systems which include linear and nonlinear input/output relations in the case of input's space being a set of the function which has the closed interval. The contents will be of popular interest to researchers, engineers, and graduate students who specialize in system theory.

In order to discuss control problems of continuous time finite dimensional dynamical systems clearly, we will discuss realization theory of our systems by rewriting the suitable form for continuous time dynamical systems from the obtained results in discrete time dynamical system (Matsuo and Hasegawa 2003).

In the monograph (Hasegawa 2013), new control problems of discrete time dynamical systems were proposed with a point of the following view:

After a state's being controlled to any state except the equilibrium state, we may have a result which means that the state can be only controlled to the same state once every n sampling times in an n dimensional canonical system. That means that any state does not remain static on different time zone. From the point of view that control is, roughly speaking, putting something in motion to our satisfaction, the change of the state in discrete time system may not be considered good for the concept of control. Therefore, the state control problem is not suitable for discrete time dynamical systems except the equilibrium state control. Thus, in monograph (Hasegawa 2013), it was shown that a state control problem is nonsense except equilibrium state control problem in discrete time dynamical systems and it was also shown that the control problem is suitable for the output control which is the fixed value output control or the tracking output control. Additionally, it was shown that their's solutions are obtained by using the non linear programming. Therefore, without considering that many results obtained in continuous time optimal control had been only converted to appropriate forms in discrete time optimal control and noting of using the special properties in discrete time case, we discussed the control problems newly.

Using the method thus allowed control inputs to be induced by characteristic phenomena of discrete time canonical finite dimensional dynamical systems. By virtue of this approach, the monograph (Hasegawa 2013, 2015) provided new control laws and their extensions which can also be more applicable for nonlinear dynamical systems.

We want to treat that the idea of control in discrete time case will be applied in continuous time case. Namely, we want to discuss the control problems in the same way as in the discrete time case. It is well known that a state of dynamical systems may be easily changed into another state by the free motion or inputs except equilibrium state and also known that the output value of a state is very different from the output value at the preceding state. If we want to construct a control system to be smooth and gradual, we must consider output control problem. Therefore, in the sense of input and output, control problem of output is more practical than control problems of input and state which are used in the usual control problem.

From the very new point, the problem will be treated in the sense of time domain approach. A new method which produces manipulated inputs will be proposed in the sense of equilibrium state control and output control and be represented as a state at the preceding time and a desired output value at the same point in time.

In analysis of state space approach, control problem may have become a theme of technology after 1960 for the purpose of efficiency in the field of economy, industrial technology, and others on the development of digital computers and mathematical programming.

Usual modern control design requires the solution of complicated nonlinear matrix equations; on the other hand, it is lacking in some aspects. The designed performance obtained by solving matrix design equations means that it is often possible to design a control system that works in theory without gaining any engineering intuition about the problem.

But our proposal provides a sort of intuition which means the closeness to input, output, and the state. And we will solve our control problems as algebraically as possible for the first time.

Note that we could discuss our control problems in the sense of nonlinear programming in the monograph (Hasegawa 2013, 2015).

Our proposal needs a computer-aided design which is an essential feature of modern controls.

Be based on input/output control, our control problems for a given dynamical system with input and output can be roughly stated as the following three problems:

1. *Equilibrium state control*

Find an input sequence that will bring an arbitrary state of the system to the equilibrium state (especially in linear system, the zero state) within the size of input values.

2. *Fixed value output control*

Find an input sequence that will bring an arbitrary output of the system to fixed value output within the size of input values.

3. *Tracking output control*

Find an input sequence that will bring an arbitrary output of the system to a desired trajectory output within the size of input values.

It is worth to remember that the development of control problem has been strongly stimulated by linear system theory well connected with the development of digital computers and related mathematics, for example, mathematical programming. However, such development of nonlinear dynamical systems has not occurred yet because there has been no suitable mathematical method for nonlinear systems, for example, without utilizing characteristic phenomena of discrete time and finite dimensional dynamical systems such as the fact in our system theory.

In this monograph, regarding the output sequence to be controlled as the equations to be expressed by terms of input, we identify our control problem as a problem of finding the unique inputs which produce the specified output. If we have not obtained the unique input, we will obtain the unique input by introducing the performance function for inputs to be treated as the square norm, namely, in the sense of energy. Our method intensively takes a positive attitude toward using computers and mathematical programming. Consequently, we will introduce a method based on least square norm.

As already mentioned, the usual control problems have been mainly discussed in linear systems. On the other hand, there are few developments for nonlinear systems. Our recent monograph *Realization Theory of Discrete-Time Dynamical Systems* (T. Matsuo and Y. Hasegawa, Lecture Notes in Control and Information Science, Vol. 296, Springer, 2003) indicated that any input/output map of nonlinear dynamical systems can be characterized by Hankel matrix or Input/output matrix, which are very similar to Hankel matrix in linear systems. The monograph also presented that obtaining a dynamical system which describes a given input/output map is equal to determining the rank of the matrix of the input/output map and the coefficients of a linear combination of column vectors in the matrix. We know that the reachability means the controllability in discrete time systems and that the reachability is completely determined by the rank of a matrix. And for observability, we may be similar in concept. This insight leads to the ability of discussing fruitful control problems, especially for both linear and nonlinear dynamical systems.

For the continuous time dynamical systems, this monograph also presents that obtaining a dynamical system which describes a given input/output map is equal to determining the rank of the matrix of the input/output map and the coefficients of a linear combination of column vectors in the matrix. We show that canonical dynamical systems are determined by the reachable (or quasi-reachable) and observable (or distinguishable). This insight of continuous time case leads to the ability of discussing fruitful control problems.

The content of this monograph may be one of the things that Tsuyoshi Matsuo and R. E. Kalman aimed for.

Gifu, Japan
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Yasumichi Hasegawa

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We also wish to thank Prof. R. E. Kalman for his suggestions. He stimulated us to research these problems of system theory directly as well as through his works. In July, 2016, he sadly passed away.

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