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# Recoverable Mutual Exclusion with Abortability

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**Abstract** Recent advances in non-volatile main memory (NVRAM) technology have spurred research on designing algorithms that are resilient to process crashes. This paper is a fuller version of our conference paper [21], which presents the first Recoverable Mutual Exclusion (RME) algorithm that supports abortability. Our algorithm uses only the read, write, and CAS operations, which are commonly supported by multiprocessors. It satisfies FCFS and other standard properties.

Our algorithm is also adaptive. On DSM and Relaxed-CC multiprocessors, a process incurs  $O(\min(k, \log n))$  RMRs in a passage and  $O(f + \min(k, \log n))$  RMRs in an attempt, where n is the number of processes that the algorithm is designed for, k is the point contention of the passage or the attempt, and f is the number of times that p crashes during the attempt. On a Strict CC multiprocessor, the passage and attempt complexities are O(n) and O(f + n).

Attiya et al. proved that, with any mutual exclusion algorithm, a process incurs at least  $\Omega(\log n)$  RMRs in a passage, if the algorithm uses only the read, write, and CAS operations [3]. This lower bound implies that the worst-case RMR complexity of our algorithm is optimal for the DSM and Relaxed CC multiprocessors.

**Keywords** concurrent algorithm, synchronization, mutual exclusion, recoverable algorithm, fault tolerance, non-volatile main memory, shared memory, multi-core algorithms

## 1 Introduction

Recent advances in non-volatile main memory (NVRAM) technology [13][28][32][33] have spurred research on designing algorithms that are resilient to process crashes. NVRAM is byte-addressable,

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so it replaces main memory, directly interfacing with the processor. This development is exciting because, if a process crashes and subsequently restarts, there is now hope that the process can somehow recover from the crash by consulting the contents of the NVRAM and resume its computation.

To leverage this advantage given by the NVRAM, there has been keen interest in reexamining the important distributed computing problems for which algorithms were designed in the past for the traditional (crash-free) model of an asynchronous shared memory multiprocessor. The goal is to design new algorithms that guarantee good properties even if processes crash at arbitrary points in the execution of the algorithm and subsequently restart and attempt to resume the execution of the algorithm. The challenge in designing such "recoverable" algorithms stems from the fact that when a process crashes, even though the shared variables that are stored in the NVRAM are unaffected, the crash wipes out the contents of the process' cache and CPU registers, including its program counter. So, when the process subsequently restarts, it can't have a precise knowledge of exactly where it crashed. For instance, if the last instruction that a process executes before a crash is a compare & swap (CAS) on a shared variable X, when it subsequently restarts, it can't tell whether the crash occurred just before or just after executing the CAS instruction and, if it did crash after the CAS, it won't know the response of the CAS (because the crash wipes out the register the CAS's response went into). The "recover" method, which a process is expected to execute when it restarts, has the arduous task of ensuring that the process can still somehow resume the execution of the algorithm seamlessly.

The mutual exclusion problem, formulated to enable multiple processes to share a resource that supports only one process at a time [8], has been thoroughly studied for over half a century for the traditional (crash-free) model, but its exploration for the crash-restart model is fairly recent. In the traditional version of the problem, each process p is initially in the "remainder" section. When pbecomes interested in acquiring the resource, it executes the try<sub>n</sub>() method; and when this method completes, p is in the "critical section" (CS). To give up the  $\hat{CS}$ , p invokes the  $exit_p()$  method; and when this method completes, p is back in the remainder section. An algorithm to this problem specifies the code for the try and exit methods so that at most one process is in the CS at any time and other desirable properties (such as starvation freedom, bounded exit, and First-Come-First-served, or FCFS) are also satisfied. Golab and Ramaraju were the first to reformulate this problem for the crash-restart model as Recoverable Mutual Exclusion (RME). In the RME problem, a process p can crash at any time and subsequently restart [12]. If p crashes while in try, CS, or exit, p's cache and registers (aka local variables) are wiped out and p returns to the remainder section (i.e., crash resets p's program counter to its remainder section). When p restarts after a crash, it is required to invoke a new method, named  $recover_p()$ , whose job is to "repair" the adverse effects of the crash and send p to where it belongs. In particular, if p crashed while in the CS,  $recover_p()$ puts p back in the CS (by returning IN\_CS). On the other hand, if p crashed while executing  $try_n()$ ,  $recover_n()$  has a choice—it can either roll p back to the Remainder (by returning IN\_REM) or put it in the CS (by returning IN\_CS). Similarly, if p crashed while executing  $exit_p()$ ,  $recover_p()$  has a choice of returning either IN\_REM or IN\_CS.

Golab and Ramaraju made a crucial observation that if p crashes while in the CS, then no other process should be allowed into the CS until p restarts and reenters the CS. This *Critical Section Reentry* (CSR) requirement was strengthed by Jayanti and Joshi's *Bounded CSR* requirement: if p crashes while in the CS, when p subsequently restarts and executes the recover method, the recover method should put p back into the CS in a bounded number of its own steps [20]. There has been a flurry of research on RME algorithms in the recent years [5][7][10][11][12][17][18][20][21][22].

Orthogonal to this development of recoverable algorithms, motivated by the needs of real time systems and database systems, Scott and Scherer advocated the need for mutual exclusion algorithms to support the "abort" feature, whereby a process in the try section can quickly quit the algorithm, if it so desires [30]. More specifically, if p receives an abort signal from the environment while executing the try method, the try method should complete in a bounded number of p's steps and either launch p into the CS or send p back to the remainder section. In the past two decades, there has been a lot of research on abortable mutual exclusion algorithms for the traditional (crash-free) model [].

The possibility of crashes, together with the CSR requirement, renders abortability even more important in the crash-restart model, yet there have been no *abortable* recoverable algorithms until the conference publication of the algorithm in this submission [21]. There has since been one more algorithm, by Katzan and Morrison [22], and we will soon compare the two algorithms.

## 1.1 RMR complexity.

Remote Memory Reference (RMR) complexity is the standard complexity metric used for comparing mutual exclusion algorithms, so we explain it here. This metric is explained for the two prevalent models of multiprocessors— $Distributed\ Shared\ Memory\ (DSM)$  and  $Cache-Coherent\ (CC)$  multiprocessors—as follows. In DSM, shared memory is partitioned into n portions, one per process, and each shared variable resides in exactly one of the n partitions. A step in which a process p executes an instruction on a shared variable X is considered an RMR if and only if X is not in p's partition of the shared memory.

In CC, the shared memory is remote to all processes, but every process has a local cache. A step in which a process p executes an instruction op on a shared variable X is considered an RMR if and only if op is read and X is not in p's cache, or op is any non-read operation (such as a write or CAS). If p reads X when X is not present in p's cache, X is brought into p's cache. If a process q performs a non-read operation op while X is in p's cache, X's copy in p's cache is deleted in the  $Strict\ CC\ model$ , but in the  $Relaxed\ CC\ model$  it is deleted only if op changes X's value. Thus, if X is in p's cache and q performs an unsuccessful CAS on X, then X continues to remain in p's cache in the relaxed CC model.

A passage of a process p starts when p leaves the remainder section and completes at the earliest subsequent time when p returns to the remainder (note that p returns to the remainder either because of a crash or because of a normal return from try, exit or recover methods). An attempt of p starts when p leaves the remainder and completes at the earliest subsequent time when p returns to the remainder "normally," i.e., not because of a crash. Note that each attempt includes one or more passages.

The RMR complexity of a passage (respectively, attempt) of a process p is the number of RMRs that p incurs in that passage (respectively, attempt).

# 1.2 Adaptive complexity.

A process is *active* if it is in the CS, or executing the try, exit, or recover methods, or crashed while in try, CS, exit, or recover and has not subsequently invoked the recover method. The *point contention* at any time t is the number of active processes at t. The point contention of a passage (respectively, attempt) is the maximum point contention at any time in that passage (respectively,

attempt). An algorithm is adaptive if the RMR complexity r of each passage (or attempt) of a process p is a function of that passage's (or attempt's) point contention k such that r = O(1) if k = O(1).

#### 1.3 Our contribution.

We present the first abortable RME algorithm. Our algorithm is based on the ideas underlying two earlier CAS-based algorithms—one that is recoverable but not abortable [20] and another that is abortable but not recoverable [16]. Our algorithm uses only the read, write, and CAS operations, which are commonly supported by multiprocessors. It satisfies FCFS and other standard properties (starvation-freedom, bounded exit, bounded CSR, and bounded abort). The algorithm's space complexity—the number of words of memory used—is O(n).

Our algorithm is also adaptive. On DSM and Relaxed CC multiprocessors, a process p incurs  $O(\min(k, \log n))$  RMRs in a passage and  $O(f + \min(k, \log n))$  RMRs in an attempt, where n is the number of processes that the algorithm is designed for, k is the point contention of the passage or the attempt, and f is the number of times that p crashes during the attempt. On a Strict CC multiprocessor, the passage and attempt complexities are O(n) and O(f + n).

Attiya et al. proved that, with any mutual exclusion algorithm (even if the algorithm does not have to satisfy recoverability or abortability), a process incurs at least  $\Omega(\log n)$  RMRs in a passage, if the algorithm uses only the read, write, and CAS operations [3]. This lower bound implies that the worst-case RMR complexity of our algorithm is optimal for the DSM and Relaxed CC multiprocessors.

### 1.4 Comparison to Katzan and Morrison's algorithm.

To the best of our knowledge, there is only one other abortable RME algorithm, published recently by Katzan and Morrison [22]. They achieve sublogarithmic complexity: a process incurs at most  $O(\min(k, \log n/\log\log n))$  RMRs in a passage and  $O(f + \min(k, \log n/\log\log n))$  in an attempt. Furthermore, they achieve these bounds for even the Strict CC multiprocessor.

On the other hand, our work has the following merits. Unlike the CAS instruction employed in our algorithm, the fetch&add instruction, which their algorithm employs to beat Attiya et al's lower bound and achieve sublogarithmic complexity, is not commonly supported by current machines. Their algorithm does not satisfy FCFS and has a higher space complexity of  $O(n \log^2 n/\log \log n)$ . Their algorithm is stated to satisfy starvation-freedom if the total number of crashes in the run is finite. In contrast, our algorithm guarantees that each attempt completes even in the face of infinitely many crashes in the run, provided that there are only finitely many crashes during each attempt.

Finally, Katzan and Morrison correctly point out a shortcoming in our conference paper: our algorithm there admits starvation if there are infinitely many aborts in a run. The algorithm in this submission has been revised to eliminate this shortcoming.

### 1.5 Related Research.

All of the works on RME prior to the conference version of our paper [21] has focused on designing algorithms that do not provide abortability as a capability. Golab and Ramaraju [12] formalized the

RME problem and designed several algorithms by adapting traditional mutual exclusion algorithms. Ramaraju [27], Jayanti and Joshi [20], and Jayanti et al. [17] designed RME algorithms that support the First-Come-First-Served property [23]. Golab and Hendler [10] presented an algorithm that has sub-logarithmic RMR complexity on CC machines. Jayanti et al. [18] presented a unified algorithm that has a sub-logarithmic RMR complexity on both CC and DSM machines. In another work, Golab and Hendler [11] presented an algorithm that has the ideal O(1) passage complexity, but this result assumes that all processes in the system crash simultaneously. Recently, Dhoked and Mittal [7] present an RME algorithm whose RMR complexity adapts to the number of crashes, and Chan and Woelfel [5] present an algorithm which has an O(1) amortized RMR complexity. Recently Katzan and Morrison [22] gave an abortable RME algorithm that incurs sub-logarithmic RMR on CC and DSM machines.

When it comes to abortability for classical mutual exclusion problem, Scott [29] and Scott and Scherer [31] designed abortable algorithms that build on the queue-based algorithms [6][25]. Jayanti [16] designed an algorithm based on read, write, and comparison primitives having  $O(\log n)$  RMR complexity which is also optimal [3]. Lee [24] designed an algorithm for CC machines that uses the Fetch-and-Add and Fetch-and-Store primitives. Alon and Morrison [1] designed an algorithm for CC machines that has a sub-logarithmic RMR complexity and uses the read, write, Fetch-And-Store, and comparison primitives. Recently, Jayanti and Jayanti [19] designed an algorithm for the CC and DSM machines that has a constant amortized RMR complexity and uses the read, write, and Fetch-And-Store primitives. While the works mentioned so far have been deterministic algorithms, randomized versions of classical mutual exclusion with abortability exist. Pareek and Woelfel [26] give a sublogarithmic RMR complexity randomized algorithm and Giakkoupis and Woelfel [9] give an O(1) expected amortized RMR complexity randomized algorithm.

#### 2 Specification of the problem

In this section, we rigorously specify the Abortable RME problem by defining what an abortable RME algorithm is, modeling the algorithm's runs, and stating the properties that these runs must satisfy.

#### 2.1 Abortable RME algorithm

An Abortable Recoverable Mutual Exclusion algorithm, abbreviated Abortable RME algorithm, is a tuple  $(\mathcal{P}, \mathcal{X}, \mathit{Vals}, \mathcal{F}, \mathit{OP}, \Delta, \mathcal{M})$ , where

- $-\mathcal{P}$  is a set of processes. Each process  $p \in \mathcal{P}$  has a set of registers, including a program counter, denoted  $PC_p$ , which points to an instruction in p's code.
- $\mathcal{X}$  is a set of variables, which includes a Boolean variable AbortSignal[p], for each  $p \in \mathcal{P}$ . No process except p can invoke any operation on AbortSignal[p], and p can only invoke a read operation on AbortSignal[p].
  - Intuitively, the "environment" sets ABORTSIGNAL[p] to true when it wishes to communicate to p that it should abort its attempt to acquire the CS and return to the Remainder.
- Vals is a set of values (that each variable in  $\mathcal{X}$  can possibly take on). For example, on a 64-bit machine, Vals would be the set of all 64-bit integers.

- $-\mathcal{F}$  is a function that assigns a value from *Vals* to each variable in  $\mathcal{X}$ . For all  $X \in \mathcal{X}$ ,  $\mathcal{F}(X)$  is X's initial value.
- OP is a set of operations that each variable in  $\mathcal{X} \{ABORTSIGNAL[p] \mid p \in \mathcal{P}\}$  supports. For the algorithm in this paper,  $OP = \{read, write, CAS\}$ , where CAS(X, r, s), when executed by a process p (and X is a variable and r, s are p's registers), compares the values of X and r; if they are equal, the operation writes in X the value in s and returns true; otherwise, the operation returns false, leaving X unchanged.
- $-\Delta$  is a partition of  $\mathcal{X}$  into  $|\mathcal{P}|$  sets, named  $\Delta(p)$ , for each  $p \in \mathcal{P}$ . Intuitively,  $\Delta(p)$  is the set of variables that reside locally at process p's partition on a DSM machine, but has no relevance on a CC machine.
- $\mathcal{M}$  is a set of methods, which includes three methods per process  $p \in \mathcal{P}$ , named  $\mathtt{try}_p()$ ,  $\mathtt{exit}_p()$ , and  $\mathtt{recover}_p()$ , such that:
  - In any instruction of any method, at most one operation is performed and it is performed on a single variable from  $\mathcal{X}$ .
  - The methods  $\mathtt{try}_p()$  and  $\mathtt{recover}_p()$  return a value from  $\{\mathtt{IN\_CS},\mathtt{IN\_REM}\},$  and  $\mathtt{exit}_p()$  has no return value.
  - None of  $try_p()$ ,  $exit_p()$ , or  $recover_p()$  calls itself or the other two. (This assumption simplifies the model, but is not limiting in any way because it does not preclude the use of helper methods each of which can call itself or the other helper methods.)

### 2.2 Abstract sections of code and abstract variables

For each process  $p \in \mathcal{P}$ , we model p's code outside of the methods in  $\mathcal{M}$  to consist of two disjoint sections, named  $\mathtt{remainder}_p()$  and  $\mathtt{cs}_p()$ . Furthermore, we introduce the following *abstract* variables, which are not in  $\mathcal{X}$  and not accessed by the methods in  $\mathcal{M}$ , but are helpful in defining the problem.

- $status_p \in \{good, recover-from-try, recover-from-cs, recover-from-exit, recover-from-rem\}$ . Informally,  $status_p$  models p's "recovery status". If  $status_p \neq good$ , it means that p is still recovering from a crash, and in this case, the value of  $status_p$  reveals the section of code where p most recently crashed.
- Cache<sub>p</sub> holds a set of pairs of the form (X, v), where  $X \in \mathcal{X}$  and  $v \in Vals$ . Informally, if (X, v) is present in the cache, X is in p's cache and v is its current value. This abstract variable helps define what operations count as remote memory references (RMR) on CC machines.

## 2.3 Run, Fair Run, Passage, Attempt

A state of a process p is a function that assigns a value to each of p's registers, including  $PC_p$ , and a value to each of  $status_p$ , AbortSignal[p], and Cache $_p$ .

A **configuration** is a function that assigns a state to each process in  $\mathcal{P}$  and a value to each variable in  $\mathcal{X}$ . (Intuitively, a configuration is a snapshot of the states of processes and values of variables at a point in time.)

An initial configuration is a configuration where, for each  $p \in \mathcal{P}$ ,  $PC_p = \mathtt{remainder}_p()$ ,  $status_p = good$ , AbortSignal[p] = false, and  $Cache_p = \emptyset$ ; and, for each  $X \in \mathcal{X}$ ,  $X = \mathcal{F}(X)$ .

A **run** is a finite sequence  $C_0, \alpha_1, C_1, \alpha_2, C_2, \dots \alpha_k, C_k$ , or an infinite sequence  $C_0, \alpha_1, C_1, \alpha_2, C_2, \dots$  such that:

- 1.  $C_0$  is an initial configuration and, for each i,  $C_i$  is a configuration and  $\alpha_i$  is either (p, normal) or (p, crash), for some  $p \in \mathcal{P}$ .
  - We call each triple  $(C_{i-1}, \alpha_i, C_i)$  a step; it is a normal step of p if  $\alpha_i = (p, normal)$ , and a crash step of p if  $\alpha_i = (p, crash)$ .
- 2. For each normal step  $(C_{i-1}, (p, normal), C_i)$ ,  $C_i$  is the configuration that results when p executes an enabled instruction of its code, explained as follows:
  - If  $PC_p = \text{remainder}_p()$  and  $status_p = good$  in  $C_{i-1}$ , then p invokes either  $\text{try}_p()$  or  $\text{recover}_p()$ .
  - If  $PC_p = \mathtt{remainder}_p()$  and  $status_p \neq good$  in  $C_{i-1}$ , then p invokes  $\mathtt{recover}_p()$ .
  - If  $PC_p = cs_p()$ , then p invokes  $exit_p()$ .
  - Otherwise, p executes the instruction that  $PC_p$  points to in  $C_{i-1}$ .
    - If this instruction returns IN\_CS (resp., IN\_REM),  $PC_p$  is set to  $cs_p()$  (resp., remainder<sub>p</sub>()). If the instruction causes p to return from  $recover_p()$ ,  $status_p$  is set to good in  $C_i$ .
    - If p performs a read on X and X is not present in CACHE<sub>p</sub> in  $C_{i-1}$ , then (X, v) is inserted in CACHE<sub>p</sub>, where v is X's value in  $C_{i-1}$ .
    - In the Strict-CC model, if p performs a non-read operation on X, X is removed from Cache, for all  $q \in \mathcal{P}$ .
    - In the Relaxed-CC model, if p performs a non-read operation on X that changes X's value, X is removed from  $CACHE_q$ , for all  $q \in \mathcal{P}$ .
- 3. For each crash step  $(C_{i-1}, (p, crash), C_i)$ , we have:
  - In  $C_i$ ,  $PC_p$  is set to remainder<sub>p</sub>() and all other registers of p are set to arbitrary values, and CACHE<sub>p</sub> is set to  $\emptyset$ .
  - If  $status_p \neq good$  in  $C_{i-1}$ , then  $status_p$  remains unchanged in  $C_i$ . Otherwise, if (in  $C_{i-1}$ ) p is in  $try_p()$  (respectively,  $cs_p()$ ,  $exit_p()$ , or  $recover_p()$ ), then  $status_p$  is set in  $C_i$  to  $recover\_from\_try$  (respectively,  $recover\_from\_cs$ ,  $recover\_from\_exit$ , or  $recover\_from\_rem$ ).

A run  $R = C_0, \alpha_1, C_1, \alpha_2, C_2, \ldots$  is **fair** if and only if either R is finite or, for all configurations  $C_i$  and for all processes  $p \in \mathcal{P}$ , the following condition is satisfied: unless  $PC_p = \mathtt{remainder}_p()$  and  $status_p = good$  in  $C_i$ , p has a step in the suffix of R from  $C_i$ .

Thus, in a fair run, a crashed process eventually restarts, no process stays in the CS forever, and no process permanently ceases to take steps when it is outside the Remainder section.

A passage of a process p is a contiguous sequence  $\sigma$  of steps in a run such that p leaves  $\mathtt{remainder}_p()$  in the first step of  $\sigma$  and the last step of  $\sigma$  is the earliest subsequent step in the run where p reenters  $\mathtt{remainder}_p()$  (either because p crashes or because p's method returns IN\_REM).

An **attempt** of a process p is a maximal contiguous sequence  $\sigma$  of steps in a run such that p leaves  $\mathtt{remainder}_p()$  in the first step of  $\sigma$  with  $status_p = good$  and the last step of  $\sigma$  is the earliest subsequent normal step in the run that causes p to reenter  $\mathtt{remainder}_p()$  (which would be a return from  $\mathtt{exit}_p$ , or a return of IN\_REM from  $\mathtt{try}_p$  or  $\mathtt{recover}_p$ ).

#### 2.4 Remote Memory Reference (RMR) and Point Contention

A step of p is an **RMR on a DSM machine** if and only if it is a normal step in which p performs an operation on some variable that is not in  $\Delta(p)$ .

A step of p is an RMR on a Strict or Relaxed CC machine if and only if it is a normal step in which p performs a non-read operation, or p reads some variable that is not present in p's cache.

The **point contention** at a configuration C is the number of processes p such that  $(PC_p \neq \text{remainder}_p) \vee (status_p \neq good)$  in C.

#### 2.5 Desirable properties

We now state the desirable properties of an abortable RME algorithm, which we divide into three groups—general, recovery-related, and abort-related.

# General properties:

- P1 Mutual Exclusion: At most one process is in the CS in any configuration of any run.
- P2 Bounded Exit: There is an integer b such that if in any run any process p invokes and executes  $exit_p()$  without crashing, the method completes in at most b steps of p.
- P3 Weak Starvation Freedom (WSF): In every fair infinite run in which there are only finitely many crash steps, if a process p is in the Try section in a configuration, p is in a different section in a later configuration.
- P4 Starvation Freedom (SF): In every fair infinite run in which every attempt contains only finitely many crash steps, if a process p is in the Try section in a configuration, p is in a different section in a later configuration.
  - We note that SF implies WSF.
- P5 First-Come-First-Served (FCFS): There is an integer b such that in any run, if A and A' are attempts by any distinct processes p and p', respectively, p performs at least b consecutive normal steps in A before the attempt A' starts, and p neither receives an abort signal nor subsequently crashes in  $\operatorname{try}_p()$  in A, then p' does not enter the CS in A' before p enters the CS in A.

#### Recovery related properties:

- P6 Critical Section Reentry (CSR) [12]: In any run, if a process p crashes while in the CS, no other process enters the CS until p subsequently reenters the CS.
- P7 Bounded Recovery to CS: There is an integer b such that if in any run any process p executes  $\overline{\mathtt{recover}_p()}$  without crashing and with  $status_p = recover\text{-}from\text{-}cs$ , the method completes in at most b steps of p and returns IN\_CS.
- P8 Bounded Recovery to Exit: There is an integer b such that if in any run any process p executes  $recover_p()$  without crashing and with  $status_p = recover-from-exit$ , the method completes in at most b steps of p.
- P9 Fast Recovery to Remainder: There is an absolute constant b, i.e., a constant independent of  $|\mathcal{P}|$ , such that if in any run any process p executes  $\mathsf{recover}_p()$  without crashing and with  $status_p \in \{good, recover\text{-}from\text{-}rem\}$ , the method completes in at most b steps of p.
- P10 Bounded Recovery to Remainder: There is an integer b such that if in any run  $\mathtt{recover}_p()$ , executed by a process p with  $status_p = recover-from-try$ , returns IN\_REM, p must have completed that execution of  $\mathtt{recover}_p()$  in at most b of its steps.

## Abort related properties:

For any run R, any configuration C of R, and any process p, define the predicate  $\beta(R, p, C)$  as true if and only if in configuration C it is the case that AbortSignal[p] is true and either p is in  $\mathtt{try}_p()$  or p is in  $\mathtt{recover}_p()$  with  $status_p = recover-from-try$ .

- P11 Bounded Abort: There is an integer b such that, for each R, C, p, if  $\beta(R, p, C)$  is true, AbortSignal[p] stays true for ever (i.e., stays true in the suffix R' of the run from C), and p executes steps without crashing (i.e., p has no crash steps in R'), then p enters either the CS or the remainder in at most b of its steps (in R').
- P12 No Trivial Aborts: In any run, if AbortSignal[p] is false when a process p invokes  $\mathsf{try}_p()$ , AbortSignal[p] remains false forever, and p executes steps without crashing, then  $\mathsf{try}_p()$  does not return IN\_REM.

## 3 The Algorithm

We present our abortable RME algorithm in Figure 1. The algorithm is designed for the set of processes  $\mathcal{P} = \{1, 2, ..., n\}$ . All the shared variables used by our algorithm are stored in NVRAM. Variables with a subscript of p to their name are local to process p, and are stored in p's registers.

### 3.1 Shared variables and their purpose

We describe below the role played by each shared variable used in the algorithm.

- Token is an unbounded positive integer. A process p reads this variable at the beginning of  $\mathsf{try}_p()$  to obtain its token and then increments, thereby ensuring that processes that invoke the try method later will get a strictly bigger token.
- CSSTATUS and SEQ: These two shared variables are used in conjunction, with SEQ holding an unbounded integer and CSSTATUS holding a pair, which is either (true, p) (for some  $p \in \mathcal{P}$ ) or (false, SEQ). If CSSTATUS = (true, p), it means that p owns the CS and, if CSSTATUS = (false, SEQ), it means that no process owns the CS. If SEQ has a value s while p is the CS, when exiting the CS p increments SEQ to s+1 and writes (0, s+1) in CSSTATUS. As we explain later, this act is crucial to ensuring that no process will be made the owner of the CS after it has moved back to the remainder.
- Go[p] has one of three values -1, 0, or p's token. The algorithm ensures that Go[p] = -1 whenever p is in the remainder "normally", i.e., not because of a crash but because the try, exit, or recover method returned normally. If Go[p] = 0, it means that p is made the owner of CS, hence p has the permission to enter the CS. After p obtains a token in  $\text{try}_p()$ , p writes its token in Go[p] and, subsequently when p must wait for its turn to enter the CS, it spins until either Go[p] turns 0 or it receives a signal to abort.
- REGISTRY is a min-array object [15] of n locations that supports two operations: REGISTRY[p].write(v), which can only be executed by process p, writes v in REGISTRY[p]; and REGISTRY.findmin() returns the minimum value in the array. After p obtains a token t in  $\text{try}_p()$ , it announces its interest to capture the CS by writing the pair (p, t) in REGISTRY[p], and when no longer interested, it takes itself out by writing (p,  $\infty$ ) in REGISTRY[p]. The "less than" relation on pairs is defined as follows: (p, t) of t and only if t and only if t and t are all of the second states of the second states t.
  - It turns out that the REGISTRY object has an implementation, using only read, write, and CAS operations, with three nice properties [15]: it is linearizable, wait-free, and idempotent, i.e., if p crashes while executing the method REGISTRY[p].write(v) and reexecutes the method once more

```
Persistent variables (stored in NVRAM)
   REGISTRY[1... |\mathcal{P}|]: A min-array; initially REGISTRY[p] = (p, \infty), for all p \in \mathcal{P}.
   CSSTATUS \in \{0\} \times (\{0\} \cup \mathbb{N}^+) \cup \{1\} \times \mathcal{P}; initially (0,1).
   SEQ \in \mathbb{N}; initially 1.
   \forall p \in \mathcal{P}, \text{Go}[p] \in \mathbb{N}^+ \cup \{-1, 0\}, \text{ initially } \perp.
   Token \in \mathbb{N}, initially 1.
1. Remainder Section
      procedure try_n():
2. tok_p \leftarrow TOKEN
3. CAS(TOKEN, tok_p, tok_p + 1)
4. Go[p] \leftarrow tok_p
5. REGISTRY[p].write((p, tok_p))
     \mathtt{promote}_p(false)
      wait till Go[p] = 0 \lor ABORTSIGNAL[p]
8. if Go[p] = 0: return IN_CS
9. return abort_p()
10. Critical Section
      \mathbf{procedure}\ \mathtt{exit}_p() \colon
11. \overline{\text{REGISTRY}[p].\text{write}((p,\infty))}
12. s_p \leftarrow \text{Seq}
13. SEQ \leftarrow s_p + 1
14. CSSTATUS \leftarrow (0, s_p + 1)
15. promote_n(false)
16. Go[p] \leftarrow -1
      procedure recover_p():
17. if Go[p] = -1: return IN REM
18. return abort<sub>p</sub>()
      procedure abort_p():
19. \overline{\text{REGISTRY}[p].\text{write}((p,\infty))}
20. promote _{p}(true)
21. if CSSTATUS = (1, p): return IN_CS
22. Go[p] \leftarrow -1
23. return IN_REM
      procedure promote_p(boolean flag_p):
24. (b_p, s_p) \leftarrow \text{CSSTATUS}; if b_p = 1: { peer_p \leftarrow s_p; go to Line 27 }

25. (peer_p, tok_p) \leftarrow \text{REGISTRY.findmin}(); if tok_p = \infty \land flag_p: peer_p \leftarrow p else if tok_p = \infty: return
26. if \neg CAS(CSSTATUS, (0, s_p), (1, peer_p)): return
27. g_p \leftarrow \text{Go}[peer_p]; if g_p \in \{-1, 0\}: return
28. if CSSTATUS \neq (1, peer_p): return
29. CAS(Go[peer_p], g_p, 0)
```

Fig. 1: Abortable RME Algorithm for CC and DSM machines. Code for process p.

upon restart, the effect is the same as executing the method once without ever crashing. The implementation uses only O(n) variables and has only a logarithmic RMR complexity on a DSM or a Relaxed CC machine: Registry.findmin() incurs O(1) RMRs and Registry[p].write(v) incurs  $O(\min(k, \log n))$  RMRs, where k is the maximum point contention during the execution of Registry[p].write(v). The idempotence property of the implementation makes it suitable for use in our algorithm [20].

#### 3.2 Informal description

In this section we present an intuitive understanding of the algorithm that explains the lines of code and, more importantly, draws attention to potential race conditions and how the algorithm overcomes them.

## Understanding $try_n()$

After a process p invokes  $\operatorname{try}_p()$ , it reads and then attempts to increments Token (Lines 2, 3). The attempt to increment serves two purposes. First, if a different process q invokes  $\operatorname{try}_q()$  later, it gets a strictly larger token, which helps realize FCFS. Second, if p were to abort its curent attempt A, it will obtain a strictly larger token in its next attempt A', which, as we will see, helps ensure that any process q that might attempt to release p from its busy-wait in the attempt A will not accidentally release p from its busy-wait in the attempt A'. Process p writes its token in  $\operatorname{GO}[p]$  (Line 4), where it will later busy-wait until some process changes  $\operatorname{GO}[p]$  to 0, and then announces its interest in the CS by changing Registry [p] from  $(p, \infty)$  to  $(p, \operatorname{its})$  token (Line 5). It then calls the  $\operatorname{promote}_p()$  procedure, which is crucial to ensuring livelock-freedom (Line 6).

# Understanding $promote_p()$

The promote<sub>n</sub>() procedure's purpose is to push a waiting process into the CS, if the CS is unoccupied. To this end, p reads CSSTATUS (Line 24). If it finds that the CS is already owned (i.e.,  $b_p = 1$ ), since it is possible that the owner  $peer_p$  is still busywaiting unaware of its ownership, p jumps to Line 27, where the code to release  $peer_p$  starts. On the other hand, if the CS is unoccupied (i.e.,  $b_p = 0$ ), it executes Line 25 to find out the process that has the smallest token in the REGISTRY, i.e., the process  $peer_p$  that has been waiting the longest. Since  $promote_p()$  is called from p's Line 6, at which point REGISTRY[p] has a finite token number for p, at Line 25 we have  $tok_p \neq \infty$ . So, p proceeds to Line 26, where it attempts to launch  $peer_p$  into the CS. If p's CAS fails, it means that someone else must have succeeded in launching a process into the CS between p's Line 24 and Line 26; in this case p has no further role to play, so it returns from the procedure. On the other hand, if p's CAS succeeds, which means that  $peer_p$  has been made the CS owner, p has a responsibility to release  $peer_p$  from its busywait, i.e., p must write 0 in  $Go[peer_p]$ . However, there is potential for a nasty race condition here, as explained by the following scenario: some process different from preleases  $peer_p$  from its busywait;  $peer_p$  enters the CS and then exits to the remainder; some other process q is now in the CS;  $peer_p$  executes the try method once more and proceeds up to the point of busy-waiting. Recall that p is poised to write 0 in  $Go[peer_p]$ . If p does this writing,  $peer_p$  will be released from its busywait, so  $peer_p$  proceeds to the CS, where q is already present. So, mutual exclusion is violated! Our algorithm averts this disaster by exploiting the fact that, while  $peer_p$ busywaits,  $Go[peer_p]$ 's value is never the same between different attempts of  $peer_p$ . Specifically, p reads  $Go[peer_p]$  (Line 27); if  $g_p$  is -1 or 0, it means that  $peer_p$  is not busywaiting, so p has no role to play, hence it returns. If things have moved on and  $peer_p$  no longer owns the CS, then too p has

no role to play, hence it returns (Line 28). Otherwise, there are two possibilities: either  $Go[peer_p]$  is still  $g_p$  or it has changed. In the former case,  $peer_p$  must be busywaiting, so it is imperative that p takes the responsibility to release  $peer_p$  (by changing  $Go[peer_p]$  to 0). In the latter case,  $peer_p$  requires no help from p, so p must not change  $Go[peer_p]$  (in order to avoid the race condition described above). This is precisely what the CAS at Line 29 accomplishes.

## The rest of $try_p()$

Upon returning from  $promote_p()$ , p busywaits until it reads a 0 in Go[p] or it receives a request to abort (Line 7). If p reads a 0 in Go[p], p infers that it owns the CS, so  $try_p()$  returns IN\_CS (Line 8). If p receives a request to abort, it calls  $abort_p()$  (Line 9), which we describe next.

## Understanding abort<sub>p</sub>()

To abort, p writes  $(p, \infty)$  to make it known to all that it has no interest in capturing the CS (Line 19). If any process will invoke the promote procedure after this point, it will not find p in REGISTRY, so it will not attempt to launch p into the CS. Does this mean that p can now return to the remainder section? The answer is a thundering no because there are two nasty race conditions that need to be overcome.

First, it is possible that, before p performed Line 19, some process q performed its Line 25 to find p in REGISTRY, and then successfully launched p into the CS (by writing (1, p) in CSSTATUS). Taking care of this scenario is easy: p can read CSSTATUS and if p finds that it owns the CS, it can abort by simply returning IN\_CS.

The second potential race is more subtle and harder to overcome. As in the earlier scenario, suppose that, before p performed Line **19**, some process q performed its Line **25** to find p in REGISTRY (i.e.,  $peer_q = p$ ). Furthermore, suppose that q is now at Line **26** and CSSTATUS =  $(0, s_q)$ . So, after performing Line **19**, if p naively returns to the remainder and then q performs Line **26**, we would be in a situation where p has been made the CS owner after it was back in the remainder!

To overcome the above two race conditions, p calls  $promote_n(true)$  (Line 20).

The parameter true conveys that the call is made by p while aborting, and has the following impact on how p executes  $promote_p()$ : if p finds the CS to be unoccupied at Line 24 and finds REGISTRY to be empty at Line 25, to preempt the second race condition discussed above (where some process q is poised to launch p into the CS), p will attempt to launch itself into the CS (by setting  $peer_p$  to p at Line 25 and attempting to change CSSTATUS to  $(1, peer_p)$ ). The key insight is that, after p performs the CAS at Line 26, only two possibilities remain: either p is already launched into the CS (i.e., CSSTATUS = (1,p)) or it is guaranteed that no process will launch p into the CS. In the former case,  $abort_p()$  returns IN\_CS at Line 21; and in the latter case, since it is safe for p to return to the remainder,  $abort_p()$  returns IN\_REM at Line 23 after setting Go[p] to -1 at Line 22 (in order to respect the earlier mentioned invariant that Go[p] = -1 whenever p returns to the remainder normally).

## Understanding $exit_p()$

There are two routes by which p might enter the CS. One is the "normal" route where p executes  $\operatorname{try}_p()$  without aborting or crashing, and  $\operatorname{try}_p()$  returns IN\_CS, thereby sending p to the CS. The second route is where p receives an abort signal, calls at Line  $\mathbf{9}$  abort $_p()$ , which returns IN\_CS at Line  $\mathbf{21}$ , causing  $\operatorname{try}_p()$  also to return IN\_CS at Line  $\mathbf{9}$ . When p is in the CS, p's announcement in Registry [p] (made at Line  $\mathbf{5}$ ), would no longer be there if it entered the CS by the second route (because of Line  $\mathbf{19}$ ), but it would still be there if it entered the CS by the first route. So, when p exits the CS, it removes its announcement in Registry [p] (Line  $\mathbf{11}$ ). It then increments the

number in SEQ and gives up its ownership of the CS by changing CSSTATUS from (1, p) to (0, SEQ) (Lines 12, 13, 14). To launch a waiting process, if any, into the just vacated CS, p then executes  $promote_p()$  (Line 15), and returns to the remainder after setting Go[p] to -1 at Line 16 (in order to respect the earlier mentioned invariant that Go[p] = -1 whenever p returns to the remainder normally).

# Understanding $recover_p()$

Process p executes  $\mathtt{recover}_p()$  when it restarts after a crash. If  $\mathrm{Go}[p]$  has -1, p infers that either  $\mathtt{recover}_p()$  was called when  $\mathit{status}_p = \mathit{good}$  or the most recent crash had occured early in  $\mathtt{try}_p()$ , so  $\mathtt{recover}_p()$  simply sends p back to the remainder (Line 17). Otherwise,  $\mathtt{recover}_p()$  simply calls  $\mathtt{abort}_p()$  (Line 17), which does the needful. In particular, if p was in the CS at the most recent crash, then CSSTATUS would have (1,p), which causes  $\mathtt{abort}_p()$  to send p back to the CS. Otherwise,  $\mathtt{abort}_p()$  extricates p from the algorithm, sending it either to the CS or to the remainder.

#### 4 Proof of Correctness

Figure 2 presents the invariant satisfied by the Abortable RME algorithm given in Figure 1. We have written the 13 statements comprising the invariant with the following conventions. All statements about process p are universally quantified, i.e.,  $\forall p \in \mathcal{P}$  is implicit (these are Statements 3 through 11, and Statement 13). The program counter for a process p, i.e.,  $PC_p$ , can take any of the values from the set [1,29]. However, when a call to procedure  $\operatorname{promote}_p()$  is made by p and p is executing one of the steps from Lines 24-29, for clearly conveying where the call was made from, we prefix the value of  $PC_p$  with the line number from where  $\operatorname{promote}_p()$  was called, along with the scope resolution operator from C++, namely, "::". Thus,  $PC_p = \mathbf{6}$ ::27 means p called  $\operatorname{promote}_p()$  from Line  $\mathbf{6}$  and is now executing Line 27 in that call. Sometimes, in the interest of brevity, we use the range operator, i.e., [a,b], to convey something more than just saying the range of values from a to b (inclusive). That is, if  $PC_p \in [6,8]$ , we also mean that  $PC_p$  could take on values from  $[\mathbf{6}$ ::24,  $\mathbf{6}$ ::29] because there is a call to  $\operatorname{promote}_p()$  at Line  $\mathbf{6}$ . Similarly,  $PC_p \in [5,6]$  means that  $PC_p$  takes on values from  $[\mathbf{6}$ ::24,  $\mathbf{6}$ ::29] because from  $[\mathbf{6}$ ::29] because, again, there is a call to  $\operatorname{promote}_p()$  at Line  $\mathbf{6}$ .

**Lemma 1 (Mutual Exclusion)** At most one process is in the CS in any configuration of any run.

Proof Suppose there is a configuration C such that two distinct processes p and q are in the CS, i.e.,  $PC_p = PC_q = 10$ . By Condition 5, CSSTATUS = (1, p) and CSSTATUS = (1, q) in C, which means CSSTATUS has two different values in the same configuration, a contradiction.

**Lemma 2 (Bounded Exit)** There is an integer b such that if in any run any process p invokes and executes  $exit_p()$  without crashing, the method completes in at most b steps of p.

Proof As explained earlier, the call to REGISTRY[p].write() at Line 11 takes  $O(\log n)$  steps. From an inspection of the algorithm we see that the rest of the execution of  $\mathtt{exit}_p()$  completes in a constant number of steps (this includes the execution of REGISTRY.findmin() invoked from a call to  $\mathtt{promote}_p()$ ). It follows that for a certain constant c, the execution of  $\mathtt{exit}_p()$  completes in at most  $c \log n$  steps in a run, if p invokes and executes it without crashing.

#### Conditions:

```
1. Token > 1
  2. (CSSTATUS = (0, Seq)) \vee (\exists q \in \mathcal{P}, CSSTATUS = (1, q))
 3. (-1 \le Go[p] < Token) \land (PC_p = 5 \Rightarrow Go[p] = tok_p) \land (PC_p \in [6, 8] \Rightarrow Go[p] \in \{0, tok_p\})
           \land (PC_p \in \{9\text{-}16, 18\text{-}22, 24\text{-}29\} \Rightarrow Go[p] \neq -1)
            \wedge ((PC_p \in \{2\text{-}4,23\} \lor (PC_p \in \{1,17\} \land status_p \in \{good, recover\text{-}from\text{-}rem\})) \Rightarrow Go[p] = -1)
 4. (\exists t \in [1, \text{Token} - 1] \cup \{\infty\}, \text{Registry}[p] = (p, t))
           5. (((PC_p \in [6,8] \land Go[p] = 0) \lor PC_p \in [10,14] \lor status_p = recover-from-cs) \Rightarrow CSSTATUS = (1,p))
            \wedge ((PC_p \in \{5, 22\} \cup [15, 16] \vee Go[p] = -1) \Rightarrow CSSTATUS \neq (1, p))
  6. This condition states what values local variables of process p take on.
       (PC_p = 3 \Rightarrow 1 \le tok_p \le \text{Token}) \land (PC_p \in [4, 8] \Rightarrow 1 \le tok_p < \text{Token})
           \land (PC_p = 13 \Rightarrow s_p = \text{SeQ}) \land (PC_p = 14 \Rightarrow s_p = \text{SeQ} - 1)
           \land \; (PC_p \in [\mathbf{6} :: \mathbf{24}, \mathbf{6} :: \mathbf{29}] \cup [\mathbf{15} :: \mathbf{24}, \mathbf{15} :: \mathbf{29}] \Rightarrow flag_p = false) \; \land \; (PC_p \in [\mathbf{20} :: \mathbf{24}, \mathbf{20} :: \mathbf{29}] \Rightarrow flag_p = true)
           \land (PC_p \in [26, 29] \Rightarrow peer_p \in \mathcal{P})
            \land (PC_p \in [2, 16] \Rightarrow status_p = good)
  7. (PC_p = 8 \Rightarrow (Go[p] = 0 \lor abort was requested)) \land (PC_p = 9 \Rightarrow abort was requested)
 8. PC_p \in \{25, 26\} \Rightarrow (s_p \leq \text{SEQ} \land (\forall q, PC_q \in \{13, 14\} \Rightarrow s_p \leq s_q))
 9. ((PC_p = 25 \land \text{CSSTATUS} = (0, s_p)) \Rightarrow
                                     \forall q, (\text{Registry}[q] \neq (q, \infty) \Rightarrow (PC_q \in \{6\text{-}9, 18, 19\} \lor (PC_q \in \{1, 17\} \land \text{Go}[q] \neq -1))))
            \land ((PC_p = 26 \land \text{CSSTATUS} = (0, s_p)) \Rightarrow (PC_{peer_p} \in [6, 8] \cup \{18\text{-}20, \textbf{20} :: \textbf{24}\} 
 \lor (PC_{peer_p} \in \{\textbf{20} :: \textbf{25}, \textbf{20} :: \textbf{26}\} \land s_{peer_p} = s_p) 
                                                                     \vee (PC_{peer_p} \in \{1, 17\} \wedge Go[peer_p] \neq -1)))
10. PC_p = \{28, 29\} \Rightarrow 1 \le g_p < \text{Token}
11. PC_p = 29 \Rightarrow ((PC_{peer_p} \in \{3, 4\} \Rightarrow 1 \leq g_p < tok_{peer_p}) \land (PC_{peer_p} = 5 \Rightarrow 1 \leq g_p < Go[peer_p])
\land ((PC_{peer_p} \in \{6, 7, 8\} \land g_p = Go[peer_p]) \Rightarrow CSSTATUS = (1, peer_p)))
12. If a process is registered, some q is either in CS or can be counted on to launch a waiting process into CS.
       \min(\text{Registry}) \neq (*, \infty) \Rightarrow \exists q, (\text{CSSTatus} = (1, q))
                                                               \lor (PC_q \in \{1, 17\} \land Go[q] \neq -1) \lor PC_q \in \{6, 15, 18\text{-}20, 24\}
                                                               \lor (PC_q \in \{25, 26\} \land \text{CSSTATUS} = (0, s_q)))
13. If p has the ownership of CS but Go[p] \neq 0, then there is some q that can be counted on to set Go[p] to 0.
       (\operatorname{CSStatus} = (1, p) \land \operatorname{Go}[p] \neq 0) \Rightarrow \exists q, (PC_q \in \{18\text{-}20, 24\} \lor (PC_q = 27 \land peer_q = p)
                                                                           \lor (PC_q \in \{28, 29\} \land peer_q = p \land g_q = Go[p])
                                                                           \vee (PC_q \in \{1, 17\} \wedge Go[q] \neq -1))
```

Fig. 2: Invariant of the Abortable RME Algorithm from Figure 1.

**Lemma 3 (First Come First Served)** There is an integer b such that in any run, if A and A' are attempts by any distinct processes p and p', respectively, p performs at least b consecutive normal steps in A before the attempt A' starts, and p neither receives an abort signal nor subsequently crashes in  $try_p()$  in A, then p' does not enter the CS in A' before p enters the CS in A.

Proof Let B be the earliest configuration when p has performed contiguous normal steps upto Line 7 during its attempt A in which p does not receive an abort signal and p' has not even initiated its attempt A'. Since  $PC_p = 7$ , by Condition 4, REGISTRY $[p] = (p, tok_p)$  in B. Let B' be the earliest configuration following B when p' has performed contiguous normal steps upto Line 7 during its attempt A'. Therefore, by the same argument, REGISTRY $[p'] = (p', tok_{p'})$  in B'. It follows from the premise of the lemma that  $tok_p < tok_{p'}$  (since p performed contiguous normal steps upto Line 7 even before A' started, applying Condition 6 right at the configuration when p completes Line 3,  $tok_p < TOKEN \le tok_{p'}$ ). Assume the lemma is false. Therefore, there is a

configuration C following B' such that p' entered the CS during attempt A' before p entered the CS during attempt A. Therefore,  $PC_{p'}=10$  and CSSTATUS =(1,p') in C. It follows that there is a configuration between B' and C, call it C', such that, CSSTATUS  $\neq (1,p')$  in configurations B' to the one just before C', and CSSTATUS =(1,p') in configurations C' to C. Let process p'' be the one that changed CSSTATUS to (1,p') in C'. p'' could have changed CSSTATUS this way only at Line  $\bf 26$ , since no other step sets the first bit of CSSTATUS to 1. It follows that p'' read the record of p' from the REGISTRY.findmin() it executed at Line  $\bf 25$ , or equally, p'=p'' and p' itself set  $peer_{p'}=p'$  because it found the REGISTRY to be empty (this could happen because p' either crashed in  ${\bf try}_{p'}()$  or aborted). In either case, since  $tok_p < tok_{p'}$ , the findmin() by p'' at Line  $\bf 25$  could have received  $(p', tok_{p'})$  (or  $(q, \infty)$ , for some q, denoting REGISTRY to be empty) if and only if REGISTRY[p]  $\neq (p, tok_p)$  (i.e., it is either  $(p, \infty)$  or (p, x) for a token x higher than  $tok_{p'}$ ). This implies that p either already left the CS from the attempt A or crashed some time after configuration B (and thereby removing its own entry from REGISTRY at Line  $\bf 19$ ) before p'' executed the findmin() at Line  $\bf 25$ , and hence before the configuration C is reached. Therefore, we have the lemma.

**Lemma 4 (Starvation Freedom)** In every fair infinite run in which every attempt contains only finitely many crash steps, if a process p is in the Try section in a configuration, p is in a different section in a later configuration.

Proof Suppose the claim is false. Therefore, there is a fair infinite run in which a process p starts an attempt and never leaves the  $\mathtt{try}_p()$  procedure, i.e., it forever loops in the procedure at Line 7 after a certain configuration (this follows from the fact that every attempt contains only a finitely many crash steps). Let C be the earliest configuration of the run such that p forever waits at Line 7 after C, all other processes are either waiting with p at Line 7 or are in the Remainder Section with  $status_p = good$ , and no process in the Recover, CS, or Exit Section. Such a configuration would exist because there are a finite number of processes each crashing finitely many times, and by Lemma 3 the algorithm satisfies the First Come First Served property. Therefore, without loss of generality, p be the process so that no other process can enter the CS before p enters it. It follows that CSSTATUS = (0, k), for some integer k, and REGISTRY[p] =  $(p, tok_p)$  from C onwards. By Condition 5 it follows that some process is either in CS or can be counted on to launch a waiting process into CS. This is a contradiction to our assumption that in C all the processes that are active in an attempt are waiting at Line 7.

**Lemma 5 (Bounded Recovery to CS)** There is an integer b such that if in any run any process p executes  $recover_p()$  without crashing and with  $status_p = recover$ -from-cs, the method completes in at most b steps of p, returning IN-CS.

Proof For p to execute  $\mathbf{recover}_p()$  with  $status_p = recover\text{-}from\text{-}cs$ , it must have crashed in the CS before. Let C be a configuration prior to a crash step when p is in the CS, i.e.,  $PC_p = 10$  in C. By Condition 5, CSSTATUS = (1, p), and, by Condition 3,  $Go[p] \neq -1$  in C. Without loss of generality, let C' be the first configuration of a passage following C, such that, p executes Line  $\mathbf{21}$  in this passage due to a call to  $\mathbf{abort}_p()$  from  $\mathbf{recover}_p()$  in this passage. That is all passages, if any, between C and C' ended with a crash in  $\mathbf{recover}_p()$  (or a crash within the nested call to  $\mathbf{abort}_p()$ ) before reaching and executing Line  $\mathbf{21}$ . Also note, by the description of  $status_p$ , it will retain the value recover-from-cs even up to C'. Since no other process except for p itself sets the value of Go[p] to -1 at Lines  $\mathbf{16}$  and  $\mathbf{22}$ , such a configuration is reachable in a bounded number of steps. It follows by the similar argument that CSSTATUS retains the value (1, p) up to C', because no other

process can write a value (0, k) at Line **14**, for some integer k, so that subsequently some process can perform the CAS at Line **26**. Thus starting at configuration C', p starts executing  $\mathtt{recover}_p()$  and reaches Line **21**. At Line **21** p notices that  $\mathtt{CSSTATUS} = (1, p)$  and it returns from  $\mathtt{abort}_p()$  and subsequently from  $\mathtt{recover}_p()$  with the value IN\_CS. From an inspection of the algorithm we note that this happens within a constant number of steps from C'. The claim thus follows.

**Lemma 6 (Critical Section Reentry)** In any run, if a process p crashes while in the CS, no other process enters the CS until p subsequently reenters the CS.

Proof Immediate from Lemma 1 and 5.

**Lemma 7 (Bounded Recovery to Exit)** There is an integer b such that if in any run any process p executes  $recover_p()$  without crashing and with  $status_p = recover$ -from-exit, the method completes in at most b steps of p.

**Proof** By an inspection of the algorithm, specifically that of  $recover_p()$ ,  $abort_p()$ , and  $promote_p()$ , we note that any execution path that p takes after crashing with  $status_p = recover-from-exit$ , if p executes  $recover_p()$  without crashing, then it completes the method in a constant number of steps.

**Lemma 8 (Fast Recovery to Remainder)** There is a constant b (independent of  $|\mathcal{P}|$ ) such that if in any run any process p executes  $recover_p()$  without crashing and with  $status_p \in \{good, recover\text{-from-rem}\}$ , the method completes in at most b steps of p.

Proof By Condition 3 we note that Go[p] = -1 when  $PC_p = 1$  and  $status_p \in \{good, recover-from-rem\}$ . It follows that if p executes  $recover_p()$ , it notices Go[p] = -1 at Line 17 and immediately returns to the Remainder.

**Lemma 9 (Bounded Recovery to Remainder)** There is an integer b such that if in any run  $recover_p()$ , executed by a process p with  $status_p = recover$ -from-try, returns  $IN\_REM$ , p must have completed that execution of  $recover_p()$  in at most b of its steps.

*Proof* From an inspection of the algorithm we note that any execution path that p takes when it returns IN\_REM from  $recover_p()$ , it must have done so in a constant number of steps from the latest step when it invoked  $recover_p()$ . Thus the claim follows.

**Lemma 10 (Bounded Abort)** There is an integer b such that, for each R, C, p, if  $\beta(R, p, C)$  is true, ABORTSIGNAL[p] stays true for ever (i.e., stays true in the suffix R' of the run from C), and p executes steps without crashing (i.e., p has no crash steps in R'), then p enters either the CS or the remainder in at most b of its steps (in R').

Proof For this we note that the only wait till loop that the algorithm has is at Line 7. Since ABORTSIGNAL[p] stays true for ever after C, p either notices that or sees that Go[p] = 0 at Line 7. At Line 8, if p sees that Go[p] = 0, it moves to the CS, satisfying the condition. Otherwise, it invokes  $abort_p()$  at Line 9. From an inspection of  $abort_p()$ , we note that the procedure returns within a constant number of steps (i.e.,  $O(\log n)$  steps, where  $n = |\mathcal{P}|$ ) with a value of either IN\_CS or IN\_REM. It follows that the claim holds.

**Lemma 11 (No Trivial Aborts)** In any run, if AbortSignal[p] is false when a process p invokes  $try_p()$ , AbortSignal[p] remains false forever, and p executes steps without crashing, then  $try_p()$  does not return IN\_REM.

Proof Since p executes steps without crashing and AbortSignal[p] remains false forever in the run, the only place  $\mathtt{try}_p()$  could return IN\_REM is due to the nested call to  $\mathtt{abort}_p()$  at Line 9. However, by Condition 7, we know that if p gets past the  $\mathtt{wait}$  till loop at Line 7, then  $\mathtt{Go}[p] = 0$  when  $PC_p = 8$  (since abort was not requested when  $\mathtt{try}_p()$  was invoked and AbortSignal[p] remains false forever). It follows that p returns IN\_CS in such a run.

### 4.1 RMR Complexity

We discuss the RMR complexity a process incurs per passage as follows. As described in Lemma 2 of Jayanti and Joshi's work [20], the REGISTRY.write() operation incurs  $O(\min(k, \log n))$  RMRs on both CC and DSM machines, where k is the maximum point contention during the REGISTRY.write() operation. On DSM machines, when the variable Go[p] is hosted in p's memory partition, any step of the algorithm other than REGISTRY.write() (at Lines  $\mathbf{5}$ ,  $\mathbf{11}$ ,  $\mathbf{19}$ ) incurs a constant RMR. Therefore, on DSM machines our algorithm incurs  $O(\min(k, \log n))$  RMR per passage. On CC machines, similarly, it would be tempting to believe that all these other operations incur constant RMRs, however, it is not so due to the following. On Strict-CC machines where a failed CAS could incur an RMR, the RMR complexity shoots up to O(n) for the following reason. There could be n/2 processes that are waiting to execute Line  $\mathbf{29}$  to perform a CAS on Go[p]. Out of these processes only one succeeds and the rest fail. However, each failed CAS still incurs an RMR. Therefore, on Strict-CC machines our algorithm incurs O(n) RMR per passage. To summarize, the algorithm incurs  $O(\min(k, \log n))$  RMRs per passage on DSM and Relaxed-CC machines and O(n) RMRs per passage on Strict-CC machines.

For an attempt having f failures, the implementation of REGISTRY taken from Jayanti and Joshi's work [20] would incur  $O(f + \min(k, \log n))$  RMRs for the REGISTRY.write() operation. Therefore, the algorithm incurs  $O(f + \log n)$  RMRs per attempt on DSM and Relaxed-CC machines and O(f + n) RMRs per attempt on Strict-CC machines in the presence of f crashes in an attempt.

#### 4.2 Proof of Invariant

**Lemma 12** The algorithm in Figure 1 satisfies the invariant (i.e., the conjunction of all the conditions) stated in Figure 2, i.e., the invariant holds in every configuration of every run of the algorithm.

Proof The proof is by induction, but it is omitted because of the page limitation on the submission. The full version of this paper, including this proof, can be found at http://people.csail.mit.edu/siddhartha/archive.html

#### 4.3 Main theorem

The theorem below summarizes the result of our paper.

**Theorem 1** The algorithm in Figure 1 is an abortable recoverable mutual exclusion algorithm for n processes and satisfies properties P1-P12 stated in Section 2. A process incurs  $O(\min(k, \log n))$  RMRs per passage on DSM and Relaxed-CC machines and O(n) RMRs per passage on Strict-CC machines. In presence of f crashes during an attempt, a process incurs  $O(f + \min(k, \log n))$  RMRs

per attempt on DSM and Relaxed-CC machines and O(f + n) RMRs per attempt on Strict-CC machines.

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