# Achieving Starvation-Freedom in Multi-Version Transactional Memory Systems \*

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#### **Abstract**

Software Transactional Memory systems (STMs) have garnered significant interest as an elegant alternative for addressing synchronization and concurrency issues with multi-threaded programming in multi-core systems. For STMs to be efficient, they must guarantee some progress properties. This work explores the notion of one of the progress property, i.e., *starvation-freedom*, in STMs. An STM system is said to be starvation-free if every thread invoking a transaction gets the opportunity to take a step (due to the presence of a fair scheduler) such that the transaction eventually commits.

A few *starvation-free* algorithms have been proposed in the literature in context of single-version STMs. These algorithms are priority based i.e. if two transactions are in conflict, then the transaction with lower priority will abort. A transaction running for a long time will eventually have the highest priority and hence commit. But the drawback with this approach is that if a set of high-priority transactions become slow, then they can cause several other transactions to abort. So, we propose multi-version *starvation-free* STM system which addresses this issue.

Multi-version STMs maintain multiple-versions for each transactional object. By storing multiple versions, these systems can achieve greater concurrency. In this paper, we propose multi-version starvation-free STM, KSFTM, which as the name suggests achieves starvation-freedom while storing K-versions of each t-object. Here K is an input parameter fixed by the application programmer depending on the requirement. Our algorithm is dynamic which can support different values of K ranging from one to infinity. If K is infinite, then there is no limit on the number of versions. But a separate garbage-collection mechanism is required to collect unwanted versions. On the other hand, when K is one, it becomes the same as a single-version starvation-free STM system. We prove the correctness and starvation-freedom property of the KSFTM algorithm.

To the best of our knowledge, this is the first multi-version STM system that satisfies *starvation-freedom*. We implement *KSFTM* and compare its performance with single-version *starvation-free* STM system (*SV-SFTM*) which works on the priority principle. Our experiments show that *KSFTM* gives an average speedup on the worst-case time to commit of a transaction by a factor of 1.22, 1.89, 23.26 and 13.12 times over *PKTO*, *SV-SFTM*, NOrec STM and ESTM respectively for counter application. *KSFTM* performs 1.5 and 1.44 times better than *PKTO* and *SV-SFTM* but 1.09 times worse than NOrec for low contention KMEANS application of STAMP benchmark whereas *KSFTM* performs 1.14, 1.4 and 2.63 times better than *PKTO*, *SV-SFTM* and NOrec for LABYRINTH application of STAMP benchmark which has high contention with long-running transactions.

### 1 Introduction

STMs [13, 25] are a convenient programming interface for a programmer to access shared memory without worrying about consistency issues. STMs often use an optimistic approach for concurrent execution of *transactions* 

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(a piece of code invoked by a thread). In optimistic execution, each transaction reads from the shared memory, but all write updates are performed on local memory. On completion, the STM system *validates* the reads and writes of the transaction. If any inconsistency is found, the transaction is *aborted*, and its local writes are discarded. Otherwise, the transaction is committed, and its local writes are transferred to the shared memory. A transaction that has begun but has not yet committed/aborted is referred to as *live*.

A typical STM is a library which exports the following methods: *stm-begin* which begins a transaction, *stm-read* which reads a *transactional object* or *t-object*, *stm-write* which writes to a *t-object*, *stm-tryC* which tries to commit the transaction. Typical code for using STMs is as shown in Algorithm 1 which shows how an insert of a concurrent linked-list library is implemented using STMs.

**Correctness:** Several *correctness-criteria* have been proposed for STMs such as opacity [11], local opacity [18, 19]. All these *correctness-criteria* require that all the transactions including aborted ones appear to execute sequentially in an order that agrees with the order of non-overlapping transactions. Unlike the correctness-criteria for traditional databases, such as serializability, strict-serializability [22], the correctness-criteria for STMs ensure that even aborted transactions read correct values. This ensures that programmers do not see any undesirable side-effects due to the reads by transaction that get aborted later such as divide-by-zero, infinite-loops, crashes etc. in the application due to concurrent executions. This additional requirement on aborted transactions is a fundamental requirement of STMs which differentiates STMs from databases as observed by Guerraoui & Kapalka [11]. Thus in this paper, we focus on optimistic executions with the *correctness-criterion* being *local opacity* [19].

**Starvation Freedom:** In the execution shown in Algorithm 1, there is a possibility that the transaction which a thread tries to execute gets aborted again and again. Every time, it executes the transaction, say  $T_i$ ,  $T_i$  conflicts with some other transaction and hence gets aborted. In other words, the thread is effectively starving because it is not able to commit  $T_i$  successfully.

A well known blocking progress condition associated with concurrent programming is starvation-freedom [15, chap 2], [14]. In the context of STMs, starvation-freedom ensures that every aborted transaction that is retried infinitely often eventually commits. It can be defined as: an STM system is said to be *starvation-free* if a thread invoking a transaction  $T_i$  gets the opportunity to retry  $T_i$  on every abort (due to the presence of a fair underlying scheduler with bounded termination) and  $T_i$  is not *parasitic*, i.e.,  $T_i$  will try to commit given a chance then  $T_i$  will eventually commit. Parasitic transactions [4] will not commit even when given a chance to commit possibly because they are caught in an infinite loop or some other error.

**Algorithm 1** Insert(LL, e): Invoked by a thread to insert an element e into a linked-list LL. This method is implemented using transactions.

```
1: retry = 0;
 2: while (true) do
       id = \text{stm-begin}(retry);
 4:
 5:
       v = stm\text{-}read(id, x); /* reads the value of x as v */
 6:
 7:
 8:
       stm\text{-}write(id, x, v'); /* writes a value v' to x */
 9:
10:
11:
       ret = stm-tryC(id); /* stm-tryC can return commit or abort */
12:
       if (ret == commit) then
13:
            break;
14:
15:
       else
            retry++;
16:
       end if
17:
18: end while
```

Wait-freedom is another interesting progress condition for STMs in which every transaction commits regardless of the nature of concurrent transactions and the underlying scheduler [14]. But it was shown by Guerraoui and Kapalka [4] that it is not possible to achieve wait-freedom in dynamic STMs in which data sets of transactions are not known in advance. So in this paper, we explore the weaker progress condition of starvation-freedom for transactional memories while assuming that the data sets of the transactions are not known in advance.

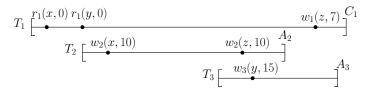


Figure 1: Limitation of Single-version Starvation Free Algorithm

**Related work on the starvation-free STMs:** Starvation-freedom in STMs has been explored by a few researchers in literature such as Gramoli et al. [9], Waliullah and Stenstrom [27], Spear et al. [26]. Most of these systems work by assigning priorities to transactions. In case of a conflict between two transactions, the transaction with lower priority is aborted. They ensure that every aborted transaction, on being retried a sufficient number of times, will eventually have the highest priority and hence will commit. We denote such an algorithm as *single-version starvation-free STM* or *SV-SFTM*.

Although SV-SFTM guarantees starvation-freedom, it can still abort many transactions spuriously. Consider the case where a transaction  $T_i$  has the highest priority. Hence, as per SV-SFTM,  $T_i$  cannot be aborted. But if it is slow (for some reason), then it can cause several other conflicting transactions to abort and hence, bring down the efficiency and progress of the entire system.

Fig 1 illustrates this problem. Consider the execution:  $r_1(x,0)r_1(y,0)w_2(x,10)w_2(z,10)w_3(y,15)w_1(z,7)$ . It has three transactions  $T_1$ ,  $T_2$  and  $T_3$ . Let  $T_1$  has the highest priority. After reading y, suppose  $T_1$  becomes slow. Next  $T_2$  and  $T_3$  want to write to x,z and y respectively and commit. But  $T_2$  and  $T_3$ 's write operations are in conflict with  $T_1$ 's read operations. Since  $T_1$  has higher priority and has not committed yet,  $T_2$  and  $T_3$  have to abort. If these transactions are retried and again conflict with  $T_1$  (while it is still live), they will have to abort again. Thus, any transaction with the priority lower than  $T_1$  and conflicts with it has to abort. It is as if  $T_1$  has locked the t-objects x,y and does not allow any other transaction, write to these t-objects and to commit.

**Multi-version starvation-free STM:** A key limitation of single-version STMs is limited concurrency. As shown above, it is possible that one long transaction conflicts with several transactions causing them to abort. This limitation can be overcome by using multi-version STMs where we store multiple versions of the data item (either unbounded versions with garbage collection, or bounded versions where the oldest version is replaced when the number of versions exceeds the bound).

Several multi-version STMs have been proposed in the literature [17, 20, 8, 23] that provide increased concurrency. But none of them provide starvation-freedom. Furthermore, achieving starvation-freedom while using only bounded versions is especially challenging given that a transaction may rely on the oldest version that is removed. In that case, it would be necessary to about that transaction, making it harder to achieve starvation-freedom.

A typical code using STMs is as shown in Algorithm 1. It shows the overview of a concurrent *insert* method which inserts an element e into a linked-list LL. It consists of a loop where the thread creates a transaction. This transaction executes the code to insert an element e in a linked-list LL using stm-read and stm-write operations. (The result of stm-write operation are stored locally.) At the end of the transaction, the thread calls stm-tryC. At this point, the STM checks if the given transaction can be committed while satisfying the required safety properties (e.g., serializability [22], opacity [11]). If yes, then the transaction is committed. At this time, any updates done by the transaction are reflected in the shared memory. Otherwise, it is aborted. In this case, all the updates made by the transaction are discarded. If the given transaction is aborted, then the invoking thread may retry that transaction again like Line 16 in Algorithm 1.

The advantage of multi-version STMs, is that they allow greater concurrency by allowing more transactions to commit. Consider the execution shown in Fig 1. Suppose this execution used multiple versions for each t-object. Then it is possible for all the three transactions to commit. Transactions  $T_2$  and  $T_3$  create a new version corresponding to each t-object x, z and y and return commit. Since multiple versions are being used,  $T_1$  need not abort as well.  $T_1$  reads the initial value of z, and returns commit. So, by maintaining multiple versions all the transactions  $T_1$ ,  $T_2$ , and  $T_3$  can commit with equivalent serial history as  $T_1T_2T_3$  or  $T_1T_3T_2$ . Thus multiple versions can help with starvation-freedom without sacrificing on concurrency. This motivated us to develop a multi-version starvation-free STM system.

Although multi-version STMs provide greater concurrency, they suffer from the cost of garbage collection. One way to avoid this is to use bounded-multi-version STMs, where the number of versions is bounded to be at most K. Thus, when  $(K+1)^{th}$  version is created, the oldest version is removed. Bounding the number of versions can hinder with starvation freedom: a transaction needing to read a version that is currently removed must be aborted.

This paper addresses this gap by developing a starvation-free algorithm for bounded MVSTMs. Our approach is different from the approach used in *SV-SFTM* to provide starvation-freedom in single version STMs (the policy of aborting lower priority transactions in case of conflict) as it does not work for MVSTMs. As part of the derivation of our final starvation-free algorithm, we consider an algorithm (*PKTO*) that considers this approach and show that it is insufficient to provide starvation freedom.

### Contributions of the paper:

- We propose a multi-version starvation-free STM system as *K-version starvation-free STM* or *KSFTM* for a given parameter K. Here K is the number of versions of each t-object and can range from 1 to  $\infty$ . To the best of our knowledge, this is the first starvation-free MVSTM. We develop *KSFTM* algorithm in a step-wise manner starting from MVTO [17] as follows:
  - First, in SubSection 3.3, we use the standard idea to provide higher priority to older transactions.
     Specifically, we propose priority-based K-version STM algorithm Priority-based K-version MVTO or PKTO. This algorithm guarantees the safety properties of strict-serializability and local opacity. However, it is not starvation-free.
  - We analyze *PKTO* to identify the characteristics that will help us to achieve preventing a transaction from getting aborted forever. This analysis leads us to the development of *starvation-free K-version TO* or *SFKTO* (SubSection 3.5), a multi-version starvation-free STM obtained by revising *PKTO*. But SFKTO does not satisfy correctness, i.e., strict-serializability, and local opacity.
  - Finally, we extend SFKTO to develop KSFTM (SubSection 3.6) that preserves the starvation-freedom, strict-serializability, and local opacity. Our algorithm works on the assumption that any transaction that is not deadlocked, terminates (commits or aborts) in a bounded time.
- Our experiments (Section 4) show that *KSFTM* gives an average speedup on the worst-case time to commit of a transaction by a factor of 1.22, 1.89, 23.26 and 13.12 times over *PKTO*, *SV-SFTM*, NOrec STM [6] and ESTM [7] respectively for counter application. *KSFTM* performs 1.5 and 1.44 times better than *PKTO* and *SV-SFTM* but 1.09 times worse than NOrec for low contention KMEANS application of STAMP [21] benchmark whereas *KSFTM* performs 1.14, 1.4 and 2.63 times better than *PKTO*, *SV-SFTM* and NOrec for LABYRINTH application of STAMP benchmark which has high contention with long-running transactions.

# 2 System Model and Preliminaries

Following [12, 19], we assume a system of n processes/threads,  $p_1, \ldots, p_n$  that access a collection of transactional objects (or t-objects) via atomic transactions. Each transaction has a unique identifier. Within a transaction, processes can perform transactional operations or methods: stm-begin that begins a transaction, stm-write(x, v) operation that updates a t-object x with value v in its local memory, the stm-read(x) operation tries to read x, stm-try(v)0 that tries to commit the transaction and returns v0. For the sake of presentation simplicity, we assume that the values taken as arguments by stm-write operations are unique.

Operations stm-read and stm-tryC() may return  $\mathscr{A}$ , in which case we say that the operations forcefully abort. Otherwise, we say that the operations have successfully executed. Each operation is equipped with a unique transaction identifier. A transaction  $T_i$  starts with the first operation and completes when any of its operations return  $\mathscr{A}$  or  $\mathscr{C}$ . We denote any operation that returns  $\mathscr{A}$  or  $\mathscr{C}$  as terminal operations. Hence, operations stm-tryC() and stm-tryA() are terminal operations. A transaction does not invoke any further operations after terminal operations.

For a transaction  $T_k$ , we denote all the t-objects accessed by its read operations as  $rset_k$  and t-objects accessed by its write operations as  $wset_k$ . We denote all the operations of a transaction  $T_k$  as  $T_k.evts$  or  $evts_k$ .

**History:** A *history* is a sequence of *events*, i.e., a sequence of invocations and responses of transactional operations. The collection of events is denoted as H.evts. For simplicity, we only consider *sequential* histories here: the invocation of each transactional operation is immediately followed by a matching response. Therefore, we treat each transactional operation as one atomic event, and let  $<_H$  denote the total order on the transactional operations incurred by H. With this assumption, the only relevant events of a transaction  $T_k$  is of the types:  $r_k(x, v)$ ,  $r_k(x, \mathscr{A})$ ,  $w_k(x, v)$ ,  $stm-tryC_k(\mathscr{C})$  (or  $c_k$  for short),  $stm-tryC_k(\mathscr{A})$ ,  $stm-tryA_k(\mathscr{A})$  (or  $a_k$  for short). We identify a history H as tuple  $\langle H.evts, <_H \rangle$ .

Let H|T denote the history consisting of events of T in H, and  $H|p_i$  denote the history consisting of events of  $p_i$  in H. We only consider well-formed histories here, i.e., no transaction of a process begins before the previous transaction invocation has completed (either commits or aborts). We also assume that every history has an initial committed transaction  $T_0$  that initializes all the t-objects with value 0.

The set of transactions that appear in H is denoted by H.txns. The set of committed (resp., aborted) transactions in H is denoted by H.committed (resp., H.aborted). The set of incomplete or live transactions in H is denoted by H.incomp = H.live = (H.txns - H.committed - H.aborted).

For a history H, we construct the *completion* of H, denoted as  $\overline{H}$ , by inserting  $stm\text{-}tryA_k(\mathscr{A})$  immediately after the last event of every transaction  $T_k \in H.live$ . But for  $stm\text{-}tryC_i$  of transaction  $T_i$ , if it released the lock on first t-object successfully that means updates made by  $T_i$  is consistent so,  $T_i$  will immediately return commit. **Transaction orders:** For two transactions  $T_k, T_m \in H.txns$ , we say that  $T_k$  precedes  $T_m$  in the real-time order of H, denote  $T_k \prec_H^{RT} T_m$ , if  $T_k$  is complete in H and the last event of  $T_k$  precedes the first event of  $T_m$  in H. If neither  $T_k \prec_H^{RT} T_m$  nor  $T_m \prec_H^{RT} T_k$ , then  $T_k$  and  $T_m$  overlap in H. We say that a history is t-sequential if all the transactions are ordered by this real-time order. Note that from our earlier assumption all the transactions of a single process are ordered by real-time.

**Sub-history:** A *sub-history* (SH) of a history (H) denoted as the tuple  $\langle SH.evts, <_{SH} \rangle$  and is defined as: (1)  $<_{SH} \subseteq <_H$ ; (2)  $SH.evts \subseteq H.evts$ ; (3) If an event of a transaction  $T_k \in H.txns$  is in SH then all the events of  $T_k$  in H should also be in SH.

For a history H, let R be a subset of H.txns. Then H.subhist(R) denotes the sub-history of H that is formed from the operations in R.

**Valid and legal history:** A successful read  $r_k(x,v)$  (i.e.,  $v \neq \mathscr{A}$ ) in a history H is said to be *valid* if there exist a transaction  $T_j$  that wrote v to x and *committed* before  $r_k(x,v)$ . Formally,  $\langle r_k(x,v) \text{ is valid} \Leftrightarrow \exists T_j : (c_j <_H r_k(x,v)) \land (w_j(x,v) \in T_j.evts) \land (v \neq \mathscr{A}) \rangle$ . The history H is valid if all its successful read operations are valid.

We define  $r_k(x,v)$ 's lastWrite as the latest commit event  $c_i$  preceding  $r_k(x,v)$  in H such that  $x \in wset_i$  ( $T_i$  can also be  $T_0$ ). A successful read operation  $r_k(x,v)$ , is said to be legal if the transaction containing  $r_k$ 's lastWrite also writes v onto x:  $\langle r_k(x,v) |$  is legal  $\Leftrightarrow (v \neq \mathscr{A}) \wedge (H.lastWrite(r_k(x,v)) = c_i) \wedge (w_i(x,v) \in T_i.evts) \rangle$ . The history H is legal if all its successful read operations are legal. From the definitions we get that if H is legal then it is also valid.

**Opacity and Strict Serializability:** We say that two histories H and H' are *equivalent* if they have the same set of events. Now a history H is said to be *opaque* [11, 12] if it is valid and there exists a t-sequential legal history S such that (1) S is equivalent to  $\overline{H}$  and (2) S respects  $\prec_H^{RT}$ , i.e.,  $\prec_H^{RT} \subset \prec_S^{RT}$ . By requiring S being equivalent to  $\overline{H}$ , opacity treats all the incomplete transactions as aborted. We call S an (opaque) *serialization* of S.

Along same lines, a valid history H is said to be strictly serializable if H.subhist(H.committed) is opaque. Unlike opacity, strict serializability does not include aborted or incomplete transactions in the global serialization order. An opaque history H is also strictly serializable: a serialization of H.subhist(H.committed) is simply the subsequence of a serialization of H that only contains transactions in H.committed.

Serializability is commonly used criterion in databases. But it is not suitable for STMs as it does not consider the correctness of *aborted* transactions as shown by Guerraoui & Kapalka [11]. Opacity, on the other hand, considers the correctness of *aborted* transactions as well. Similarly, local opacity (described below) is another correctness-criterion for STMs but is not as restrictive as opacity.

**Local opacity:** For a history H, we define a set of sub-histories, denoted as H.subhistSet as follows: (1) For each aborted transaction  $T_i$ , we consider a subhist consisting of operations from all previously committed transactions and including all successful operations of  $T_i$  (i.e., operations which did not return  $\mathscr{A}$ ) while immediately putting commit after last successful operation of  $T_i$ ; (2) for last committed transaction  $T_l$  considers all the previously committed transactions including  $T_l$ .

A history H is said to be *locally-opaque* [18, 19] if all the sub-histories in H.subhistSet are opaque. It must be seen that in the construction of sub-history of an aborted transaction  $T_i$ , the *subhist* will contain operations from only one aborted transaction which is  $T_i$  itself and no other live/aborted transactions. Similarly, the sub-history of *committed* transaction  $T_l$  has no operations of aborted and live transactions. Thus in local opacity, no aborted or live transaction can cause another transaction to abort. It was shown that local opacity [18, 19] allows greater concurrency than opacity. Any history that is opaque is also locally-opaque but not necessarily the vice-versa. On the other hand, a history that is locally-opaque is also strict-serializable, but the vice-versa need not be true.

**Graph Characterization of Local Opacity:** To prove correctness of STM systems, it is useful to consider graph characterization of histories. In this section, we describe the graph characterization developed by Kumar et al [17] for proving opacity which is based on characterization by Bernstein and Goodman [2]. We extend this characterization for LO.

Consider a history H which consists of multiple versions for each t-object. The graph characterization uses the notion of *version order*. Given H and a t-object x, we define a version order for x as any (non-reflexive) total order on all the versions of x ever created by committed transactions in H. It must be noted that the version order may or may not be the same as the actual order in which the version of x are generated in x. A version order of x, denoted as x0 is the union of the version orders of all the t-objects in x1.

Consider the history  $H2: r_1(x,0)r_2(x,0)r_1(y,0)r_3(z,0)w_1(x,5)w_3(y,15)w_2(y,10)w_1(z,10)c_1c_2r_4(x,5)$   $r_4(y,10)w_3(z,15)c_3r_4(z,10)$ . Using the notation that a committed transaction  $T_i$  writing to x creates a version  $x_i$ , a possible version order for  $H2 \ll_{H2}$  is:  $\langle x_0 \ll x_1 \rangle, \langle y_0 \ll y_2 \ll y_3 \rangle, \langle z_0 \ll z_1 \ll z_3 \rangle$ .

We define the graph characterization based on a given version order. Consider a history H and a version order  $\ll$ . We then define a graph (called opacity graph) on H using  $\ll$ , denoted as  $OPG(H, \ll) = (V, E)$ . The vertex set V consists of a vertex for each transaction  $T_i$  in  $\overline{H}$ . The edges of the graph are of three kinds and are defined as follows:

- 1. real-time (real-time) edges: If  $T_i$  commits before  $T_j$  starts in H, then there is an edge from  $v_i$  to  $v_j$ . This set of edges are referred to as rt(H).
- 2. rf(reads-from) edges: If  $T_j$  reads x from  $T_i$  in H, then there is an edge from  $v_i$  to  $v_j$ . Note that in order for this to happen,  $T_i$  must have committed before  $T_j$  and  $c_i <_H r_j(x)$ . This set of edges are referred to as rf(H).
- 3. mv(multiversion) edges: The mv edges capture the multiversion relations and is based on the version order. Consider a successful read operation  $r_k(x,v)$  and the write operation  $w_j(x,v)$  belonging to transaction  $T_j$  such that  $r_k(x,v)$  reads x from  $w_j(x,v)$  (it must be noted  $T_j$  is a committed transaction and  $c_j <_H r_k$ ). Consider a committed transaction  $T_i$  which writes to x,  $w_i(x,u)$  where  $u \neq v$ . Thus the versions created  $x_i, x_j$  are related by  $\ll$ . Then, if  $x_i \ll x_j$  we add an edge from  $v_i$  to  $v_j$ . Otherwise  $(x_j \ll x_i)$ , we add an edge from  $v_k$  to  $v_i$ . This set of edges are referred to as  $mv(H, \ll)$ .

Using the construction, the  $OPG(H2, \ll_{H2})$  for history H2 and  $\ll_{H2}$  is shown in Fig 14. The edges are annotated. The only mv edge from T4 to T3 is because of t-objects y, z. T4 reads value 5 for z from T1 whereas T3 also writes 15 to z and commits before  $r_4(z)$ .

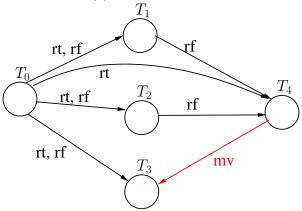


Figure 2:  $OPG(H2, \ll_{H2})$ 

Kumar et al [17] showed that if a version order  $\ll$  exists for a history H such that  $OPG(H, \ll_H)$  is acyclic, then H is opaque. This is captured in the following result.

**Result 1** A valid history H is opaque iff there exists a version order  $\ll_H$  such that  $OPG(H, \ll_H)$  is acyclic.

This result can be easily extended to prove LO as follows

**Theorem 2** A valid history H is locally-opaque iff for each sub-history sh in H.subhistSet there exists a version order  $\ll_{sh}$  such that  $OPG(sh, \ll_{sh})$  is acyclic. Formally,  $\langle (H \text{ is locally-opaque}) \Leftrightarrow (\forall sh \in H. \text{subhist} Set, \exists \ll_{sh}: OPG(sh, \ll_{sh}) \text{ is acyclic}) \rangle$ .

**Proof.** To prove this theorem, we have to show that each sub-history sh in H.subhistSet is valid. Then the rest follows from Result 9. Now consider a sub-history sh. Consider any read operation  $r_i(x, v)$  of a transaction  $T_i$ . It

is clear that  $T_i$  must have read a version of x created by a previously committed transaction. From the construction of sh, we get that all the transaction that committed before  $r_i$  are also in sh. Hence sh is also valid.

Now, proving sh to be opaque iff there exists a version order  $\ll_{sh}$  such that  $OPG(sh, \ll_{sh})$  is acyclic follows from Result 9.

# 3 The Working of *KSFTM* Algorithm

In this section, we propose *K-version starvation-free STM* or *KSFTM* for a given parameter K. Here K is the number of versions of each t-object and can range from 1 to  $\infty$ . When K is 1, it boils down to single-version *starvation-free* STM. If K is  $\infty$ , then *KSFTM* uses unbounded versions and needs a separate garbage collection mechanism to delete old versions like other MVSTMs proposed in the literature [17, 20]. We denote *KSFTM* using unbounded versions as *UVSFTM* and *UVSFTM* with garbage collection as *UVSFTM-GC*.

Next, we describe some *starvation-freedom* preliminaries in SubSection 3.1 to explain the working of *KSFTM* algorithm. To explain the intuition behind the *KSFTM* algorithm, we start with the modification of MVTO [2, 17] algorithm in SubSection 3.3. We then make a sequence of modifications to it to arrive at *KSFTM* algorithm.

#### 3.1 Starvation-Freedom Preliminaries

In this section, we start with the definition of *starvation-freedom*. Then we describe the invocation of transactions by the application. Next, we describe the data structures used by the algorithms.

**Definition 1** *Starvation-Freedom:* A STM system is said to be starvation-free if a thread invoking a non-parasitic transaction  $T_i$  gets the opportunity to retry  $T_i$  on every abort, due to the presence of a fair scheduler, then  $T_i$  will eventually commit.

As explained by Herlihy & Shavit [14], a fair scheduler implies that no thread is forever delayed or crashed. Hence with a fair scheduler, we get that if a thread acquires locks then it will eventually release the locks. Thus a thread cannot block out other threads from progressing.

**Assumption about Scheduler:** In order for starvation-free algorithm *KSFTM* (described in SubSection 3.6) to work correctly, we make the following assumption about the fair scheduler:

**Assumption 1** Bounded-Termination: For any transaction  $T_i$ , invoked by a thread  $Th_x$ , the fair system scheduler ensures, in the absence of deadlocks,  $Th_x$  is given sufficient time on a CPU (and memory etc.) such that  $T_i$  terminates (either commits or aborts) in bounded time.

While the bound for each transaction may be different, we use L to denote the maximum bound. In other words, in time L, every transaction will either abort or commit due to the absence of deadlocks.

In our algorithm, we will ensure that it is deadlock free using standard techniques from the literature. In other words, each thread is in a position to make progress. We assume that the scheduler provides sufficient CPU time to complete (either commit or abort) within a bounded time.

As explained by Herlihy & Shavit [14], a fair scheduler implies that no thread is forever delayed or crashed. Hence with a fair scheduler, we get that if a thread acquires locks then it will eventually release the locks. Thus a thread cannot block out other threads from progressing.

**Transaction Invocation:** Transactions are invoked by threads. Suppose a thread  $Th_x$  invokes a transaction  $T_i$ . If this transaction  $T_i$  gets *aborted*,  $Th_x$  will reissue it, as a new incarnation of  $T_i$ , say  $T_j$ . The thread  $Th_x$  will continue to invoke new incarnations of  $T_i$  until an incarnation commits.

When the thread  $Th_x$  invokes a transaction, say  $T_i$ , for the first time then the STM system assigns  $T_i$  a unique timestamp called *current timestamp or CTS*. If it aborts and retries again as  $T_j$ , then its CTS will change. However,

in this case, the thread  $Th_x$  will also pass the CTS value of the first incarnation  $(T_i)$  to  $T_j$ . By this,  $Th_x$  informs the STM system that,  $T_j$  is not a new invocation but is an incarnation of  $T_i$ .

We denote the CTS of  $T_i$  (first incarnation) as *Initial Timestamp or ITS* for all the incarnations of  $T_i$ . Thus, the invoking thread  $Th_x$  passes  $cts_i$  to all the incarnations of  $T_i$  (including  $T_j$ ). Thus for  $T_j$ ,  $its_j = cts_i$ . The transaction  $T_j$  is associated with the timestamps:  $\langle its_j, cts_j \rangle$ . For  $T_i$ , which is the initial incarnation, its ITS and CTS are the same, i.e.,  $its_i = cts_i$ . For simplicity, we use the notation that for transaction  $T_j$ , j is its CTS, i.e.,  $cts_j = j$ .

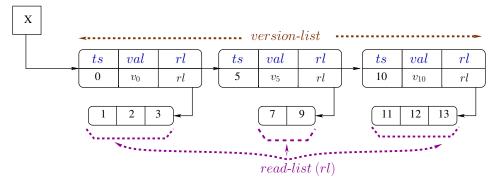


Figure 3: Data Structures for Maintaining Versions

We also assume that in the absence of other concurrent conflicting transactions, every transaction will commit. In other words, if a transaction is executing in a system where other concurrent conflicting transactions are not present then it will not self-abort. If transactions can self-abort then providing *starvation-freedom* is impossible. **Common Data Structures and STM Methods:** Here we describe the common data structures used by all the algorithms proposed in this section. For each t-object, the algorithms maintain multiple versions in version-list (or vlist) using list. Similar to versions in MVTO [17], each version of a t-object is a tuple denoted as vTuple and consists of three fields: (1) timestamp, (or ts) of the transaction that created this version which normally is the CTS; (2) the value (or val) of the version; (3) a list, called read-list (or rl), consisting of transactions ids (can be CTS as well) that read from this version. The read-list of a version is initially empty. Fig 3 illustrates this structure. For a t-object x, we use the notation x[t] to access the version with timestamp t. Depending on the algorithm considered, the fields change of this structure.

The algorithms have access to a global atomic counter,  $G_{-}tCntr$  used for generating timestamps in the various transactional methods. We assume that the STM system exports the following methods for a transaction  $T_i$ : (1) stm-begin(t) where t is provided by the invoking thread,  $Th_x$ . From our earlier assumption, it is the CTS of the first incarnation. In case  $Th_x$  is invoking this transaction for the first time, then t is null. This method returns a unique timestamp to  $Th_x$  which is the CTS/id of the transaction. (2) stm- $read_i(x)$  tries to read t-object x. It returns either value v or  $\mathscr{A}$ . (3) stm- $write_i(x,v)$  operation that updates a t-object x with value v locally. It returns ok. (4) stm- $tryC_i()$  tries to commit the transaction and returns  $\mathscr{C}$  if it succeeds. Otherwise, it returns  $\mathscr{A}$ . Correctness Criteria: For ease of exposition, we initially consider strict-serializability as correctness-criterion to illustrate the correctness of the algorithms. But strict-serializability does not consider the correctness of aborted transactions and as a result not a suitable correctness-criterion for STMs. Finally, we show that the proposed STM algorithm KSFTM satisfies local opacity, a correctness-criterion for STMs (described in Section 2). We denote the set of histories generated by an STM algorithm, say A, as gen(A).

#### 3.2 Motivation for Starvation Freedom in Multi-Version Systems

In this section, first we describe the starvation freedom solution used for single version i.e. SV-SFTM algorithm and then the drawback of it.

#### 3.2.1 Illustration of SV-SFTM

Forward-oriented optimistic concurrency control protocol (FOCC), is a commonly used optimistic algorithm in databases [28, Chap 4]. In fact, several STM Systems are also based on this idea. In a typical STM system (also in database optimistic concurrency control algorithms), a transaction execution is divided can be two phases - a read/local-write phase and try-Commit phase (also referred to as validation phase in databases). The various algorithms differ in how the try-Commit phase executes. Let the write-set or wset and read-set or rset of a  $t_i$ 

denotes the set of t-objects written & read by  $t_i$ . In FOCC a transaction  $t_i$  in its try-Commit phase is validated against all live transactions that are in their read/local-write phase as follows:  $\langle wset(t_i) \cap (\forall t_j : rset^n(t_j)) = \Phi \rangle$ . This implies that the wset of  $t_i$  can not have any conflict with the current rset of any transaction  $t_j$  in its read/local-write phase. Here  $rset^n(t_j)$  implies the rset of  $t_j$  till the point of validation of  $t_i$ . If there is a conflict, then either  $t_i$  or  $t_j$  (all transactions conflicting with  $t_i$ ) is aborted. A commonly used approach in databases is to abort  $t_i$ , the validating transaction.

In SV-SFTM we use t ss which are monotonically in increasing order. We implement the t ss using atomic counters. Each transaction  $t_i$  has two time-stamps: (i) current time-stamp or CTS: this is a unique t s alloted to  $t_i$  when it begins; (ii) initial time-stamp or ITS: this is same as CTS when a transaction  $t_i$  starts for the first time. When  $t_i$  aborts and re-starts later, it gets a new CTS. But it retains its original CTS as ITS. The value of ITS is retained across aborts. For achieving starvation freedom, SV-SFTM uses ITS with a modification to FOCC as follows: a transaction  $t_i$  in try-Commit phase is validated against all other conflicting transactions, say  $t_j$  which are in their read/local-write phase. The ITS of  $t_i$  is compared with the ITS of any such transaction  $t_j$ . If ITS of  $t_i$  is smaller than ITS of all such  $t_j$ , then all such  $t_j$  are aborted while  $t_i$  is committed. Otherwise,  $t_i$  is aborted. We show that SV-SFTM satisfies opacity and starvation-free.

**Theorem 3** Any history generated by SV-SFTM is opaque.

#### **Theorem 4** SV-SFTM ensure starvation-freedom.

We prove the correctness by showing that the conflict graph [28, Chap 3], [18] of any history generated by SV-SFTM is acyclic. We show starvation-freedom by showing that for each transaction  $t_i$  there eventually exists a global state in which it has the smallest ITS.

Fig 4 shows the a sample execution of SV-SFTM. It compares the execution of FOCC with SV-SFTM. The execution on the left corresponds to FOCC, while the execution one the right is of SV-SFTM for the same input. It can be seen that each transaction has two tss in SV-SFTM. They correspond to CTS, ITS respectively. Thus, transaction  $T_{1,1}$  implies that CTS and ITS are 1. In this execution, transaction  $T_3$  executes the read operation  $T_3(z)$  and is aborted due to conflict with  $T_2$ . The same happens with  $T_{3,3}$ . Transaction  $T_5$  is re-execution of  $T_3$ . With FOCC  $T_5$  again aborts due to conflict with  $T_4$ . In case of SV-SFTM,  $T_{5,3}$  which is re-execution of  $T_{3,3}$  has the same ITS 3. Hence, when  $T_{4,4}$  validates in SV-SFTM, it aborts as  $T_{5,3}$  has lower ITS. Later  $T_{5,3}$  commits.

It can be seen that ITSs prioritizes the transactions under conflict and the transaction with lower ITS is given higher priority.

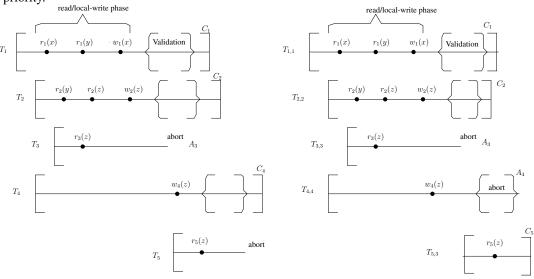


Figure 4: Sample execution of SV-SFTM

#### 3.2.2 Drawback of SV-SFTM

Figure 5 is representing history H:  $r_1(x,0)r_1(y,0)w_2(x,10)w_3(y,15)a_2a_3c_1$  It has three transactions  $T_1$ ,  $T_2$  and  $T_3$ .  $T_1$  is having lowest time stamp and after reading it became slow.  $T_2$  and  $T_3$  wants to write to x and y respectively but when it came into validation phase, due to  $r_1(x)$ ,  $r_1(y)$  and not committed yet,  $T_2$  and  $T_3$  gets

aborted. However, when we are using multiple version  $T_2$  and  $T_3$  both can commit and  $T_1$  can also read from  $T_0$ . The equivalent serial history is  $T_1T_2T_3$ .



Figure 5: Pictorial representation of execution under SFTM

#### 3.2.3 Data Structures and Pseudocode of SV-SFTM

We start with data-structures that are local to each transaction. For each transaction  $T_i$ :

- $rset_i$ (read-set): It is a list of data tuples  $(d\_tuples)$  of the form  $\langle x, val \rangle$ , where x is the t-object and v is the value read by the transaction  $T_i$ . We refer to a tuple in  $T_i$ 's read-set by  $rset_i[x]$ .
- $wset_i$ (write-set): It is a list of  $(d\_tuples)$  of the form  $\langle x, val \rangle$ , where x is the t-object to which transaction  $T_i$  writes the value val. Similarly, we refer to a tuple in  $T_i$ 's write-set by  $wset_i[x]$ .

In addition to these local structures, the following shared global structures are maintained that are shared across transactions (and hence, threads). We name all the shared variable starting with 'G'.

• GtCntr (counter): This a numerical valued counter that is incremented when a transaction begins.

For each transaction  $T_i$  we maintain the following shared time-stamps:

- $G\_lock_i$ : A lock for accessing all the shared variables of  $T_i$ .
- $G_{i}ts_{i}$  (initial timestamp): It is a time-stamp assigned to  $T_{i}$  when it was invoked for the first time.
- $G\_cts_i$  (current timestamp): It is a time-stamp when  $T_i$  is invoked again at a later time. When  $T_i$  is created for the first time, then its  $G\_cts$  is same as its its.
- G- $valid_i$ : This is a boolean variable which is initially true (T). If it becomes false (F) then  $T_i$  has to be aborted.
- $G\_state_i$ : This is a variable which states the current value of  $T_i$ . It has three states: live, commit or abort.

For each data item x in history H, we maintain:

- x.val (value): It is the successful previous closest value written by any transaction.
- x.rl (readList): It is the read list consists of all the transactions that have read x.

**Algorithm 2** STM init(): Invoked at the start of the STM system. Initializes all the data items used by the STM System

- 1:  $G_{-}tCntr = 1$ ;
- 2: for all data item x used by the STM System do
- 3: add  $\langle 0, nil \rangle$  to  $x.val; /* T_0$  is initializing x \*/
- 4: end for;

**Algorithm 5** STM  $write_i(x, val)$ : A Transaction  $T_i$  writes into local memory

- 1: Append the  $d\_tuple\langle x, val \rangle$  to  $wset_i$ ./\* If same dataitem then overwrite the tuple \*/
- 2: return ok;

**Algorithm 3** STM stm-begin(its): Invoked by a thread to start a new transaction  $T_i$ . Thread can pass a parameter its which is the initial timestamp when this transaction was invoked for the first time. If this is the first invocation then its is nil. It returns the tuple  $\langle id, G\_cts \rangle$ 

```
1: i = \text{unique-id}; /* An unique id to identify this transaction. It could be same as G\_cts. */
2: if (its == nil) then
3: G\_its_i = G\_cts_i = G\_tCntr.get\&Inc();
4: /*G\_tCntr.get\&Inc() returns the current value of G\_tCntr and atomically increments it by 1. */
5: else
6: G\_its_i = its;
7: G\_cts_i = G\_tCntr.get\&Inc();
8: end if
9: rset_i = wset_i = null;
10: G\_state_i = live;
11: G\_valid_i = T;
12: return \langle i, G\_cts_i \rangle
```

### **Algorithm 4** STM read(i, x): Invoked by a transaction $T_i$ to read x. It returns either the value of x or $\mathscr{A}$

```
1: if (x \in wset_i) then /* Check if x is in wset_i */
       return wset_i[x].val;
2:
3: else if (x \in rset_i) then /* Check if x is in rset_i */
       return rset_i[x].val;
5: else/* x is not in rset_i and wset_i */
       lock x;
6:
       lock G\_lock_i;
7:
       if (G\_valid_i == F) then
8:
9:
            return abort(i);
       end if
10:
11:
       val = x.val;
       add T_i to x.rl;
12:
       unlock G\_lock_i;
       unlock x;
14:
       return val;
15:
16: end if
```

**Algorithm 6** STM findLLTS(TSet): Find the lowest its value among all the live transactions in TSet.

```
1: min\_its = \infty

2: for all (T_j \in TSet) do

3: if ((G\_its_j < min\_its) && (G\_state_j == live)) then

4: min\_its = G\_its_j;

5: end if

6: end for

7: return min\_its;
```

**Algorithm 7** STM stm-tryC(): Returns  $\mathscr C$  on commit else return Abort  $\mathscr A$ 

```
1: lock G\_lock_i
 2: if (G\_valid_i == F) then return abort(i);
 3: end if
 4: TSet = null \ /* TSet storing transaction Ids */
 5: for all (x \in wset_i) do
       lock x in pre-defined order;
       for all (T_i \in x.rl) do
 7:
            TSet = TSet \cup \{T_i\}
 8:
 9:
       end for
10: end for/* x \in wset_i */
11: TSet = TSet \cup \{T_i\} /* Add current transaction T_i into TSet */
12: for all (T_k \in TSet) do
       lock G.lock_k in pre-defined order; /* Note: Since T_i is also in TSet, G.lock_i is also locked */
13:
14: end for
15: if (G\_valid_i == F) then return abort(i);
16: else
       if (G_its_i = findLLTS(TSet)) then /* Check if T_i has lowest its among all live transactions in
17:
    TSet */
            for all (T_i \in TSet) do /* (T_i \neq T_i) */
18:
19:
                G_{\text{-}}valid_{i} = F
                unlock G\_lock_i;
20:
            end for
21:
22:
       else
23:
            return abort(i);
       end if
24:
25: end if
26: for all (x \in wset_i) do
       replace the old value in x.val with newValue;
27:
       x.rl = \text{null};
28:
29: end for
30: G_{-state_i} = commit;
31: unlock all variables locked by T_i;
32: return \mathscr{C};
```

**Algorithm 8** abort(i): Invoked by various STM methods to abort transaction  $T_i$ . It returns  $\mathscr{A}$ 

```
1: G\_valid_i = F;

2: G\_state_i = \texttt{abort};

3: unlock all variables locked by T_i;

4: return \mathscr{A};
```

**Simplifying Assumptions:** We next describe the main idea behind the starvation-free STM algorithm *KSFTM* through a sequence of algorithms. For ease of exposition, we make two simplifying assumptions (1) We assume that in the absence of other concurrent conflicting transactions, every transaction will commit. In other words, if a transaction is executed in a system by itself, it will not self-abort. (2) We initially consider strict-serializability as correctness-criterion to illustrate the correctness of the algorithms. But strict-serializability does not consider the correctness of aborted transactions and as a result not a suitable correctness-criterion for STMs. Finally, we show

that the proposed STM algorithm KSFTM satisfies local opacity, a correctness-criterion for STMs. We denote the set of histories generated by an STM algorithm, say A, as gen(A).

### 3.3 Priority-based MVTO Algorithm

In this subsection, we describe a modification to the multi-version timestamp ordering (MVTO) algorithm [2, 17] to ensure that it provides preference to transactions that have low ITS, i.e., transactions that have been in the system for a longer time. We denote the basic algorithm which maintains unbounded versions as *Priority-based MVTO* or *PMVTO* (akin to the original MVTO). We denote the variant of *PMVTO* that maintains *K* versions as *PKTO* and the unbounded versions variant with garbage collection as *PMVTO-GC*. In this sub-section, we specifically describe *PKTO*. But most of these properties apply to *PMVTO* and *PMVTO-GC* as well.

stm-begin(t): A unique timestamp ts is allocated to  $T_i$  which is its CTS (i from our assumption). The timestamp ts is generated by atomically incrementing the global counter G-tCntr. If the input t is null, then  $cts_i = its_i = ts$  as this is the first incarnation of this transaction. Otherwise, the non-null value of t is assigned as  $its_i$ .

stm-read(x): Transaction  $T_i$  reads from a version of x in the shared memory (if x does not exist in  $T_i$ 's local buffer) with timestamp j such that j is the largest timestamp less than i (among the versions x), i.e., there exists no version of x with timestamp k such that j < k < i. After reading this version of x,  $T_i$  is stored in x[j]'s read-list. If no such version exists then  $T_i$  is aborted.

stm-write(x,v):  $T_i$  stores this write to value x locally in its  $wset_i$ . If  $T_i$  ever reads x again, this value will be returned.

stm-tryC: This operation consists of three steps. In Step 1, it checks whether  $T_i$  can be *committed*. In Step 2, it performs the necessary tasks to mark  $T_i$  as a *committed* transaction and in Step 3,  $T_i$  return commits.

- 1. Before  $T_i$  can commit, it needs to verify that any version it creates does not violate consistency. Suppose  $T_i$  creates a new version of x with timestamp i. Let j be the largest timestamp smaller than i for which version of x exists. Let this version be x[j]. Now,  $T_i$  needs to make sure that any transaction that has read x[j] is not affected by the new version created by  $T_i$ . There are two possibilities of concern:
  - (a) Let  $T_k$  be some transaction that has read x[j] and k > i ( $k = \text{CTS of } T_k$ ). In this scenario, the value read by  $T_k$  would be incorrect (w.r.t strict-serializability) if  $T_i$  is allowed to create a new version. In this case, we say that the transactions  $T_i$  and  $T_k$  are in *conflict*. So, we do the following:
    - (i) if  $T_k$  has already *committed* then  $T_i$  is *aborted*;
    - (ii) if  $T_k$  is live and  $its_k$  is less than  $its_i$ . Then again  $T_i$  is aborted;
    - (iii) If  $T_k$  is still live with  $its_i$  less than  $its_k$  then  $T_k$  is aborted.
  - (b) The previous version x[j] does not exist. This happens when the previous version x[j] has been overwritten. In this case,  $T_i$  is aborted since *PKTO* does not know if  $T_i$  conflicts with any other transaction  $T_k$  that has read the previous version.
- 2. After Step 1, we have verified that it is ok for  $T_i$  to commit. Now, we have to create a version of each t-object x in the wset of  $T_i$ . This is achieved as follows:
  - (a)  $T_i$  creates a  $vTuple \ \langle i, wset_i.x.v, null \rangle$ . In this tuple, i (CTS of  $T_i$ ) is the timestamp of the new version;  $wset_i.x.v$  is the value of x is in  $T_i$ 's wset, and the read-list of the vTuple is null.
  - (b) Suppose the total number of versions of x is K. Then among all the versions of x,  $T_i$  replaces the version with the smallest timestamp with  $vTuple\ \langle i, wset_i.x.v, null \rangle$ . Otherwise, the vTuple is added to x's vlist.
- 3. Transaction  $T_i$  is then *committed*.

The algorithm described here is only the main idea. The actual implementation will use locks to ensure that each of these methods are linearizable [16]. It can be seen that *PKTO* gives preference to the transaction having lower ITS in Step 1a. Transactions having lower ITS have been in the system for a longer time. Hence, *PKTO* gives preference to them.

#### 3.4 Pseudocode of PKTO

**Algorithm 9** STM init(): Invoked at the start of the STM system. Initializes all the t-objects used by the STM System

```
1: G\_tCntr = 1;

2: for all x in \mathscr{T} do /* All the t-objects used by the STM System */

3: add \langle 0, 0, nil \rangle to x.v1; /* T_0 is initializing x */

4: end for;
```

**Algorithm 10** STM stm-begin(its): Invoked by a thread to start a new transaction  $T_i$ . Thread can pass a parameter its which is the initial timestamp when this transaction was invoked for the first time. If this is the first invocation then its is nil. It returns the tuple  $\langle id, G\_cts \rangle$ 

```
1: i = \text{unique-id}; /* An unique id to identify this transaction. It could be same as G_cts */
 2: /* Initialize transaction specific local and global variables */
 3: if (its == nil) then
        /* G_tCntr.get&Inc() returns the current value of G_tCntr and atomically increments it */
        G_{-its_i} = G_{-cts_i} = G_{-t}Cntr.get\&Inc();
5:
 6: else
 7:
        G_{-}its_i = its;
        G_{-}cts_{i} = G_{-}tCntr.get\&Inc();
8:
9: end if
10: rset_i = wset_i = null;
11: G_{-}state_{i} = live;
12: G_{-}valid_{i} = T;
13: return \langle i, G\_cts_i \rangle
```

**Algorithm 11** STM read(i, x): Invoked by a transaction  $T_i$  to read t-object x. It returns either the value of x or  $\mathscr{A}$ 

```
1: if (x \in rset_i) then /* Check if the t-object x is in rset_i */
2:
        return rset_i[x].val;
 3: else if (x \in wset_i) then /* Check if the t-object x is in wset_i */
        return wset_i[x].val;
 5: else/* t-object x is not in rset_i and wset_i */
        lock x; lock G\_lock_i;
 6:
        if (G_{-}valid_{i} == F) then return abort(i);
 7:
        end if
 8:
        /* findLTS: From x.vl, returns the largest ts value less than G_{\cdot}cts_{i}. If no such version exists, it returns
   nil */
        curVer = findLTS(G\_cts_i, x);
10:
        if (curVer == nil) then return abort(i); /* Proceed only if curVer is not nil */
11:
12:
        val = x[curVer].v; add \langle x, val \rangle to rset_i;
13:
        add T_i to x[curVer].rl;
14:
15:
        unlock G\_lock_i; unlock x;
        return val;
16:
17: end if
```

**Algorithm 12** STM  $write_i(x, val)$ : A Transaction  $T_i$  writes into local memory

```
1: Append the d\_tuple\langle x, val \rangle to wset_i.
```

2: return ok;

#### **Algorithm 13** STM stm-tryC(): Returns ok on commit else return Abort

```
1: /* The following check is an optimization which needs to be performed again later */
 2: lock G\_lock_i;
 3: if (G\_valid_i == F) then
        return abort(i);
 5: end if
 6: unlock G\_lock_i;
 7: largeRL = allRL = nil; /* Initialize larger read list (largeRL), all read list (allRL) to nil */
 8: for all x \in wset_i do
        lock x in pre-defined order;
        /* findLTS: returns the version with the largest ts value less than G_{-}cts_{i}. If no such version exists, it
    returns nil. */
        prevVer = findLTS(G\_cts_i, x); /* prevVer: largest version smaller than G\_cts_i */
11:
        if (prevVer == nil) then /* There exists no version with ts value less than G_cts_i */
            lock G\_lock_i; return abort(i);
13:
14:
        /* getLar: obtain the list of reading transactions of x[prevVer].rl whose G_{-}cts is greater than G_{-}cts_{i} */
15:
        largeRL = largeRL \cup getLar(G\_cts_i, x[prevVer].rl);
17: end for/* x \in wset_i */
18: relLL = largeRL \cup T_i; /* Initialize relevant Lock List (relLL) */
19: for all (T_k \in relLL) do
        lock G.lock_k in pre-defined order; /* Note: Since T_i is also in relLL, G.lock_i is also locked */
20:
21: end for
22: /* Verify if G_{-}valid_{i} is false */
23: if (G\_valid_i == F) then
24:
        return abort(i);
25: end if
26: abortRL = nil /* Initialize abort read list (abortRL) */
27: /* Among the transactions in T_k in largeRL, either T_k or T_i has to be aborted */
28: for all (T_k \in largeRL) do
        if (isAborted(T_k)) then /* Transaction T_k can be ignored since it is already aborted or about to be aborted
    */
            continue;
30:
        end if
31:
        if (G_{-}its_i < G_{-}its_k) \wedge (G_{-}state_k == live) then
32:
33:
            /* Transaction T_k has lower priority and is not yet committed. So it needs to be aborted */
            abortRL = abortRL \cup T_k; /* Store T_k in abortRL */
34:
35:
        else/* Transaction T_i has to be aborted */
            return abort(i);
36:
37:
        end if
38: end for
Algorithm 14 is Aborted (T_k): Verifies if T_i is already aborted or its G_valid flag is set to false implying that T_i
will be aborted soon
 1: if (G\_valid_k == F) \lor (G\_state_k == abort) \lor (T_k \in abortRL) then
 2:
        return T;
 3: else
        return F;
 5: end if
Algorithm 15 abort(i): Invoked by various STM methods to abort transaction T_i. It returns \mathscr{A}
 1: G_{\text{-}}valid_i = F; G_{\text{-}}state_i = \text{abort};
 2: unlock all variables locked by T_i;
```

We have the following property on the correctness of *PKTO*.

3: return  $\mathscr{A}$ ;

```
39: /* Store the current value of the global counter as commit time and increment it */
40: comTime = G_{-}tCntr.get\&Inc();
41: for all T_k \in abortRL do /* Abort all the transactions in abortRL */
        G-valid_k = F;
42:
43: end for
44: /* Having completed all the checks, T_i can be committed */
45: for all (x \in wset_i) do
        newTuple = \langle G\_cts_i, wset_i[x].val, nil \rangle; /* Create new v_tuple: G_cts, val, r1 for x */
46:
47:
        if (|x.vl| > k) then
            replace the oldest tuple in x.vl with newTuple; /* x.vl is ordered by timestamp */
48:
49:
        else
            add a newTuple to x.vl in sorted order;
50:
        end if
51:
52: end for/* x \in wset_i */
53: G_{-}state_{i} = commit;
54: unlock all variables;
55: return \mathscr{C}:
```

**Property 5** Any history generated by PKTO is strict-serializable.

Consider a history H generated by PKTO. Let the *committed* sub-history of H be CSH = H.subhist(H.committed). It can be shown that CSH is opaque with the equivalent serialized history SH' is one in which all the transactions of CSH are ordered by their CTSs. Hence, H is strict-serializable.

**Possibility of Starvation in** *PKTO***:** As discussed above, *PKTO* gives priority to transactions having lower ITS. But a transaction  $T_i$  having the lowest ITS could still abort due to one of the following reasons: (1) Upon executing stm-read(x) method if it does not find any other version of x to read from. This can happen if all the versions of x present have a timestamp greater than  $cts_i$ . (2) While executing Step 1a(i), of the stm-tryC method, if  $T_i$  wishes to create a version of x with timestamp i. But some other transaction, say  $T_k$  has read from a version with timestamp j and j < i < k. In this case,  $T_i$  has to abort if  $T_k$  has already committed.

This issue is not restricted only to *PKTO*. It can occur in *PMVTO* (and *PMVTO-GC*) due to the point (2) described above.

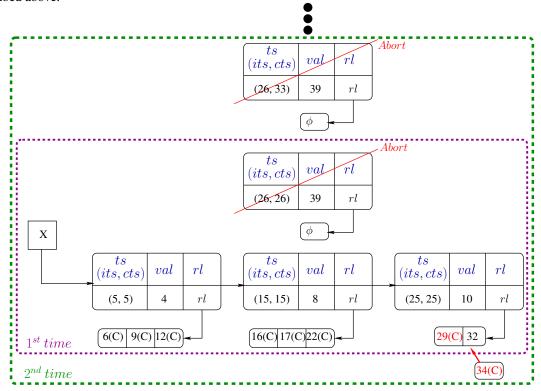


Figure 6: Pictorial representation of execution under PKTO

We illustrate this problem in PKTO with Fig 6. Here transaction  $T_{26}$ , with ITS 26 is the lowest among all the live transactions, starves due to Step 1a.(i) of the stm-tryC. First time,  $T_{26}$  gets aborted due to higher timestamp transaction  $T_{29}$  in the read-list of x[25] has committed. We have denoted it by a '(C)' next to the version. The second time,  $T_{26}$  retries with same ITS 26 but new CTS 33. Now when  $T_{33}$  comes for commit, suppose another transaction  $T_{34}$  in the read-list of x[25] has already committed. So this will cause  $T_{33}$  (another incarnation of  $T_{26}$ ) to abort again. Such scenario can possibly repeat again and again and thus causing no incarnation of  $T_{26}$  to ever commit leading to its starvation.

Garbage Collection in *UVSFTM-GC* and *PMVTO-GC*: Having multiple versions to increase the performance and to decrease the number of aborts, leads to creating too many versions which are not of any use and hence occupying space. So, such garbage versions need to be taken care of. Hence we come up with a garbage collection over these unwanted versions. This technique help to conserve memory space and increases the performance in turn as no more unnecessary traversing of garbage versions by transactions is necessary. We have used a global, i.e., across all transactions a list that keeps track of all the live transactions in the system. We call this list as *live-list*. Each transaction at the beginning of its life cycle creates its entry in this *live-list*. Under the optimistic approach of STM, each transaction in the shared memory performs its updates in the *stm-tryC* phase. In this phase, each transaction performs some validations, and if all the validations are successful then the transaction make changes or in simple terms creates versions of the corresponding t-object in the shared memory. While creating a version every transaction, check if it is the least timestamp live transaction present in the system by using *live-list* data structure, if yes then the current transaction deletes all the version of that t-object and create one of its own. Else the transaction does not do any garbage collection or delete any version and look for creating a new version of next t-object in the write set, if at all. Fig 10 and Fig 11 show that both *UVSFTM-GC* and *PMVTO-GC* performs better than *UVSFTM* and *PMVTO* across all workloads.

## 3.5 Modifying PKTO to Obtain SFKTO: Trading Correctness for Starvation-Freedom

Our goal is to revise PKTO algorithm to ensure that starvation-freedom is satisfied. Specifically, we want the transaction with the lowest ITS to eventually commit. Once this happens, the next non-committed transaction with the lowest ITS will commit. Thus, from induction, we can see that every transaction will eventually commit. **Key Insights For Eliminating Starvation in PKTO:** To identify the necessary revision, we first focus on the effect of this algorithm on two transactions, say  $T_{50}$  and  $T_{60}$  with their CTS values being 50 and 60 respectively. Furthermore, for the sake of discussion, assume that these transactions only read and write t-object x. Also, assume that the latest version for x is with ts 40. Each transaction first reads x and then writes x (as part of the stm-tryC operation). We use  $r_{50}$  and  $r_{60}$  to denote their read operations while  $w_{50}$  and  $w_{60}$  to denote their stm-tryC operations. Here, a read operation will not fail as there is a previous version present.

Now, there are six possible permutations of these statements. We identify these permutations and the action that should be taken for that permutation in Table 1. In all these permutations, the read operations of a transaction come before the write operations as the writes to the shared memory occurs only in the stm-tryC operation (due to optimistic execution) which is the final operation of a transaction.

S. No	Sequence	Action
1.	$r_{50}, w_{50}, r_{60}, w_{60}$	$T_{60}$ reads the version written by $T_{50}$ . No conflict.
2.	$r_{50}, r_{60}, w_{50}, w_{60}$	Conflict detected at $w_{50}$ . Either abort $T_{50}$ or $T_{60}$ .
3.	$r_{50}, r_{60}, w_{60}, w_{50}$	Conflict detected at $w_{50}$ . Hence, abort $T_{50}$ .
4.	$r_{60}, r_{50}, w_{60}, w_{50}$	Conflict detected at $w_{50}$ . Hence, abort $T_{50}$ .
5.	$r_{60}, r_{50}, w_{50}, w_{60}$	Conflict detected at $w_{50}$ . Either abort $T_{50}$ or $T_{60}$ .
6.	$r_{60}, w_{60}, r_{50}, w_{50}$	Conflict detected at $w_{50}$ . Hence, abort $T_{50}$ .

Table 1: Permutations of operations

From this table, it can be seen that when a conflict is detected, in some cases, algorithm PKTO must abort  $T_{50}$ . In case both the transactions are live, PKTO has the option of aborting either transaction depending on their ITS. If  $T_{60}$  has lower ITS then in no case, PKTO is required to abort  $T_{60}$ . In other words, it is possible to ensure that the transaction with lowest ITS and the highest CTS is never aborted. Although in this example, we considered only one t-object, this logic can be extended to cases having multiple operations and t-objects.

Next, consider Step 1b of PKTO algorithm. Suppose a transaction  $T_i$  wants to read a t-object but does not find a version with a timestamp smaller than i. In this case,  $T_i$  has to abort. But if  $T_i$  has the highest CTS, then it will certainly find a version to read from. This is because the timestamp of a version corresponds to the timestamp of the transaction that created it. If  $T_i$  has the highest CTS value then it implies that all versions of all the t-objects

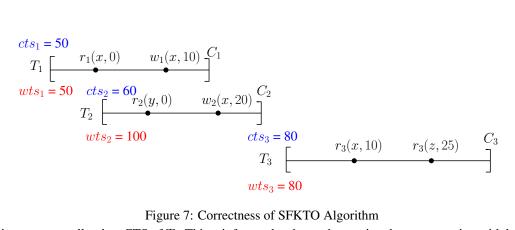


Figure 7: Correctness of SFKTO Algorithm

have a timestamp smaller than CTS of  $T_i$ . This reinforces the above observation that a transaction with lowest ITS and highest CTS is not aborted.

To summarize the discussion, algorithm PKTO has an in-built mechanism to protect transactions with lowest ITS and highest CTS value. However, this is different from what we need. Specifically, we want to protect a transaction  $T_i$ , with lowest ITS value. One way to ensure this: if transaction  $T_i$  with lowest ITS keeps getting aborted, eventually it will achieve the highest CTS. Once this happens, PKTO ensures that  $T_i$  cannot be further aborted. In this way, we can ensure the liveness of all transactions.

The working of starvation-free algorithm: To realize this idea and achieve starvation-freedom, we consider another variation of MVTO, Starvation-Free MVTO or SFMVTO. We specifically consider SFMVTO with K versions, denoted as SFKTO.

A transaction  $T_i$  instead of using the current time as  $cts_i$ , uses a potentially higher timestamp, Working Timestamp - WTS or  $wts_i$ . Specifically, it adds  $C*(cts_i-its_i)$  to  $cts_i$ , i.e.,

$$wts_i = cts_i + C * (cts_i - its_i); (1)$$

where, C is any constant greater than 0. In other words, when the transaction  $T_i$  is issued for the first time,  $wts_i$ is same as  $cts_i = its_i$ ). However, as transaction keeps getting aborted, the drift between  $cts_i$  and  $wts_i$  increases. The value of  $wts_i$  increases with each retry.

Furthermore, in SFKTO algorithm, CTS is replaced with WTS for stm-read, stm-write and stm-tryC operations of PKTO. In SFKTO, a transaction  $T_i$  uses  $wts_i$  to read a version in stm-read. Similarly,  $T_i$  uses  $wts_i$ in stm-tryC to find the appropriate previous version (in Step 1b) and to verify if  $T_i$  has to be aborted (in Step 1a). Along the same lines, once  $T_i$  decides to commit and create new versions of x, the timestamp of x will be same as its  $wts_i$  (in Step 3). Thus the timestamp of all the versions in vlist will be WTS of the transactions that created

Now, we have the following property about SFKTO algorithm.

#### **Property 6** SFKTO algorithm ensures starvation-freedom.

While the proof of this property is somewhat involved, the key idea is that the transaction with lowest ITS value, say  $T_{low}$ , will eventually have highest WTS value than all the other transactions in the system. Moreover, after a certain duration, any new transaction arriving in the system (i.e., whose ITS value sufficiently higher than that of  $T_{low}$ ) will have a lower WTS value than  $T_{low}$ . This will ensure that  $T_{low}$  will not be aborted. In fact, this property can be shown to be true of SFMVTO as well.

The drawback of SFKTO: Although SFKTO satisfies starvation-freedom, it, unfortunately, does not satisfy strict-serializability. Specifically, it violates the real-time requirement. PKTO uses CTS for its working while SFKTO uses WTS. It can be seen that CTS is close to the real-time execution of transactions whereas WTS of a transaction  $T_i$  is artificially inflated based on its ITS and might be much larger than its CTS. We illustrate this with an example. Consider the history H1 as shown in Fig 7:  $r_1(x,0)r_2(y,0)w_1(x,10)$ 

 $C_1w_2(x,20)C_2r_3(x,10)r_3(z,25)C_3$  with CTS as 50, 60 and 80 and WTS as 50, 100 and 80 for  $T_1,T_2,T_3$  respectively. Here  $T_1,T_2$  are ordered before  $T_3$  in real-time with  $T_1 \prec_{H1}^{RT} T_3$  and  $T_2 \prec_{H1}^{RT} T_3$  although  $T_2$  has a higher WTS than  $T_3$ .

Here, as per SFKTO algorithm,  $T_3$  reads x from  $T_1$  since  $T_1$  has the largest WTS (50) smaller than  $T_3$ 's WTS (80). It can be verified that it is possible for SFKTO to generate such a history. But this history is not strict-serializable. The only possible serial order equivalent to H1 and legal is  $T_1T_3T_2$ . But this violates real-time order as  $T_3$  is serialized before  $T_2$  but in H1,  $T_2$  completes before  $T_3$  has begun. Since H1 is not strict-serializable, it is not locally-opaque as well. Naturally, this drawback extends to SFMVTO as well.

#### 3.6 Design of KSFTM: Regaining Correctness while Preserving Starvation-Freedom

In this section, we discuss how principles of *PKTO* and SFKTO can be combined to obtain *KSFTM* that provides both correctness (strict-serializability and locally-opaque) as well as *starvation-freedom*. To achieve this, we first understand why the initial algorithm, *PKTO* satisfies strict-serializability. This is because CTS was used to create the ordering among committed transactions. CTS is closely associated with real-time. In contrast, SFKTO uses WTS which may not correspond to the real-time, as WTS may be significantly larger than CTS as shown by *H*1 in Fig 7.

One straightforward way to modify SFKTO is to delay a committing transaction, say  $T_i$  with WTS value  $wts_i$  until the real-time (G\_tCntr) catches up to  $wts_i$ . This will ensure that value of WTS will also become same as the real-time thereby guaranteeing strict-serializability. However, this is unacceptable, as in practice, it would require transaction  $T_i$  locking all the variables it plans to update and wait. This will adversely affect the performance of the STM system.

We can allow the transaction  $T_i$  to commit before its  $wts_i$  has caught up with the actual time if it does not violate the real-time ordering. Thus, to ensure that the notion of real-time order is respected by transactions in the course of their execution in SFKTO, we add extra time constraints. We use the idea of timestamp ranges. This notion of timestamp ranges was first used by Riegel et al. [24] in the context of multi-version STMs. Several other researchers have used this idea since then such as Guerraoui et al. [10], Crain et al. [5], Aydonat & Abdelrahman [11].

Thus, in addition to ITS, CTS and WTS, each transaction  $T_i$  maintains a timestamp range: Transaction Lower Timestamp Limit or  $tltl_i$ , and Transaction Upper Timestamp Limit or  $tutl_i$ . When a transaction  $T_i$  begins,  $tltl_i$  is assigned  $cts_i$  and  $tutl_i$  is assigned a largest possible value which we denote as infinity. When  $T_i$  executes a method m in which it reads a version of a t-object x or creates a new version of x in stm-tryC,  $tltl_i$  is incremented while  $tutl_i$  gets decremented t.

We require to serialize all the transactions based on their WTS while maintaining their real-time order. On executing m,  $T_i$  is ordered w.r.t to other transactions that have created a version of x based on increasing order of WTS. For all transactions  $T_j$  which also have created a version of x and whose  $wts_j$  is less than  $wts_i$ ,  $tltl_i$  is incremented such that  $tutl_j$  is less than  $tltl_i$ . Note that all such  $T_j$  are serialized before  $T_i$ . Similarly, for any transaction  $T_k$  which has created a version of x and whose  $wts_k$  is greater than  $wts_i$ ,  $tutl_i$  is decremented such that it becomes less than  $tltl_k$ . Again, note that all such  $T_k$  is serialized after  $T_i$ .

Note that in the above discussion,  $T_i$  need not have created a version of x. It could also have read the version of x created by  $T_j$ . After the increments of  $tltl_i$  and the decrements of  $tutl_i$ , if  $tltl_i$  turns out to be greater than  $tutl_i$  then  $T_i$  is aborted. Intuitively, this implies that  $T_i$ 's WTS and real-time orders are out of sync and cannot be reconciled.

Finally, when a transaction  $T_i$  commits: (1)  $T_i$  records its commit time (or  $comTime_i$ ) by getting the current value of  $G_t$ Cntr and incrementing it by incrVal which is any value greater than or equal to 1. Then  $tutl_i$  is set to  $comTime_i$  if it is not already less than it. Now suppose  $T_i$  occurs in real-time before some other transaction,  $T_k$  but does not have any conflict with it. This step ensures that  $tutl_i$  remains less than  $tltl_k$  (which is initialized with  $cts_k$ ); (2) Ensure that  $tltl_i$  is still less than  $tutl_i$ . Otherwise,  $T_i$  is aborted.

We illustrate this technique with the history H1 shown in Fig 7. When  $T_1$  starts its  $cts_1=50, tltl_1=50, tutl_1=\infty$ . Now when  $T_1$  commits, suppose  $G\_tCntr$  is 70. Hence,  $tutl_1$  reduces to 70. Next, when  $T_2$  commits, suppose  $tutl_2$  reduces to 75 (the current value of  $G\_tCntr$ ). As  $T_1, T_2$  have accessed a common t-object x in a conflicting manner,  $tltl_2$  is incremented to a value greater than  $tutl_1$ , say 71. Next, when  $T_3$  begins,  $tltl_3$  is assigned  $cts_3$  which is 80 and  $tutl_3$  is initialized to  $\infty$ . When  $T_3$  reads 10 from  $T_1$ , which is  $r_3(x, 10), tutl_3$  is reduced to a value less than  $tltl_2(=71)$ , say 70. But  $tltl_3$  is already at 80. Hence, the limits of  $T_3$  have crossed and thus causing  $T_3$  to abort. The resulting history consisting of only committed transactions  $T_1T_2$  is strict-serializable.

Based on this idea, we next develop a variation of SFKTO, *K-version Starvation-Free STM System* or *KSFTM*. To explain this algorithm, we first describe the structure of the version of a t-object used. It is a slight variation of the t-object used in *PKTO* algorithm. It consists of: (1) timestamp, *ts* which is the WTS of the transaction that created this version (and not CTS like *PKTO*); (2) the value of the version; (3) a list, called read-list, consisting of transactions ids (could be CTS as well) that read from this version; (4) version real-time timestamp or vrt which is the tutl of the transaction that created this version. Thus a version has information of WTS and tutl of the transaction that created it.

<sup>&</sup>lt;sup>1</sup>Technically  $\infty$ , which is assigned to  $tutl_i$ , cannot be decremented. But here as mentioned earlier, we use  $\infty$  to denote the largest possible value that can be represented in a system.

Now, we describe the main idea behind stm-begin, stm-read, stm-write and stm-tryC operations of a transaction  $T_i$  which is an extension of PKTO. Note that as per our notation i represents the CTS of  $T_i$ . stm-begin(t): A unique timestamp ts is allocated to  $T_i$  which is its CTS (i from our assumption) which is generated by atomically incrementing the global counter G-tCntr. If the input t is null then  $cts_i = its_i = ts$  as this is the first incarnation of this transaction. Otherwise, the non-null value of t is assigned to  $its_i$ . Then, WTS is computed by Eq.(1). Finally, tltl and tutl are initialized:  $tltl_i = cts_i$ ,  $tutl_i = \infty$ . stm-read(x): Transaction  $T_i$  reads from a version of x with timestamp y such that y is the largest timestamp less than  $wts_i$  (among the versions x), i.e. there exists no version x0 such that y1 stored in y2 is true. If no such y2 exists then y3 is aborted. Otherwise, after reading this version of x3, x4 is stored in y5 str. Then we modify tltl, tutl as follows:

- 1. The version x[j] is created by a transaction with  $wts_j$  which is less than  $wts_i$ . Hence,  $tltl_i = max(tltl_i, x[j].vrt+1)$ .
- 2. Let p be the timestamp of smallest version larger than i. Then  $tutl_i = min(tutl_i, x[p].vrt 1)$ .
- 3. After these steps, abort  $T_i$  if tltl and tutl have crossed, i.e.,  $tltl_i > tutl_i$ .

stm-write(x, v):  $T_i$  stores this write to value x locally in its  $wset_i$ . stm-tryC: This operation consists of multiple steps:

- 1. Before  $T_i$  can commit, we need to verify that any version it creates is updated consistently.  $T_i$  creates a new version with timestamp  $wts_i$ . Hence, we must ensure that any transaction that read a previous version is unaffected by this new version. Additionally, creating this version would require an update of that and tutl of  $T_i$  and other transactions whose read-write set overlaps with that of  $T_i$ . Thus,  $T_i$  first validates each t-object x in its wset as follows:
  - (a)  $T_i$  finds a version of x with timestamp j such that j is the largest timestamp less than  $wts_i$  (like in stm-read). If there exists no version of x with a timestamp less than  $wts_i$  then  $T_i$  is aborted. This is similar to Step 1b of the stm-tryC of PKTO algorithm.
  - (b) Among all the transactions that have previously read from j suppose there is a transaction  $T_k$  such that  $j < wts_i < wts_k$ . Then (i) if  $T_k$  has already committed then  $T_i$  is aborted; (ii) Suppose  $T_k$  is live, and  $its_k$  is less than  $its_i$ . Then again  $T_i$  is aborted; (iii) If  $T_k$  is still live with  $its_i$  less than  $its_k$  then  $T_k$  is aborted.
    - This step is similar to Step 1a of the stm-tryC of PKTO algorithm.
  - (c) Next, we must ensure that  $T_i$ 's that and tutal are updated correctly w.r.t to other concurrently executing transactions. To achieve this, we adjust that, tutal as follows: (i) Let j be the ts of the largest version smaller than  $wts_i$ . Then  $tltl_i = max(tltl_i, x[j].vrt + 1)$ . Next, for each reading transaction,  $T_r$  in x[j].read-list, we again set,  $tltl_i = max(tltl_i, tutl_r + 1)$ . (ii) Similarly, let p be the ts of the smallest version larger than  $wts_i$ . Then,  $tutl_i = min(tutl_i, x[p].vrt 1)$ . (Note that we don't have to check for the transactions in the read-list of x[p] as those transactions will have that higher than x[p].vrt due to stm-read.) (iii) Finally, we get the commit time of this transaction from G-tCntr:  $comTime_i = G\_tCntr.add\&Get(incrVal)$  where incrVal is any constant  $\geq 1$ . Then,  $tutl_i = min(tutl_i, comTime_i)$ . After performing these updates, abort  $T_i$  if that and tutal have crossed, i.e.,  $tltl_i > tutl_i$ .
- 2. After performing the tests of Step 1 over each t-objects x in  $T_i$ 's wset, if  $T_i$  has not yet been aborted, we proceed as follows: for each x in  $wset_i$  create a vTuple  $\langle wts_i, wset_i.x.v, null, tutl_i \rangle$ . In this tuple,  $wts_i$  is the timestamp of the new version;  $wset_i.x.v$  is the value of x is in  $T_i$ 's wset; the read-list of the vTuple is null; vrt is  $tutl_i$  (actually it can be any value between  $tltl_i$  and  $tutl_i$ ). Update the vlist of each t-object x similar to Step 2 of stm-tryC of PKTO.
- 3. Transaction  $T_i$  is then committed.

Step 1c.(iii) of stm-tryC ensures that real-time order between transactions that are not in conflict. It can be seen that locks have to be used to ensure that all these methods to execute in a linearizable manner (i.e., atomically).

#### 3.7 Data Structures and Pseudocode of KSFTM

The STM system consists of the following methods:  $init(), stm\text{-}begin(), read(i, x), write_i(i, x, v)$  and stm-tryC(i). We assume that all the t-objects are ordered as  $x_1, x_2, ... x_n$  and belong to the set  $\mathscr{T}$ . We describe the data-structures used by the algorithm.

We start with structures that local to each transaction. Each transaction  $T_i$  maintains a  $rset_i$  and  $wset_i$ . In addition it maintains the following structures (1)  $comTime_i$ : This is value given to  $T_i$  when it terminates which is assigned a value in stm-tryC method. (2) A series of lists: smallRL, largeRL, allRL, prevVL, nextVL, relLL, abortRL. The meaning of these lists will be clear with the description of the pseudocode. In addition to these local structures, the following shared global structures are maintained that are shared across transactions (and hence, threads). We name all the shared variable starting with 'G'.

• GtCntr (counter): This a numerical valued counter that is incremented when a transaction begins and terminates.

For each transaction  $T_i$  we maintain the following shared time-stamps:

- $G\_lock_i$ : A lock for accessing all the shared variables of  $T_i$ .
- $G\_its_i$  (initial timestamp): It is a time-stamp assigned to  $T_i$  when it was invoked for the first time without any aborts. The current value of  $G\_tCntr$  is atomically assigned to it and then incremented. If  $T_i$  is aborted and restarts later then the application assigns it the same  $G\_its$ .
- $G\_cts_i$  (current timestamp): It is a time-stamp when  $T_i$  is invoked again at a later time after an abort. Like  $G\_its$ , the current value of  $G\_tCntr$  is atomically assigned to it and then incremented. When  $T_i$  is created for the first time, then its  $G\_cts$  is same as its  $G\_its$ .
- $G\_wts_i$  (working timestamp): It is the time-stamp that  $T_i$  works with. It is either greater than or equal to  $T_i$ 's  $G\_cts_i$ . It is computed as follows:  $G\_wts_i = G\_cts_i + C * (G\_cts_i G\_its_i)$ .
- $G\_valid_i$ : This is a boolean variable which is initially true. If it becomes false then  $T_i$  has to be aborted.
- $G\_state_i$ : This is a variable which states the current value of  $T_i$ . It has three states: live, committed or aborted.
- $G\_tltl_i$ ,  $G\_tutl_i$  (transaction lower & upper time limits): These are the time-limits described in the previous section used to keep the transaction WTS and real-time orders in sync.  $G\_tltl_i$  is  $G\_cts$  of  $T_i$  when transaction begins and is a non-decreasing value. It continues to increase (or remains same) as  $T_i$  reads t-objects and later terminates.  $G\_tutl_i$  on the other hand is a non-increasing value starting with  $\infty$  when the  $T_i$  is created. It reduces (or remains same) as  $T_i$  reads t-objects and later terminates. If  $T_i$  commits then both  $G\_tltl_i$  &  $G\_tutl_i$  are made equal.

Two transactions having the same ITS are said to be incarnations. No two transaction can have the same CTS. For simplicity, we assume that no two transactions have the same WTS as well. In case, two transactions have the same WTS, one can use the tuple  $\langle \text{WTS}, \text{CTS} \rangle$  instead of WTS. But we ignore such cases. For each t-object x in  $\mathscr{T}$ , we maintain:

- x.vl (version list): It is a list consisting of version tuples or vTuple of the form  $\langle ts, val, rl, vrt \rangle$ . The details of the tuple are explained below.
- ts (timestmp): Here ts is the  $G_wts_i$  of a committed transaction  $T_i$  that has created this version.
- val: The value of this version.
- rl (readList): rl is the read list consists of all the transactions that have read this version. Each entry in this list is of the form  $\langle rts \rangle$  where rts is the  $G_-wts_j$  of a transaction  $T_j$  that read this version.
- vrt (version real-time timestamp): It is the  $G_{\perp}$ tutl value (which is same as  $G_{\perp}$ tltl) of the transaction  $T_i$  that created this version at the time of commit of  $T_i$ .

**Algorithm 16** STM init(): Invoked at the start of the STM system. Initializes all the t-objects used by the STM System

```
1: G\_tCntr = 1; /* Global Transaction Counter */
2: for all x in \mathscr{T} do /* All the t-objects used by the STM System */
3: /* T_0 is creating the first version of x: ts = 0, val = 0, rl = nil, vrt = 0 */
4: add \langle 0, 0, nil, 0 \rangle to x.vl;
5: end for;
```

**Algorithm 17** STM stm-begin(its): Invoked by a thread to start a new transaction  $T_i$ . Thread can pass a parameter its which is the initial timestamp when this transaction was invoked for the first time. If this is the first invocation then its is nil. It returns the tuple  $\langle id, G\_wts, G\_cts \rangle$ 

```
1: i = \text{unique-id}; /* An unique id to identify this transaction. It could be same as G_cts */
 2: /* Initialize transaction specific local & global variables */
 3: if (its == nil) then
        G_{\cdot}its_{i} = G_{\cdot}wts_{i} = G_{\cdot}tcs_{i} = G_{\cdot}tCntr.qet\&Inc(); /*G_{\cdot}tCntr.qet\&Inc()  returns the current value of
    G_tCntr and atomically increments it */
5: else
        G_{-}its_{i} = its;
 6:
        G\_cts_i = G\_tCntr.qet\&Inc();
 7:
        G_{-}wts_{i} = G_{-}cts_{i} + C * (G_{-}cts_{i} - G_{-}its_{i}); /* C is any constant greater or equal to than 1 */
 8.
9: end if
10: G_{-}tltl_{i} = G_{-}cts_{i}; G_{-}tutl_{i} = comTime_{i} = \infty;
11: G\_state_i = live; G\_valid_i = T;
12: rset_i = wset_i = nil;
13: return \langle i, G_-wts_i, G_-cts_i \rangle
```

**Algorithm 18** STM read(i, x): Invoked by a transaction  $T_i$  to read t-object x. It returns either the value of x or  $\mathscr{A}$ 

```
1: if (x \in wset_i) then /* Check if the t-object x is in wset_i */
       return wset_i[x].val;
 3: else if (x \in rset_i) then /* Check if the t-object x is in rset_i */
       return rset_i[x].val;
 5: else/* t-object x is not in rset_i and wset_i */
       lock x; lock G\_lock_i;
       if (G_{-}valid_{i} == F) then return abort(i);
 7:
 8:
 9:
       /* findLTS: From x.vl, returns the largest ts value less than G_{\cdot}wts_{i}. If no such version exists, it returns
   nil */
       curVer = findLTS(G_{-}wts_i, x);
10:
       if (curVer == nil) then return abort(i); /* Proceed only if curVer is not nil */
12:
       /* findSTL: From x.vl, returns the smallest ts value greater than G_{-}wts_{i}. If no such version exists, it
   returns nil */
       nextVer = findSTL(G_{-}wts_i, x);
14:
```

```
15:
       if (nextVer \neq nil) then
           /* Ensure that G_tutl; remains smaller than nextVer's vrt */
16:
            G_{-}tutl_{i} = min(G_{-}tutl_{i}, x[nextVer].vrt - 1);
17:
       end if
18:
       /* G_{-}tltl_{i} should be greater than x[curVer].vrt */
19:
       G_{-}tltl_{i} = max(G_{-}tltl_{i}, x[curVer].vrt + 1);
20:
       if (G_{-}tltl_i > G_{-}tutl_i) then /* If the limits have crossed each other, then T_i is aborted */
21:
22:
           return abort(i);
23:
       end if
       val = x[curVer].v; add \langle x, val \rangle to rset_i;
24:
       add T_i to x[curVer].rl;
25:
       unlock G\_lock_i; unlock x;
26:
       return val;
28: end if
Algorithm 19 STM write_i(x, val): A Transaction T_i writes into local memory
 1: Append the d\_tuple\langle x, val \rangle to wset_i.
 2: return ok;
Algorithm 20 STM stm-tryC(): Returns ok on commit else return Abort
 1: /* The following check is an optimization which needs to be performed again later */
 2: lock G\_lock_i;
 3: if (G\_valid_i == F) then return abort(i);
 4: end if
 5: unlock G-lock_i;
 6: /* Initialize smaller read list (smallRL), larger read list (largeRL), all read list (allRL) to nil */
 7: smallRL = largeRL = allRL = nil;
 8: /* Initialize previous version list (prevVL), next version list (nextVL) to nil */
 9: prevVL = nextVL = nil;
10: for all x \in wset_i do
        lock x in pre-defined order;
11:
        /* findLTS: returns the version of x with the largest ts less than G_{-}wts_{i}. If no such version exists, it
12:
    returns nil. */
        prevVer = findLTS(G_{-}wts_{i}, x); /* prevVer: largest version smaller than G_{-}wts_{i} */
13:
        if (prevVer == nil) then /* There exists no version with ts value less than G_-wts_i */
            lock G\_lock_i; return abort(i);
15:
16:
        prevVL = prevVL \cup prevVer; /* prevVL stores the previous version in sorted order */
17:
        allRL = allRL \cup x[prevVer].rl; /* Store the read-list of the previous version */
        /* getLar: obtain the list of reading transactions of x[prevVer].rl whose G_{-}wts is greater than G_{-}wts_{i}
19:
        largeRL = largeRL \cup getLar(G\_wts_i,
20:
    x[prevVer].rl);
        /* getSm: obtain the list of reading transactions of x[prevVer].rl whose G\_wts is smaller than G\_wts_i
21:
22:
        smallRL = smallRL \cup getSm(G\_wts_i,
    x[prevVer].rl);
Algorithm 21 is Aborted (T_k): Verifies if T_i is already aborted or its G-valid flag is set to false implying that T_i
will be aborted soon
 1: if (G\_valid_k == F) \lor (G\_state_k == abort) \lor (T_k \in abortRL) then
 2:
 3: else
        return F;
 4:
```

5: end if

```
/* findSTL: returns the version with the smallest ts value greater than G_{-}wts_{i}. If no such version exists,
23:
    it returns nil. */
        nextVer = findSTL(G_{-}wts_{i}, x); /* nextVer: smallest version larger than G_{-}wts_{i} */
24:
        if (nextVer \neq nil)) then
25:
            nextVL = nextVL \cup nextVer; /* nextVL stores the next version in sorted order */
26:
        end if
27:
28: end for/* x \in wset_i */
29: relLL = allRL \cup T_i; /* Initialize relevant Lock List (relLL) */
30: for all (T_k \in relLL) do
        lock G.lock_k in pre-defined order; /* Note: Since T_i is also in relLL, G.lock_i is also locked */
31:
32: end for
33: /* Verify if G_{-}valid_{i} is false */
34: if (G_{-}valid_{i} == F) then return abort(i);
36: abortRL = nil /* Initialize abort read list (abortRL) */
37: /* Among the transactions in T_k in largeRL, either T_k or T_i has to be aborted */
38: for all (T_k \in largeRL) do
        if (isAborted(T_k)) then
39:
            /* Transaction T_k can be ignored since it is already aborted or about to be aborted */
40:
            continue;
41:
        end if
42:
        if (G_{-}its_i < G_{-}its_k) \wedge (G_{-}state_k == live) then
43:
44:
            /* Transaction T_k has lower priority and is not yet committed. So it needs to be aborted */
            abortRL = abortRL \cup T_k; /* Store T_k in abortRL */
45:
        else/* Transaction T_i has to be aborted */
46:
            return abort(i);
47:
        end if
48:
49: end for
50: /* Ensure that G_{-}tltl_{i} is greater than vrt of the versions in prevVL */
51: for all (ver \in prevVL) do
        x = \text{t-object of } ver;
        G\_tltl_i = max(G\_tltl_i, x[ver]. \texttt{vrt} + 1);
53:
54: end for
55: /* Ensure that vutl_i is less than vrt of versions in nextVL */
56: for all (ver \in nextVL) do
        x = \text{t-object of } ver;
57:
        G_{\text{-}}tutl_{i} = min(G_{\text{-}}tutl_{i}, x[ver].vrt - 1);
58:
59: end for
60: /* Store the current value of the global counter as commit time and increment it */
61: comTime_i = G_tCntr.add\&Get(incrVal); /* incrVal can be any constant \geq 1 */
62: G_{-}tutl_{i} = min(G_{-}tutl_{i}, comTime_{i}); /* Ensure that G_{-}tutl_{i} is less than or equal to comTime_{i} */
63: /* Abort T_i if its limits have crossed */
64: if (G_{-}tltl_{i} > G_{-}tutl_{i}) then return abort(i);
65: end if
```

```
66: for all (T_k \in smallRL) do
        if (isAborted(T_k)) then
67:
68:
            continue;
        end if
69:
        if (G_{-}tltl_{k} \geq G_{-}tutl_{i}) then /* Ensure that the limits do not cross for both T_{i} & T_{k} */
70:
             if (G\_state_k == live) then /* Check if T_k is live */
71:
72:
                 if (G_{-}its_i < G_{-}its_k) then
                     /* Transaction T_k has lower priority and is not yet committed. So it needs to be aborted */
73:
74:
                     abortRL = abortRL \cup T_k; /* Store T_k in abortRL */
75:
                 else/* Transaction T_i has to be aborted */
                     return abort(i);
76:
                 end if/* (G_{-its_i} < G_{-its_k}) */
77:
78:
            else/* (T_k is committed. Hence, T_i has to be aborted) */
79:
                 return abort(i);
            end if/* (G_{-}state_k == live) */
80:
        end if/* (G_{-}tltl_k \geq G_{-}tutl_i) */
81:
82: end for(T_k \in smallRL)
83: /* After this point T_i can't abort. */
84: G_{-}tltl_{i} = G_{-}tutl_{i};
85: /* Since T_i can't abort, we can update T_k's G_tutl */
86: for all (T_k \in smallRL) do
        if (isAborted(T_k)) then
87:
             continue;
88:
89:
        end if
        /* The following line ensure that G_{-}tltl_{k} \leq G_{-}tutl_{k} < G_{-}tltl_{i}. Note that this does not cause the limits of
90:
    T_k to cross each other because of the check in Line 70.*/
        G_{-}tutl_{k} = min(G_{-}tutl_{k}, G_{-}tltl_{i} - 1);
91:
93: for all T_k \in abortRL do /* Abort all the transactions in abortRL since T_i can't abort */
        G-valid_k = F;
94.
95: end for
96: /* Having completed all the checks, T_i can be committed */
97: for all (x \in wset_i) do
        /* Create new v_tuple: ts, val, rl, vrt for x */
98:
        newTuple = \langle G\_wts_i, wset_i[x].val, nil, G\_tltl_i \rangle;
99:
         if (|x.vl| > k) then
100:
             replace the oldest tuple in x.vl with newTuple; /* x.vl is ordered by ts */
101:
         else
102:
103:
             add a newTuple to x.vl in sorted order;
         end if
105: end for/* x \in wset_i */
106: G_{-}state_{i} = commit;
107: unlock all variables;
108: return \mathscr{C};
```

**Algorithm 22** abort(i): Invoked by various STM methods to abort transaction  $T_i$ . It returns  $\mathscr{A}$ 

- 1:  $G_{-}valid_{i} = F$ ;  $G_{-}state_{i} = abort$ ;
- 2: unlock all variables locked by  $T_i$ ;
- 3: return  $\mathscr{A}$ ;

We get the following nice properties on KSFTM. For simplicity, we assumed C and incrVal to be 0.1 and 1 respectively in our analysis. But the proof and the analysis holds for any value greater than 0.

**Theorem 7** Any history generated by KSFTM is strict-serializable and locally-opaque.

**Theorem 8** KSFTM algorithm ensures starvation-freedom.

As explained in the description Property 6, the proof of this property is somewhat involved. As expected, this proof can be extended to *UVSFTM* as well.

**Garbage Collection:** Having described the *starvation-free* algorithm, we now describe how garbage collection can be performed on the unbounded variant, UVSFTM to achieve UVSFTM-GC. This is achieved by deleting non-latest version (i.e., there exists a version with greater ts) of each t-object whose timestamp, ts is less than the CTS of smallest live transaction. It must be noted that UVSFTM (KSFTM) works with WTS which is greater or equal to CTS for any transaction. Interestingly, the same garbage collection principle can be applied for PMVTO to achieve PMVTO-GC.

To identify the transaction with the smallest CTS among live transactions, we maintain a set of all the live transactions, *live-list*. When a transaction  $T_i$  begins, its CTS is added to this *live-list*. And when  $T_i$  terminates (either commits or aborts),  $T_i$  is deleted from this *live-list*.

# 4 Experimental Evaluation

For performance evaluation of KSFTM with the state-of-the-art STMs, we implemented the the algorithms PKTO, SV-SFTM [9, 27, 26] along with KSFTM in C++  $^2$ . We used the available implementations of NOrec STM [6], and ESTM [7] developed in C++. Although, only KSFTM and SV-SFTM provide starvation-freedom, we compared with other STMs as well, to see its performance in practice.

**Experimental system:** The experimental system is a 2-socket Intel(R) Xeon(R) CPU E5-2690 v4 @ 2.60GHz with 14 cores per socket and 2 hyper-threads (HTs) per core, for a total of 56 threads. Each core has a private 32KB L1 cache and 256 KB L2 cache. The machine has 32GB of RAM and runs Ubuntu 16.04.2 LTS. In our implementation, all threads have the same base priority and we use the default Linux scheduling algorithm. This satisfies the Assumption 1 (bounded-termination) about the scheduler. We ensured that there no parasitic transactions [3] in our experiments.

**Methodology:** Here we have considered two different applications:(1) Counter application - In this, each thread invokes a single transaction which performs 10 reads/writes operations on randomly chosen t-objects. A thread continues to invoke a transaction until it successfully commits. To obtain high contention, we have taken large number of threads ranging from 50-250 where each thread performs its read/write operation over a set of 5 t-objects. We have performed our tests on three workloads stated as: (W1) Li - Lookup intensive: 90% read, 10% write, (W2) Mi - Mid intensive: 50% read, 50% write and (W3) Ui - Update intensive: 10% read, 90% write. This application is undoubtedly very flexible as it allows us to examine performance by tweaking different parameters (refer to SubSection 4.1 for details). (2) Two benchmarks from STAMP suite [21] - (a) We considered KMEANS which has low contention with short running transactions. The number of data points as 2048 with 16 dimensions and total clusters as 5. (b) We then considered LABYRINTH which has high contention with long running transactions. We considered the grid size as 64x64x3 and paths to route as 48.

To study starvation in the various algorithms, we considered *max-time*, which is the maximum time taken by a transaction among all the transactions in a given experiment to commit from its first invocation. This includes time taken by all the aborted incarnations of the transaction to execute as well. To reduce the effect of outliers, we took the average of max-time in ten runs as the final result for each application.

**Results Analysis:** Fig 8 illustrates max-time analysis of KSFTM over the above mentioned STMs for the counters application under the workloads W1, W2 and W3 while varying the number of threads from 50 to 250. For KSFTM and PKTO, we chose the value of K as 5 and C as 0.1 as the best results were obtained with these

<sup>&</sup>lt;sup>2</sup>Code is available here: https://github.com/PDCRL/KSFTM

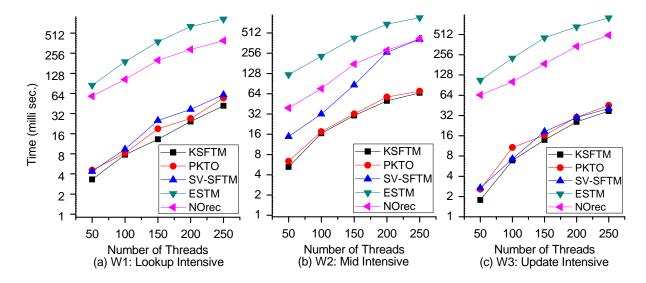


Figure 8: Performance analysis on workload W1, W2, W3

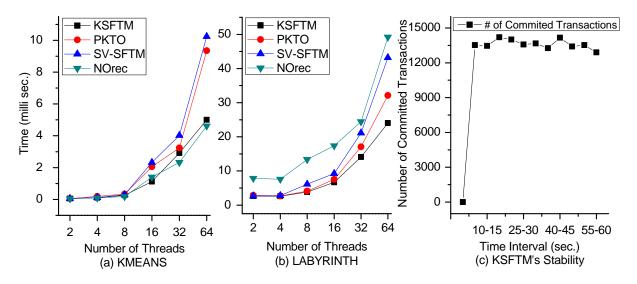


Figure 9: Performance analysis on KMEANS, LABYRINTH and KSFTM's Stability

parameters. We can see that *KSFTM* performs the best for all the three workloads. *KSFTM* gives an average speedup on max-time by a factor of 1.22, 1.89, 23.26 and 13.12 over *PKTO*, *SV-SFTM*, NOrec STM and ESTM respectively.

Fig 9(a) shows analysis of max-time for KMEANS while Fig 9(b) shows for LABYRINTH. In this analysis we have not considered ESTM as the integrated STAMP code for ESTM is not publicly available. For KMEANS, *KSFTM* performs 1.5 and 1.44 times better than *PKTO* and *SV-SFTM*. But, NOrec is performing 1.09 times better than *KSFTM*. This is because KMEANS has short running transactions have low contention. As a result, the commit time of the transactions is also low.

On the other hand for LABYRINTH, *KSFTM* again performs the best. It performs 1.14, 1.4 and 2.63 times better than *PKTO*, *SV-SFTM* and NOrec respectively. This is because LABYRINTH has high contention with long running transactions. This result in longer commit times for transactions.

Fig 9(c) shows the stability of KSFTM algorithm over time for the counter application. Here we fixed the number of threads to 32, K as 5, C as 0.1, t-objects as 1000, along with 5 seconds warm-up period on W1 workload. Each thread invokes transactions until its time-bound of 60 seconds expires. We performed the experiments on number of transactions committed over time in the increments 5 seconds. The experiment shows that over time KSFTM is stable which helps to hold the claim that KSFTM's performance will continue in same manner if time is increased to higher orders.

Maintaining multiple versions to increase the performance and to decrease the number of aborts, leads to

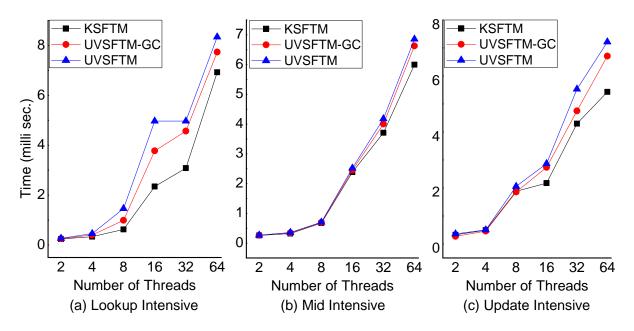


Figure 10: Time comparison among variants of KSFTM

creating too many versions which are not of any use and hence occupying space. So, such garbage versions need to be taken care of. Hence we come up with a garbage collection over these unwanted versions. This technique help to conserve memory space and increases the performance in turn as no more unnecessary traversing of garbage versions by transactions is necessary. We have used a global, i.e., across all transactions a list that keeps track of all the live transactions in the system. We call this list as *live-list*. Each transaction at the beginning of its life cycle creates its entry in this *live-list*. Under the optimistic approach of STM, each transaction in the shared memory performs its updates in the stm-tryC phase. In this phase, each transaction performs some validations, and if all the validations are successful then the transaction make changes or in simple terms creates versions of the corresponding t-object in the shared memory. While creating a version every transaction, check if it is the least timestamp live transaction present in the system by using *live-list* data structure, if yes then the current transaction deletes all the version of that t-object and create one of its own. Else the transaction does not do any garbage collection or delete any version and look for creating a new version of next t-object in the write set, if at all.

Fig 10 represents three variants of KSFTM (UVSFTM, UVSFTM-GC, and KSFTM) and Fig 11 shows the three variants of PKTO (PMVTO, PMVTO-GC, and PKTO) on all the workloads W1 W2 and W3. KSFTM outperforms UVSFTM and UVSFTM-GC by a factor of 2.1 and 1.5. Similarly, PKTO outperforms PMVTO and PMVTO-GC by a factor of 2 and 1.35. These results show that maintaining finite versions corresponding to each t-object performs better than maintaining infinite versions and garbage collection on infinite versions corresponding to each t-object.

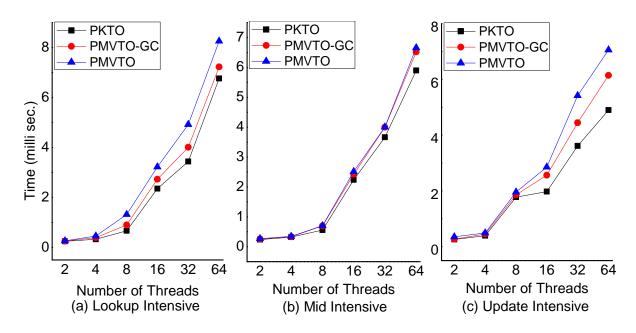


Figure 11: Time comparison among variants of PKTO

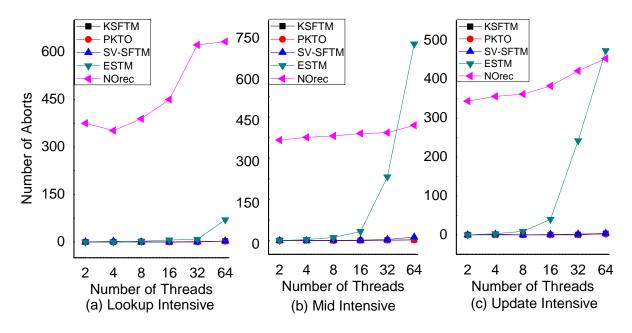


Figure 12: Abort Count on workload W1, W2, W3

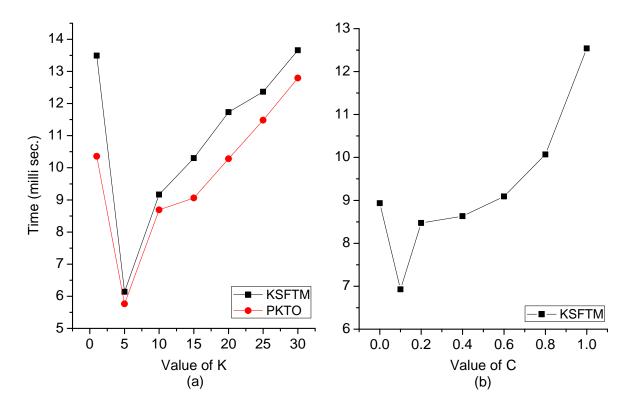


Figure 13: Best value of K and optimal value of C for KSFTM

Comparison on the basis of Abort count: Fig 12 shows the abort count comparisons of KSFTM with PKTO, ESTM, NOrec, MVTO, and SV-SFTM across all workloads (W1, W2, and W3). The number of aborts in ESTM and NOrec are high as compared to all other STM algorithms while all other algorithms (KSFTM, PKTO, MVTO, SV-SFTM) have marginally small differences among them.

Best value of K and optimal value of constant C: To identify the best value of K for KSFTM, we ran our experiment, varying value of K and keeping the number of threads as 64 on workload W1 and obtained the optimal value of K in KSFTM is 5 as shown in Fig 13.(a) for counter application. Similarly, we calculate the best value of K as 5 for PKTO on the same parameters. C, is a constant that is used to calculate WTS of a transaction. i.e.,  $wts_i = cts_i + C * (cts_i - its_i)$ ; where, C is any constant greater than 0. We run or experiments across load W1, for 64 threads and other parameters are same as defined in the methodology of Section 4, we achieve the best value of C as 0.1 for counter application. Experimental results are shown in Fig 13 (b).

#### 4.1 Pseudo code of Counter Application

OP\_LT\_SEED is defined as number of operations per transaction, T\_OBJ\_SEED is defined as number of transaction objects in the system, TRANS\_LT defines the total number of transactions to be executed in the system, and READ\_PER is the percentage of read operation which is used to define various workloads.

#### Algorithm 24 testFunc\_helper():Function invoked by threads

- 1:  $transaction\_count = 0$
- 2: while (TRANS\_LT) do
- 3: /\* Log the time at the start of every transaction \*/
- 4:  $begin\_time = time\_request()$
- 5: /\* Invoke the test function to execute a transaction \*/
- 6:  $abort\_count[thread\_id] = test\_function()$
- 7:  $transaction\_count + +$

#### **Algorithm 23** main(): The main procedure invoked by counter application

```
1: /* To log abort counts by each thread */
2: abort_count[NUMTHREADS]
3: /* To log average time taken by each transaction to commit */
4: time_taken[NUMTHREADS]
5: /* To log the time of longest running transaction by each thread, worst case time */
6: worst_time[NUMTHREADS]
7: for (i = 0 : NUMTHREADS) do
      pthread\_create(\&threads[i], NULL, testFunc\_helper, (void*)args)
9: end for
10: for (i = 0 : NUMTHREADS) do
      pthread_join(threads[i], &status)
12: end for
13: max\_worst\_time = 0.0
14: total\_abort\_count = 0
15: average\_time_taken = 0
16: for (i = 0 : NUMTHREADS) do
      if (max\_worst\_time < worst\_time[i]) then
17:
18:
          max\_worst\_time = worst\_time[i]
19:
      end if
      total\_abort\_count+ = abort\_count[i]
20:
21:
      average\_time\_taken+ = time\_taken[i]
22: end for
```

```
8: /* Log the time at the end of every transaction */
9: end_time = time_request()
10: time_taken[thread_id] + = (end_time - begin_time)
11: if (worst_time[thread_id] < (end_time - begin_time)) then
12: worst_time[thread_id] = (end_time - begin_time)
13: end if
14: TRANS_LT -= 1
15: end while
16: time_taken[thread_id] /= transaction_count
```

#### **Algorithm 25** *test\_function()*:main test function while executes a transaction

```
1: Transaction *T = new Transaction;
 2: T \rightarrow g_{-}its = NIL
 3: local\_abort\_count = 0
 4: label:
 5: while (true) do
        if (T \rightarrow g\_its != NIL) then
 6:
            its = T \rightarrow g\_its
 7:
            T = lib \rightarrow stm\text{-}begin(its)
 8:
 9:
        else
10:
            T = lib \rightarrow stm\text{-}begin(T \rightarrow g\_its)
        end if
11:
        for all (OP_LT_SEED) do
12:
            t\_obj = rand()\%T\_OBJ\_SEED
13:
            randVal = rand()\%OP\_SEED
14:
            if (randVal \le READ\_PER) then
15:
                stm-read(t\_obj, value)
                if (value == ABORTED) then
17:
                    local\_abort\_count++
18:
19:
                    goto label
                end if
20:
21:
                stm-write(t\_obj, value)
22:
            end if
23:
24:
        end for
25:
        if (lib \rightarrow stm\text{-}tryC() == ABORTED) then
            local\_abort\_count +\!+
26:
            continue
27:
        end if
28:
29.
        break
30: end while
```

# 5 Graph Characterization of Local Opacity & KSFTM Correctness

To prove correctness of STM systems, it is useful to consider graph characterization of histories. In this section, we describe the graph characterization developed by Kumar et al [17] for proving opacity which is based on characterization by Bernstein and Goodman [2]. We extend this characterization for LO.

Consider a history H which consists of multiple versions for each t-object. The graph characterization uses the notion of version order. Given H and a t-object x, we define a version order for x as any (non-reflexive) total order on all the versions of x ever created by committed transactions in H. It must be noted that the version order may or may not be the same as the actual order in which the version of x are generated in x. A version order of x, denoted as x0 is the union of the version orders of all the t-objects in x1.

Consider the history  $H2: r_1(x,0)r_2(x,0)r_1(y,0)r_3(z,0)w_1(x,5)w_3(y,15)w_2(y,10)w_1(z,10)$   $c_1c_2r_4(x,5)r_4(y,10)w_3(z,15)c_3r_4(z,10)$ . Using the notation that a committed transaction  $T_i$  writing to x creates a version  $x_i$ , a possible version order for  $H2 \ll_{H2}$  is:  $\langle x_0 \ll x_1 \rangle, \langle y_0 \ll y_2 \ll y_3 \rangle, \langle z_0 \ll z_1 \ll z_3 \rangle$ .

We define the graph characterization based on a given version order. Consider a history H and a version order  $\ll$ . We then define a graph (called opacity graph) on H using  $\ll$ , denoted as  $OPG(H, \ll) = (V, E)$ . The vertex set V consists of a vertex for each transaction  $T_i$  in  $\overline{H}$ . The edges of the graph are of three kinds and are defined as follows:

- 1. real-time(real-time) edges: If  $T_i$  commits before  $T_j$  starts in H, then there is an edge from  $v_i$  to  $v_j$ . This set of edges are referred to as rt(H).
- 2. rf(reads-from) edges: If  $T_j$  reads x from  $T_i$  in H, then there is an edge from  $v_i$  to  $v_j$ . Note that in order for this to happen,  $T_i$  must have committed before  $T_j$  and  $c_i <_H r_j(x)$ . This set of edges are referred to as

rf(H).

3. mv(multiversion) edges: The mv edges capture the multiversion relations and is based on the version order. Consider a successful read operation  $r_k(x,v)$  and the write operation  $w_j(x,v)$  belonging to transaction  $T_j$  such that  $r_k(x,v)$  reads x from  $w_j(x,v)$  (it must be noted  $T_j$  is a committed transaction and  $c_j <_H r_k$ ). Consider a committed transaction  $T_i$  which writes to x,  $w_i(x,u)$  where  $u \neq v$ . Thus the versions created  $x_i, x_j$  are related by  $\ll$ . Then, if  $x_i \ll x_j$  we add an edge from  $v_i$  to  $v_j$ . Otherwise  $(x_j \ll x_i)$ , we add an edge from  $v_k$  to  $v_i$ . This set of edges are referred to as  $mv(H, \ll)$ .

Using the construction, the  $OPG(H2, \ll_{H2})$  for history H2 and  $\ll_{H2}$  is shown in Fig 14. The edges are annotated. The only mv edge from T4 to T3 is because of t-objects y, z. T4 reads value 5 for z from T1 whereas T3 also writes 15 to z and commits before  $r_4(z)$ .

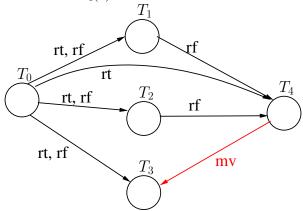


Figure 14:  $OPG(H2, \ll_{H2})$ 

Kumar et al [17] showed that if a version order  $\ll$  exists for a history H such that  $OPG(H, \ll_H)$  is acyclic, then H is opaque. This is captured in the following result.

**Result 9** A valid history H is opaque iff there exists a version order  $\ll_H$  such that  $OPG(H, \ll_H)$  is acyclic.

This result can be easily extended to prove LO as follows

**Theorem 10** A valid history H is locally-opaque iff for each sub-history sh in H.subhistSet there exists a version order  $\ll_{sh}$  such that  $OPG(sh, \ll_{sh})$  is acyclic. Formally,  $\langle (H \text{ is locally-opaque}) \Leftrightarrow (\forall sh \in H.\text{subhist}Set, \exists \ll_{sh}: OPG(sh, \ll_{sh}) \text{ is acyclic}) \rangle$ .

**Proof.** To prove this theorem, we have to show that each sub-history sh in H.subhistSet is valid. Then the rest follows from Result 9. Now consider a sub-history sh. Consider any read operation  $r_i(x,v)$  of a transaction  $T_i$ . It is clear that  $T_i$  must have read a version of x created by a previously committed transaction. From the construction of sh, we get that all the transaction that committed before  $r_i$  are also in sh. Hence sh is also valid.

Now, proving sh to be opaque iff there exists a version order  $\ll_{sh}$  such that  $OPG(sh, \ll_{sh})$  is acyclic follows from Result 9.

**Lemma 11** Consider a history H in gen(KSFTM) with two transactions  $T_i$  and  $T_j$  such that both their G-valid flags are true, there is an edge from  $T_i \to T_j$  then G-tlt $l_i < G$ -tlt $l_j$ .

**Proof.** There are three types of possible edges in MVSG.

- 1. Real-time edge: Since, transaction  $T_i$  and  $T_j$  are in real time order so  $comTime_i < G\_cts_j$ . As we know from Lemma 36  $(G\_tltl_i \leq comTime_i)$ . So,  $(G\_tltl_i \leq CTS_j)$ . We know from STM stm-begin(its) method,  $G\_tltl_j = G\_cts_j$ . Eventually,  $G\_tltl_i < G\_tltl_j$ .
- 2. Read-from edge: Since, transaction  $T_i$  has been committed and  $T_j$  is reading from  $T_i$  so, from Line 99  $stm\text{-}tryC(T_i)$ ,  $G\text{-}tltl_i = \text{vrt}_i$ . and from Line 20 STM read(j,x),  $G\text{-}tltl_j = max(G\text{-}tltl_j)$ ,  $x[curVer].\text{vrt} + 1) \Rightarrow (G\text{-}tltl_j > \text{vrt}_i) \Rightarrow (G\text{-}tltl_j > G\text{-}tltl_i)$  Hence,  $G\text{-}tltl_i < G\text{-}tltl_j$ .

- 3. Version-order edge: Consider a triplet  $w_j(x_j)r_k(x_j)w_i(x_i)$  in which there are two possibilities of version order:
  - (a)  $i \ll j \Longrightarrow G_-wts_i < G_-wts_j$

There are two possibilities of commit order:

- i.  $comTime_i <_H comTime_j$ : Since,  $T_i$  has been committed before  $T_j$  so  $G\_tltl_i = \texttt{vrt}_i$ . From Line 53 of  $stm\_tryC(T_j)$ ,  $\texttt{vrt}_i < G\_tltl(j)$ . Hence,  $G\_tltl_i < G\_tltl_j$ .
- ii.  $comTime_j <_H comTime_i$ : Since,  $T_j$  has been committed before  $T_i$  so  $G\_tltl_j = \mathtt{vrt}_j$ . From Line 58 of  $stm\_tryC(T_i)$ ,  $G\_tutl_i < \mathtt{vrt}_j$ . As we have assumed  $G\_valid_i$  is true so definitely it will execute the Line 84  $stm\_tryC(T_i)$  i.e.  $G\_tltl_i = G\_tutl_i$ . Hence,  $G\_tltl_i < G\_tltl_j$ .
- (b)  $j \ll i \Longrightarrow G_{-}wts_{i} < G_{-}wts_{i}$

Again, there are two possibilities of commit order:

- i.  $comTime_j <_H comTime_i$ : Since,  $T_j$  has been committed before  $T_i$  and  $T_k$  read from  $T_j$ . There can be two possibilities  $G_-wts_k$ .
  - A.  $G\_wts_k > G\_wts_i$ : That means  $T_k$  is in largeRL of  $T_i$ . From Line 45 to Line 47of stm-tryC(i), either transaction  $T_k$  or  $T_i$ ,  $G\_valid$  flag is set to be false. If  $T_i$  returns abort then this case will not be considered in Lemma 11. Otherwise, as  $T_j$  has already been committed and later  $T_i$  will execute the Line 99  $stm-tryC(T_i)$ , Hence,  $G\_tltl_j < G\_tltl_i$ .
  - B.  $G\_wts_k < G\_wts_i$ : That means  $T_k$  is in smallRL of  $T_i$ . From Line 17 of read(k,x),  $G\_tutl_k < vrt_i$  and from Line 20 of read(k,x),  $G\_tltl_k > vrt_j$ . Here,  $T_j$  has already been committed so,  $G\_tltl_j = vrt_j$ . As we have assumed  $G\_valid_i$  is true so definitely it will execute the Line 99  $stm\_tryC(T_i)$ ,  $G\_tltl_i = vrt_i$ . So,  $G\_tutl_k < G\_tltl_i$  and  $G\_tltl_k > G\_tltl_j$ . While considering  $G\_valid_k$  flag is true  $\to G\_tltl_k < G\_tutl_k$ .

Hence,  $G\_tltl_j < G\_tltl_k < G\_tutl_k < G\_tltl_i$ .

Therefore,  $G_{-}tltl_{i} < G_{-}tltl_{k} < G_{-}tltl_{i}$ .

ii.  $comTime_i <_H comTime_j$ : Since,  $T_i$  has been committed before  $T_j$  so,  $G.tltl_i = \mathtt{vrt}_i$ . From Line 58 of  $stm-tryC(T_j)$ ,  $G.tutl_j < \mathtt{vrt}_i$  i.e.  $G.tutl_j < G.tltl_i$ . Here,  $T_k$  read from  $T_j$ . So, From Line 17 of read(k,x),  $G.tutl_k < \mathtt{vrt}_i \to G.tutl_k < G.tltl_i$  from Line 20 of read(k,x),  $G.tltl_k > \mathtt{vrt}_j$ . As we have assumed  $G.valid_j$  is true so definitely it will execute the Line 99  $stm-tryC(T_j)$ ,  $G.tltl_j = \mathtt{vrt}_j$ .

Hence,  $G_{-}tltl_{j} < G_{-}tltl_{k} < G_{-}tutl_{k} < G_{-}tltl_{i}$ .

Therefore,  $G_{-}tltl_{i} < G_{-}tltl_{k} < G_{-}tltl_{i}$ .

**Theorem 12** Any history H gen(KSFTM) is local opaque iff for a given version order  $\ll H$ ,  $MVSG(H, \ll)$  is acyclic.

**Proof.** We are proving it by contradiction, so Assuming MVSG(H, $\ll$ ) has cycle. From Lemma 11, For any two transactions  $T_i$  and  $T_j$  such that both their G\_valid flags are true and if there is an edge from  $T_i \to T_j$  then  $G\_tltl_i$ . While considering transitive case for k transactions  $T_1, T_2, T_3...T_k$  such that G\_valid flags of all the transactions are true. if there is an edge from  $T_1 \to T_2 \to T_3 \to .... \to T_k$  then  $G\_tltl_1 < G\_tltl_2 < G\_tltl_3 < .... < G\_tltl_k$ .

Now, considering our assumption, MVSG(H, $\ll$ ) has cycle so,  $T_1 \rightarrow T_2 \rightarrow T_3 \rightarrow .... \rightarrow T_k \rightarrow T_1$  that implies  $G\_tltl_1 < G\_tltl_2 < G\_tltl_3 < .... < G\_tltl_k < G\_tltl_1$ .

Hence from above assumption,  $G_{-}tltl_1 < G_{-}tltl_1$  but this is impossible. So, our assumption is wrong. Therefore, MVSG(H, $\ll$ ) produced by KSFTM is acyclic.

 $M\_Order_H$ : It stands for method order of history H in which methods of transactions are interval (consists of invocation and response of a method) instead of dot (atomic). Because of having method as an interval, methods of different transactions can overlap. To prove the correctness (*local opacity*) of our algorithm, we need to order the overlapping methods.

Let say, there are two transactions  $T_i$  and  $T_j$  either accessing common (t-objects/G\_lock) or G\_tCntr through operations  $op_i$  and  $op_j$  respectively. If  $res(op_i) <_H inv(op_j)$  then  $op_i$  and  $op_j$  are in real-time order in H. So, the M\_Order\_H is  $op_i \to op_j$ .

If operations are overlapping and either accessing common t-objects or sharing Glock:

- 1.  $read_i(x)$  and  $read_j(x)$ : If  $read_i(x)$  acquires the lock on x before  $read_j(x)$  then the  $M\_Order_H$  is  $op_i \rightarrow op_j$ .
- 2.  $read_i(x)$  and  $stm-tryC_j()$ : If they are accessing common t-objects then, let say  $read_i(x)$  acquires the lock on x before  $stm-tryC_j()$  then the  $M\_Order_H$  is  $op_i \to op_j$ . Now if they are not accessing common t-objects but sharing  $G\_lock$  then, let say  $read_i(x)$  acquires the lock on  $G\_lock_i$  before  $stm-tryC_j()$  acquires the lock on relLL (which consists of  $G\_lock_i$  and  $G\_lock_i$ ) then the  $M\_Order_H$  is  $op_i \to op_j$ .
- 3.  $stm-tryC_i()$  and  $stm-tryC_j()$ : If they are accessing common t-objects then, let say  $stm-tryC_i()$  acquires the lock on x before  $stm-tryC_j()$  then the  $M\_Order_H$  is  $op_i \to op_j$ . Now if they are not accessing common t-objects but sharing  $G\_lock$  then, let say  $stm-tryC_i()$  acquires the lock on  $relLL_i$  before  $stm-tryC_j()$  then the  $M\_Order_H$  is  $op_i \to op_j$ .

If operations are overlapping and accessing different t-objects but sharing  $G_{-t}Cntr$  counter:

- 1. stm- $begin_i$  and stm- $begin_j$ : Both the stm-begin are accessing shared counter variable G-tCntr. If stm- $begin_i$  executes G-tCntr. get&Inc() before stm- $begin_i$  then the M- $Order_H$  is  $op_i o op_i$ .
- 2. stm-begin<sub>i</sub> and stm-tryC(j): If stm-begin<sub>i</sub> executes G-tCntr.get&Inc() before stm-tryC(j) then the M-Order<sub>H</sub> is  $op_i \rightarrow op_j$ .

Linearization: The history generated by STMs are generally not sequintial because operations of the transactions are overlapping. The correctness of STMs is defined on sequintial history, inorder to show history generated by our algorithm is correct we have to consider sequintial history. We have enough information to order the overlapping methods, after ordering the operations will have equivalent sequintial history, the total order of the operation is called linearization of the history.

Operation graph (OPG): Consider each operation as a vertex and edges as below:

- 1. Real time edge: If response of operation  $op_i$  happen before the invocation of operation  $op_j$  i.e.  $rsp(op_i) <_H inv(op_j)$  then there exist real time edge between  $op_i \to op_j$ .
- 2. Conflict edge: It is based on  $L_{-}Order_{H}$  which depends on three conflicts:
  - (a) Common *t-object*: If two operations  $op_i$  and  $op_j$  are overlapping and accessing common *t-object x*. Let say  $op_i$  acquire lock first on x then  $L_Order.op_i(x) <_H L_Order.op_j(x)$  so, conflict edge is  $op_i \to op_j$ .
  - (b) Common  $G\_valid$  flag: If two operation  $op_i$  and  $op_j$  are overlapping but accessing common  $G\_valid$  flag instead of t-object. Let say  $op_i$  acquire lock first on  $G\_valid_i$  then  $L\_Order.op_i(\mathbf{x}) <_H L\_Order.op_j(\mathbf{x})$  so, conflict edge is  $op_i \to op_j$ .
- 3. Common  $G\_tCntr$  counter: If two operation  $op_i$  and  $op_j$  are overlapping but accessing common  $G\_tCntr$  counter instead of t-object. Let say  $op_i$  access  $G\_tCntr$  counter before  $op_j$  then  $L\_Order.op_i(x) <_H$   $L\_Order.op_j(x)$  so, conflict edge is  $op_i \to op_j$ .

**Lemma 13** All the locks in history  $H(L\_Order_H)$  gen(KSFTM) follows strict partial order. So, operation graph (OPG(H)) is acyclic. If  $(op_i \rightarrow op_j)$  in OPG, then atleast one of them will definitely true:  $(Fpu_i(\alpha) < Lpl\_op_j(\alpha)) \cup (access.G\_tCntr_i < access.G\_tCntr_j) \cup (access.G\_tCntr_j) \cup (access.G\_tCntr_i < Lpl\_op_j(\alpha))$ . Here,  $\alpha$  can either be t-object or  $G\_valid$ .

**Proof.** we consider proof by induction, So we assummed there exist a path from  $op_1$  to  $op_n$  and there is an edge between  $op_n$  to  $op_{n+1}$ . As we described, while constructing OPG(H) we need to consider three types of edges. We are considering one by one:

- 1. Real time edge between  $op_n$  to  $op_{n+1}$ :
  - (a)  $op_{n+1}$  is a locking method: In this we are considering all the possible path between  $op_1$  to  $op_n$ :

```
i. (Fu\_op_1(\alpha) < Ll\_op_n(\alpha)): Here, (Fu\_op_n(\alpha) < Ll\_op_{n+1}(\alpha)). So, (Fu\_op_1(\alpha) < Ll\_op_n(\alpha)) < (Fu\_op_n(\alpha) < Ll\_op_{n+1}(\alpha)) Hence, (Fu\_op_1(\alpha) < Ll\_op_{n+1}(\alpha))
```

ii.  $(Fu\_op_1(\alpha) < Ll\_op_n(\alpha))$ : Here,  $(access.G\_tCntr_n < Ll\_op_{n+1}(\alpha))$ . As we know if any method is locking as well as accessing common counter then locking tobject first then accessing the counter after that unlocking tobject i.e.

```
So, (Ll\_op_n(\alpha)) < (access.G\_tCntr_n) < (Fu\_op_n(\alpha)).
Hence, (Fu\_op_1(\alpha) < Ll\_op_{n+1}(\alpha))
```

iii.  $(access.G\_tCntr_1) < (access.G\_tCntr_n)$ : Here,  $(access.G\_tCntr_n) < Ll\_op_{n+1}(\alpha)$ ). So,  $(access.G\_tCntr_1) < (access.G\_tCntr_n) < Ll\_op_{n+1}(\alpha)$ ). Hence,  $(access.G\_tCntr_1) < Ll\_op_{n+1}(\alpha)$ ).

iv.  $(Fu\_op_1(\alpha) < (access.G\_tCntr_n)$ : Here,  $(access.G\_tCntr_n) < Ll\_op_{n+1}(\alpha)$ ). So,  $(Fu\_op_1(\alpha) < (access.G\_tCntr_n) < Ll\_op_{n+1}(\alpha)$ ). Hence,  $(Fu\_op_1(\alpha) < Ll\_op_{n+1}(\alpha))$ 

v.  $(access.G\_tCntr_1) < Ll\_op_n(\alpha))$ : Here,  $(Fu\_op_n(\alpha) < Ll\_op_{n+1}(\alpha))$ . So,  $(access.G\_tCntr_1) < Ll\_op_n(\alpha)) < (Fu\_op_n(\alpha) < Ll\_op_{n+1}(\alpha))$ . Hence,  $(access.G\_tCntr_1) < Ll\_op_{n+1}(\alpha))$ .

vi.  $(access.G\_tCntr_1) < Ll\_op_n(\alpha)$ ): Here,  $(access.G\_tCntr_n < Ll\_op_{n+1}(\alpha))$ . As we know if any method is locking as well as accessing common counter then locking tobject first then accessing the counter after that unlocking tobject i.e.

```
So, (Ll\_op_n(\alpha)) < (access.G\_tCntr_n) < (Fu\_op_n(\alpha)). Hence, (access.G\_tCntr_1) < Ll\_op_{n+1}(\alpha)).
```

- (b)  $op_{n+1}$  is a non-locking method: Again, we are considering all the possible path between  $op_1$  to  $op_n$ :
  - i.  $(Fu\_op_1(\alpha) < Ll\_op_n(\alpha))$ : Here,  $(access.G\_tCntr_n) < (access.G\_tCntr_{n+1})$ . As we know if any method is locking as well as accessing common counter then locking tobject first then accessing the counter after that unlocking tobject i.e.

So,  $(Ll\_op_n(\alpha)) < (access.G\_tCntr_n) < (Fu\_op_n(\alpha))$ . Hence,  $(Fu\_op_1(\alpha) < (access.G\_tCntr_{n+1})$ 

- ii.  $(Fu\_op_1(\alpha) < Ll\_op_n(\alpha))$ : Here,  $(Fu\_op_n(\alpha) < (access.G\_tCntr_{n+1})$ . So,  $(Fu\_op_1(\alpha) < Ll\_op_n(\alpha)) < (Fu\_op_n(\alpha) < (access.G\_tCntr_{n+1})$ Hence,  $(Fu\_op_1(\alpha) < (access.G\_tCntr_{n+1}))$
- iii.  $(access.G\_tCntr_1) < (access.G\_tCntr_n)$ : Here,  $(access.G\_tCntr_n) < (access.G\_tCntr_{n+1})$ . So,  $(access.G\_tCntr_1) < (access.G\_tCntr_n) < (access.G\_tCntr_{n+1})$ . Hence,  $(access.G\_tCntr_1) < (access.G\_tCntr_{n+1})$ .
- iv.  $(Fu\_op_1(\alpha) < (access.G\_tCntr_n)$ : Here,  $(access.G\_tCntr_n) < (access.G\_tCntr_{n+1})$ . So,  $(Fu\_op_1(\alpha) < (access.G\_tCntr_n) < (access.G\_tCntr_{n+1})$ . Hence,  $(Fu\_op_1(\alpha) < (access.G\_tCntr_{n+1})$
- v.  $(access.G\_tCntr_1) < Ll\_op_n(\alpha)$ ): Here,  $(access.G\_tCntr_n) < (access.G\_tCntr_{n+1})$ . As we know if any method is locking as well as accessing common counter then locking tobject first then accessing the counter after that unlocking tobject i.e.

So,  $(Ll\_op_n(\alpha)) < (access.G\_tCntr_n) < (Fu\_op_n(\alpha))$ . Hence,  $(access.G\_tCntr_1) < (access.G\_tCntr_{n+1})$ .

vi.  $(access.G\_tCntr_1) < Ll\_op_n(\alpha)$ ): Here,  $(Fu\_op_n(\alpha) < (access.G\_tCntr_{n+1})$ . So,  $(access.G\_tCntr_1) < Ll\_op_n(\alpha)) < (Fu\_op_n(\alpha) < (access.G\_tCntr_{n+1})$ . Hence,  $(access.G\_tCntr_1) < (access.G\_tCntr_{n+1})$ .

- 2. Conflict edge between  $op_n$  to  $op_{n+1}$ :
  - (a)  $(Fu\_op_1(\alpha) < Ll\_op_n(\alpha))$ : Here,  $(Fu\_op_n(\alpha) < Ll\_op_{n+1}(\alpha))$ . Ref 1.(a).i.
  - (b)  $(access.G\_tCntr_1) < (access.G\_tCntr_n)$ : Here,  $(Fu\_op_n(\alpha) < Ll\_op_{n+1}(\alpha))$ . As we know if any method is locking as well as accessing common counter then locking tobject first then accessing the counter after that unlocking tobject i.e.

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So, (Ll\_op_n(\alpha)) < (access.G\_tCntr_n) < (Fu\_op_n(\alpha)). Hence, (access.G\_tCntr_1) < Ll\_op_{n+1}(\alpha)).
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(c)  $(Fu\_op_1(\alpha) < (access.G\_tCntr_n)$ : Here,  $(Fu\_op_n(\alpha) < Ll\_op_{n+1}(\alpha))$ . As we know if any method is locking as well as accessing common counter then locking tobject first then accessing the counter after that unlocking tobject i.e.

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So, (Ll\_op_n(\alpha)) < (access.G\_tCntr_n) < (Fu\_op_n(\alpha)).
Hence, (Fu\_op_1(\alpha) < Ll\_op_{n+1}(\alpha)).
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- (d)  $(access.G\_tCntr_1) < Ll\_op_n(\alpha)$ ): Here,  $(Fu\_op_n(\alpha) < Ll\_op_{n+1}(\alpha))$ . Ref 1.(a).v.
- 3. Common counter edge between  $op_n$  to  $op_{n+1}$ :
  - (a)  $(Fu\_op_1(\alpha) < Ll\_op_n(\alpha))$ : Here,  $(access.G\_tCntr_n) < (access.G\_tCntr_{n+1})$ . As we know if any method is locking as well as accessing common counter then locking tobject first then accessing the counter after that unlocking tobject i.e.

```
So, (Ll\_op_n(\alpha)) < (access.G\_tCntr_n) < (Fu\_op_n(\alpha)).
Hence, (Fu\_op_1(\alpha) < (access.G\_tCntr_{n+1}).
```

- (b)  $(access.G\_tCntr_1) < (access.G\_tCntr_n)$ : Here,  $(access.G\_tCntr_n) < (access.G\_tCntr_{n+1})$ . Ref 1.(b).iii.
- (c)  $(Fu\_op_1(\alpha) < (access.G\_tCntr_n)$ : Here,  $(access.G\_tCntr_n) < (access.G\_tCntr_{n+1})$ . Ref 1.(b).iv.
- (d)  $(access.G_tCntr_1) < Ll\_op_n(\alpha)$ ): Here,  $(access.G_tCntr_n) < (access.G_tCntr_{n+1})$ . Ref 1.(b).v

Therefore,  $OPG(H, M\_Order)$  produced by KSFTM is acyclic.

**Lemma 14** Any history H gen(KSFTM) with  $\alpha$  linearization such that it respects M\_Order $_H$  then  $(H, \alpha)$  is valid.

**Proof.** From the definition of *valid history*: If all the read operations of H is reading from the previously committed transaction  $T_j$  then H is valid.

In order to prove H is valid, we are analyzing the read(i,x). so, from Line 10, it returns the largest ts value less than  $G_{-}wts_{i}$  that has already been committed and return the value successfully. If such version created by transaction  $T_{j}$  found then  $T_{i}$  read from  $T_{j}$ . Otherwise, if there is no version whose WTS is less than  $T_{i}$ 's WTS, then  $T_{i}$  returns abort.

Now, consider the base case read(i,x) is the first transaction  $T_1$  and none of the transactions has been created a version then as we have assummed, there always exist  $T_0$  by default that has been created a version for all t-objects. Hence,  $T_1$  reads from committed transaction  $T_0$ .

So, all the reads are reading from largest ts value less than  $G_{-}wts_{i}$  that has already been committed. Hence, (H,  $\alpha$ ) is valid.

**Lemma 15** Any history H gen(KSFTM) with  $\alpha$  and  $\beta$  linearization such that both respects M\_Order $_H$  i.e. M\_Order $_H \subseteq \alpha$  and M\_Order $_H \subseteq \beta$  then  $\prec_{(H,\alpha)}^{RT} = \prec_{(H,\beta)}^{RT}$ .

**Proof.** Consider a history H gen(KSFTM) such that two transactions  $T_i$  and  $T_j$  are in real time order which respects  $M\_Order_H$  i.e.  $stm\_tryC_i < stm\_begin_j$ . As  $\alpha$  and  $\beta$  are linearizations of H so,  $stm\_tryC_i <_{(H,\alpha)} stm\_begin_j$  and  $stm\_tryC_i <_{(H,\beta)} stm\_begin_j$ . Hence in both the cases of linearizations,  $T_i$  committed before begin of  $T_j$ . So,  $\prec_{(H,\alpha)}^{RT} = \prec_{(H,\beta)}^{RT}$ .

**Lemma 16** Any history H gen(KSFTM) with  $\alpha$  and  $\beta$  linearization such that both respects M\_Order $_H$  i.e. M\_Order $_H \subseteq \alpha$  and M\_Order $_H \subseteq \beta$  then  $(H, \alpha)$  is local opaque iff  $(H, \beta)$  is local opaque.

**Proof.** As  $\alpha$  and  $\beta$  are linearizations of history H gen(KSFTM) so, from Lemma 14 (H,  $\alpha$ ) and (H,  $\beta$ ) are valid histories.

Now assuming (H,  $\alpha$ ) is local opaque so we need to show (H,  $\beta$ ) is also local opaque. Since (H,  $\alpha$ ) is local opaque so there exists legal t-sequential history S (with respect to each aborted transactions and last committed transaction while considering only committed transactions) which is equivalent to  $(\overline{H}, \alpha)$ . As we know  $\beta$  is a linearization of H so  $(\overline{H}, \beta)$  is equivalent to some legal t-sequential history S. From the definition of local opacity  $\prec_{(H,\alpha)}^{RT} \subseteq \prec_S^{RT}$ . From Lemma 15,  $\prec_{(H,\alpha)}^{RT} = \prec_{(H,\beta)}^{RT}$  that implies  $\prec_{(H,\beta)}^{RT} \subseteq \prec_S^{RT}$ . Hence,  $(H,\beta)$  is local opaque.

Now consider the other way in which  $(H, \beta)$  is local opaque and we need to show  $(H, \alpha)$  is also local opaque. We can prove it while giving the same argument as above, by exchanging  $\alpha$  and  $\beta$ .

Hence,  $(H, \alpha)$  is local opaque iff  $(H, \beta)$  is local opaque.

**Theorem 17** Any history generated by KSFTM is locally-opaque.

**Proof.** For proving this, we consider a sequential history H generated by KSFTM. We define the version order  $\ll_{vrt}$ : for two versions  $v_i, v_j$  it is defined as

$$(v_i \ll_{\text{vrt}} v_j) \equiv (v_i.\text{vrt} < v_j.\text{vrt})$$

Using this version order  $\ll_{vrt}$ , we can show that all the sub-histories in H.subhistSet are acyclic. Since the histories generated by KSFTM are locally-opaque, we get that they are also strict-serializable.

**Corollary 18** Any history generated by KSFTM is strict-serializable.

### 6 Proof of Liveness

**Proof Notations:** Let gen(KSFTM) consist of all the histories accepted by KSFTM algorithm. In the follow subsection, we only consider histories that are generated by KSFTM unless explicitly stated otherwise. For simplicity, we only consider sequential histories in our discussion below.

Consider a transaction  $T_i$  in a history H generated by KSFTM. Once it executes stm-begin method, its ITS, CTS, WTS values do not change. Thus, we denote them as  $its_i, cts_i, wts_i$  respectively for  $T_i$ . In case the context of the history H in which the transaction executing is important, we denote these variables as  $H.its_i, H.cts_i, H.wts_i$  respectively.

The other variables that a transaction maintains are: tltl, tutl, lock, valid, state. These values change as the execution proceeds. Hence, we denote them as:  $H.tltl_i, H.tutl_i, H.lock_i, H.valid_i, H.state_i$ . These represent the values of tltl, tutl, lock, valid, state after the execution of last event in H. Depending on the context, we sometimes ignore H and denote them only as:  $lock_i, valid_i, state_i, tltl_i, tutl_i$ .

We approximate the system time with the value of tCntr. We denote the sys-time of history H as the value of tCntr immediately after the last event of H. Further, we also assume that the value of C is 1 in our arguments. But, it can be seen that the proof will work for any value greater than 1 as well.

The application invokes transactions in such a way that if the current  $T_i$  transaction aborts, it invokes a new transaction  $T_j$  with the same ITS. We say that  $T_i$  is an *incarnation* of  $T_j$  in a history H if  $H.its_i = H.its_j$ . Thus the multiple incarnations of a transaction  $T_i$  get invoked by the application until an incarnation finally commits.

To capture this notion of multiple transactions with the same ITS, we define *incarSet* (incarnation set) of  $T_i$  in H as the set of all the transactions in H which have the same ITS as  $T_i$  and includes  $T_i$  as well. Formally,

$$H.incarSet(T_i) = \{T_i | (T_i = T_j) \lor (H.its_i = H.its_j)\}$$

Note that from this definition of incarSet, we implicitly get that  $T_i$  and all the transactions in its incarSet of H also belong to H. Formally,  $H.incarSet(T_i) \in H.txns$ .

The application invokes different incarnations of a transaction  $T_i$  in such a way that as long as an incarnation is live, it does not invoke the next incarnation. It invokes the next incarnation after the current incarnation has got aborted. Once an incarnation of  $T_i$  has committed, it can't have any future incarnations. Thus, the application views all the incarnations of a transaction as a single application-transaction.

We assign incNums to all the transactions that have the same ITS. We say that a transaction  $T_i$  starts afresh, if  $T_i.incNum$  is 1. We say that  $T_i$  is the nextInc of  $T_i$  if  $T_j$  and  $T_i$  have the same ITS and  $T_i$ 's incNum is  $T_j$ 's incNum + 1. Formally,  $\langle (T_i.nextInc = T_j) \equiv (its_i = its_j) \wedge (T_i.incNum = T_j.incNum + 1) \rangle$ 

As mentioned the objective of the application is to ensure that every application-transaction eventually commits. Thus, the applications views the entire incarSet as a single application-transaction (with all the transactions in the incarSet having the same ITS). We can say that an application-transaction has committed if in the corresponding incarSet a transaction in eventually commits. For  $T_i$  in a history H, we denote this by a boolean value incarCt (incarnation set committed) which implies that either  $T_i$  or an incarnation of  $T_i$  has committed. Formally, we define it as  $H.incarCt(T_i)$ 

$$H.incarCt(T_i) = \begin{cases} True & (\exists T_j : (T_j \in H.incarSet(T_i)) \land (T_j \in H.committed)) \\ False & \text{otherwise} \end{cases}$$

From the definition of incarCt we get the following observations & lemmas about a transaction  $T_i$ 

**Observation 19** Consider a transaction  $T_i$  in a history H with its incarCt being true in H. Then  $T_i$  is terminated (either committed or aborted) in H. Formally,  $\langle H, T_i : (T_i \in H.txns) \land (H.incarCt(T_i)) \implies (T_i \in H.terminated) \rangle$ .

**Observation 20** Consider a transaction  $T_i$  in a history H with its incarCt being true in H1. Let H2 be a extension of H1 with a transaction  $T_j$  in it. Suppose  $T_j$  is an incarnation of  $T_i$ . Then  $T_j$ 's incarCt is true in H2. Formally,  $\langle H1, H2, T_i, T_j : (H1 \sqsubseteq H2) \land (H1.incarCt(T_i)) \land (T_j \in H2.txns) \land (T_i \in H2.incarSet(T_j)) \Longrightarrow (H2.incarCt(T_j)) \rangle$ .

**Lemma 21** Consider a history H1 with a strict extension H2. Let  $T_i$  &  $T_j$  be two transactions in H1 & H2 respectively. Let  $T_j$  not be in H1. Suppose  $T_i$ 's incarCt is true. Then ITS of  $T_i$  cannot be the same as ITS of  $T_j$ . Formally,  $\langle H1, H2, T_i, T_j : (H1 \sqsubset H2) \land (H1.incarCt(T_i)) \land (T_j \in H2.txns) \land (T_j \notin H1.txns) \Longrightarrow (H1.its_i \neq H2.its_j) \rangle$ .

**Proof.** Here, we have that  $T_i$ 's incarCt is true in H1. Suppose  $T_j$  is an incarnation of  $T_i$ , i.e., their ITSs are the same. We are given that  $T_j$  is not in H1. This implies that  $T_j$  must have started after the last event of H1.

We are also given that  $T_i$ 's incarCt is true in H1. This implies that an incarnation of  $T_i$  or  $T_i$  itself has committed in H1. After this commit, the application will not invoke another transaction with the same ITS as  $T_i$ . Thus, there cannot be a transaction after the last event of H1 and in any extension of H1 with the same ITS of  $T_1$ . Hence,  $H1.its_i$  cannot be same as  $H2.its_i$ .

Now we show the liveness with the following observations, lemmas & theorems. We start with two observations about that histories of which one is an extension of the other. The following states that for any history, there exists an extension. In other words, we assume that the STM system runs forever and does not terminate. This is required for showing that every transaction eventually commits.

**Observation 22** Consider a history H1 generated by gen(KSFTM). Then there is a history H2 in gen(KSFTM) such that H2 is a strict extension of H1. Formally,  $\langle \forall H1: (H1 \in gen(ksftm)) \implies (\exists H2: (H2 \in gen(ksftm)) \wedge (H1 \sqsubset H2) \rangle$ .

The follow observation is about the transaction in a history and any of its extensions.

**Observation 23** Given two histories H1 & H2 such that H2 is an extension of H1. Then, the set of transactions in H1 are a subset equal to the set of transaction in H2. Formally,  $\langle \forall H1, H2 : (H1 \sqsubseteq H2) \implies (H1.txns \subseteq H2.txns) \rangle$ .

In order for a transaction  $T_i$  to commit in a history H, it has to compete with all the live transactions and all the aborted that can become live again as a different incarnation. Once a transaction  $T_j$  aborts, another incarnation of  $T_j$  can start and become live again. Thus  $T_i$  will have to compete with this incarnation of  $T_j$  later. Thus, we have the following observation about aborted & committed transactions.

**Observation 24** Consider an aborted transaction  $T_i$  in a history H1. Then there is an extension of H1, H2 in which an incarnation of  $T_i$ ,  $T_j$  is live and has  $cts_j$  is greater than  $cts_i$ . Formally,  $\langle H1, T_i : (T_i \in H1.aborted) \implies (\exists T_i, H2 : (H1 \sqsubseteq H2) \land (T_i \in H2.live) \land (H2.its_i = H2.its_j) \land (H2.cts_i < H2.cts_j)) \rangle$ .

**Observation 25** Consider an committed transaction  $T_i$  in a history H1. Then there is no extension of H1, in which an incarnation of  $T_i$ ,  $T_j$  is live. Formally,  $\langle H1, T_i : (T_i \in H1.committed) \implies (\nexists T_j, H2 : (H1 \sqsubseteq H2) \land (T_j \in H2.live) \land (H2.its_i = H2.its_j)) \rangle$ .

**Lemma 26** Consider a history H1 and its extension H2. Let  $T_i, T_j$  be in H1, H2 respectively such that they are incarnations of each other. If WTS of  $T_i$  is less than WTS of  $T_j$  then CTS of  $T_i$  is less than CTS  $T_j$ . Formally,  $\langle H1, H2, T_i, T_j : (H1 \sqsubset H2) \land (T_i \in H1.txns) \land (T_j \in H2.txns) \land (T_i \in H2.incarSet(T_j)) \land (H1.wts_i < H2.wts_j) \implies (H1.cts_i < H2.cts_j) \rangle$ 

**Proof.** Here we are given that

$$H1.wts_i < H2.wts_i \tag{2}$$

The definition of WTS of  $T_i$  is:  $H1.wts_i = H1.cts_i + C * (H1.cts_i - H1.its_i)$ . Combining this Eq.(2), we get that

$$(C+1)*H1.cts_i - C*H1.its_i < (C+1)*H2.cts_j - C*H2.its_j \xrightarrow{T_i \in H2.incarSet(T_j)} H1.cts_i < H2.cts_j.$$

**Lemma 27** Consider a live transaction  $T_i$  in a history H1 with its  $wts_i$  less than a constant  $\alpha$ . Then there is a strict extension of H1, H2 in which an incarnation of  $T_i$ ,  $T_j$  is live with WTS greater than  $\alpha$ . Formally,  $\langle H1, T_i : (T_i \in H1.live) \land (H1.wts_i < \alpha) \implies (\exists T_j, H2 : (H1 \sqsubseteq H2) \land (T_i \in H2.incarSet(T_j)) \land ((T_j \in H2.committed) \lor ((T_j \in H2.live) \land (H2.wts_j > \alpha))))\rangle.$ 

**Proof.** The proof comes the behavior of an application-transaction. The application keeps invoking a transaction with the same ITS until it commits. Thus the transaction  $T_i$  which is live in H1 will eventually terminate with an abort or commit. If it commits, H2 could be any history after the commit of  $T_2$ .

On the other hand if  $T_i$  is aborted, as seen in Observation 24 it will be invoked again or reincarnated with another CTS and WTS. It can be seen that CTS is always increasing. As a result, the WTS is also increasing. Thus eventually the WTS will become greater  $\alpha$ . Hence, we have that either an incarnation of  $T_i$  will get committed or will eventually have WTS greater than or equal to  $\alpha$ .

Next we have a lemma about CTS of a transaction and the sys-time of a history.

**Lemma 28** Consider a transaction  $T_i$  in a history H. Then, we have that CTS of  $T_i$  will be less than or equal to sys-time of H. Formally,  $\langle T_i, H1 : (T_i \in H.txns) \implies (H.cts_i \leq H.sys-time) \rangle$ .

**Proof.** We get this lemma by observing the methods of the STM System that increment the tCntr which are stm-begin and stm-tryC. It can be seen that CTS of  $T_i$  gets assigned in the stm-begin method. So if the last method of H is the stm-begin of  $T_i$  then we get that CTS of  $T_i$  is same as sys-time of H. On the other hand if some other method got executed in H after stm-begin of  $T_i$  then we have that CTS of  $T_i$  is less than sys-time of H. Thus combining both the cases, we get that CTS of  $T_i$  is less than or equal to as sys-time of H, i.e.,  $(H.cts_i \leq H.sys-time)$ 

From this lemma, we get the following corollary which is the converse of the lemma statement

**Corollary 29** Consider a transaction  $T_i$  which is not in a history H1 but in an strict extension of H1, H2. Then, we have that CTS of  $T_i$  is greater than the sys-time of H. Formally,  $\langle T_i, H1, H2 : (H1 \sqsubset H2) \land (T_i \notin H1.txns) \land (T_i \in H2.txns) \Longrightarrow (H2.cts_i > H1.sys-time) \rangle$ .

Now, we have lemma about the methods of KSFTM completing in finite time.

**Lemma 30** If all the locks are fair and the underlying system scheduler is fair then all the methods of KSFTM will eventually complete.

**Proof.** It can be seen that in any method, whenever a transaction  $T_i$  obtains multiple locks, it obtains locks in the same order: first lock relevant t-objects in a pre-defined order and then lock relevant G\_locks again in a predefined order. Since all the locks are obtained in the same order, it can be seen that the methods of *KSFTM* will not deadlock

It can also be seen that none of the methods have any unbounded while loops. All the loops in stm-tryC method iterate through all the t-objects in the write-set of  $T_i$ . Moreover, since we assume that the underlying scheduler is fair, we can see that no thread gets swapped out infinitely. Finally, since we assume that all the locks are fair, it can be seen all the methods terminate in finite time.

**Theorem 31** Every transaction either commits or aborts in finite time.

**Proof.** This theorem comes directly from the Lemma 30. Since every method of *KSFTM* will eventually complete, all the transactions will either commit or abort in finite time.

From this theorem, we get the following corollary which states that the maximum lifetime of any transaction is L.

**Corollary 32** Any transaction  $T_i$  in a history H will either commit or abort before the sys-time of H crosses  $cts_i + L$ .

The following lemma connects WTS and ITS of two transactions,  $T_i$ ,  $T_j$ .

**Lemma 33** Consider a history H1 with two transactions  $T_i, T_j$ . Let  $T_i$  be in H1.live. Suppose  $T_j$ 's WTS is greater or equal to  $T_i$ 's WTS. Then ITS of  $T_j$  is less than  $its_i + 2 * L$ . Formally,  $\langle H, T_i, T_j : (\{T_i, T_j\} \subseteq H.txns) \land (T_i \in H.live) \land (H.wts_j \ge H.wts_i) \Longrightarrow (H.its_i + 2L \ge H.its_j) \rangle$ .

**Proof.** Since  $T_i$  is live in H1, from Corollary 32, we get that it terminates before the system time, tCntr becomes  $cts_i + L$ . Thus, sys-time of history H1 did not progress beyond  $cts_i + L$ . Hence, for any other transaction  $T_j$  (which is either live or terminated) in H1, it must have started before sys-time has crossed  $cts_i + L$ . Formally  $\langle cts_j \leq cts_i + L \rangle$ .

Note that we have defined WTS of a transaction  $T_j$  as:  $wts_j = (cts_j + C*(cts_j - its_j))$ . Now, let us consider the difference of the WTSs of both the transactions.

```
\begin{array}{l} wts_j - wts_i = (cts_j + C*(cts_j - its_j)) - (cts_i + C*(cts_i - its_i)) \\ = (C+1)(cts_j - cts_i) - C(its_j - its_i) \\ \leq (C+1)L - C(its_j - its_i) \quad [\because cts_j \leq cts_i + L] \\ = 2*L + its_i - its_j \quad [\because C = 1] \end{array}
```

Thus, we have that:  $\langle (its_i + 2L - its_j) \geq (wts_j - wts_i) \rangle$ . This gives us that  $((wts_j - wts_i) \geq 0) \Longrightarrow ((its_i + 2L - its_j) \geq 0)$ .

From the above implication we get that,  $(wts_j \ge wts_i) \Longrightarrow (its_i + 2L \ge its_j)$ .

It can be seen that KSFTM algorithm gives preference to transactions with lower ITS to commit. To understand this notion of preference, we define a few notions of enablement of a transaction  $T_i$  in a history H. We start with the definition of itsEnabled as:

**Definition 2** We say  $T_i$  is its Enabled in H if for all transactions  $T_j$  with ITS lower than ITS of  $T_i$  in H have incarCt to be true. Formally,

$$H.itsEnabled(T_i) = \begin{cases} True & (T_i \in H.live) \land (\forall T_j \in H.txns : (H.its_j < H.its_i) \implies (H.incarCt(T_j))) \\ False & otherwise \end{cases}$$

The follow lemma states that once a transaction  $T_i$  becomes its Enabled it continues to remain so until it terminates.

**Lemma 34** Consider two histories H1 and H2 with H2 being a extension of H1. Let a transaction  $T_i$  being live in both of them. Suppose  $T_i$  is itsEnabled in H1. Then  $T_i$  is itsEnabled in H2 as well. Formally,  $\langle H1, H2, T_i : (H1 \sqsubseteq H2) \land (T_i \in H1.live) \land (T_i \in H2.live) \land (H1.itsEnabled(T_i)) \Longrightarrow (H2.itsEnabled(T_i)) \rangle$ .

**Proof.** When  $T_i$  begins in a history H3 let the set of transactions with ITS less than  $its_i$  be smIts. Then in any extension of H3, H4 the set of transactions with ITS less than  $its_i$  remains as smIts.

Suppose H1, H2 are extensions of H3. Thus in H1, H2 the set of transactions with ITS less than  $its_i$  will be smIts. Hence, if  $T_i$  is itsEnabled in H1 then all the transactions  $T_j$  in smIts are  $H1.incarCt(T_j)$ . It can be seen that this continues to remain true in H2. Hence in H2,  $T_i$  is also itsEnabled which proves the lemma.

The following lemma deals with a committed transaction  $T_i$  and any transaction  $T_j$  that terminates later. In the following lemma, incrVal is any constant greater than or equal to 1.

**Lemma 35** Consider a history H with two transactions  $T_i, T_j$  in it. Suppose transaction  $T_i$  commits before  $T_j$  terminates (either by commit or abort) in H. Then  $comTime_i$  is less than  $comTime_j$  by at least incrVal. Formally,  $\langle H, \{T_i, T_j\} \in H.txns : (stm-tryC_i <_H term-op_j) \implies (comTime_i + incrVal \le comTime_j) \rangle$ .

**Proof.** When  $T_i$  commits, let the value of the global tCntr be  $\alpha$ . It can be seen that in stm-begin method,  $comTime_j$  get initialized to  $\infty$ . The only place where  $comTime_j$  gets modified is at Line 61 of stm-tryC. Thus if  $T_j$  gets aborted before executing stm-tryC method or before this line of stm-tryC we have that  $comTime_j$  remains at  $\infty$ . Hence in this case we have that  $\langle comTime_i + incrVal < comTime_j \rangle$ .

If  $T_j$  terminates after executing Line 61 of stm-tryC method then  $comTime_j$  is assigned a value, say  $\beta$ . It can be seen that  $\beta$  will be greater than  $\alpha$  by at least incrVal due to the execution of this line. Thus, we have that  $\langle \alpha + incrVal \leq \beta \rangle$ 

The following lemma connects the G<sub>tll</sub> and comTime of a transaction  $T_i$ .

**Lemma 36** Consider a history H with a transaction  $T_i$  in it. Then in H,  $tltl_i$  will be less than or equal to  $comTime_i$ . Formally,  $\langle H, \{T_i\} \in H.txns : (H.tltl_i \leq H.comTime_i) \rangle$ .

**Proof.** Consider the transaction  $T_i$ . In stm-begin method,  $comTime_i$  get initialized to  $\infty$ . The only place where  $comTime_i$  gets modified is at Line 61 of stm-tryC. Thus if  $T_i$  gets aborted before this line or if  $T_i$  is live we have that  $(tltl_i \leq comTime_i)$ . On executing Line 61,  $comTime_i$  gets assigned to some finite value and it does not change after that.

It can be seen that  $tltl_i$  gets initialized to  $cts_i$  in Line 4 of stm-begin method. In that line,  $cts_i$  reads tCntr and increments it atomically. Then in Line 61,  $comTime_i$  gets assigned the value of tCntr after incrementing it. Thus, we clearly get that  $cts_i (= tltl_i \text{ initially}) < comTime_i$ . Then  $tltl_i$  gets updated on Line 20 of read, Line 53 and Line 84 of stm-tryC methods. Let us analyze them case by case assuming that  $tltl_i$  was last updated in each of these methods before the termination of  $T_i$ :

1. Line 20 of read method: Suppose this is the last line where  $tltl_i$  updated. Here  $tltl_i$  gets assigned to 1 + vrt of the previously committed version which say was created by a transaction  $T_j$ . Thus, we have the following equation,

$$tltl_i = 1 + x[j].vrt (3)$$

It can be seen that x[j].vrt is same as  $tltl_j$  when  $T_j$  executed Line 99 of stm-tryC. Further,  $tltl_j$  in turn is same as  $tutl_j$  due to Line 84 of stm-tryC. From Line 62, it can be seen that  $tutl_j$  is less than or equal to  $comTime_j$  when  $T_j$  committed. Thus we have that

$$x[j].vrt = tltl_j = tutl_j \le comTime_j$$
 (4)

It is clear that from the above discussion that  $T_j$  executed stm-tryC method before  $T_i$  terminated (i.e. stm-tryC $_j <_{H1} term$ -o $p_i$ ). From Eq.(3) and Eq.(4), we get

$$tltl_i \leq 1 + comTime_j \xrightarrow{incrVal \geq 1} tltl_i \leq incrVal + comTime_j \xrightarrow{Lemma \ 35} tltl_i \leq comTime_i$$

- 2. Line 53 of stm-tryC method: The reasoning in this case is very similar to the above case.
- 3. Line 84 of stm-tryC method: In this line,  $tltl_i$  is made equal to  $tutl_i$ . Further, in Line 62,  $tutl_i$  is made lesser than or equal to  $comTime_i$ . Thus combing these, we get that  $tltl_i \leq comTime_i$ . It can be seen that the reasoning here is similar in part to Case 1.

Hence, in all the three cases we get that  $\langle tltl_i \leq comTime_i \rangle$ .

The following lemma connects the G<sub>tutl</sub>,comTime of a transaction  $T_i$  with WTS of a transaction  $T_j$  that has already committed.

**Lemma 37** Consider a history H with a transaction  $T_i$  in it. Suppose  $tutl_i$  is less than  $comTime_i$ . Then, there is a committed transaction  $T_j$  in H such that  $wts_j$  is greater than  $wts_i$ . Formally,  $\langle H \in gen(KSFTM), \{T_i\} \in H.txns: (H.tutl_i < H.comTime_i) \implies (\exists T_j \in H.committed: H.wts_j > H.wts_i) \rangle$ .

**Proof.** It can be seen that  $G_-tutl_i$  initialized in stm-begin method to  $\infty$ .  $tutl_i$  is updated in Line 17 of read method, Line 58 & Line 62 of stm-tryC method. If  $T_i$  executes Line 17 of read method and/or Line 58 of stm-tryC method then  $tutl_i$  gets decremented to some value less than  $\infty$ , say  $\alpha$ . Further, it can be seen that in both these lines the value of  $tutl_i$  is possibly decremented from  $\infty$  because of nextVer (or ver), a version of x whose ts is greater than  $T_i$ 's WTS. This implies that some transaction  $T_j$ , which is committed in H, must have created nextVer (or ver) and  $wts_j > wts_i$ .

Next, let us analyze the value of  $\alpha$ . It can be seen that  $\alpha = x[nextVer/ver].vrt - 1$  where nextVer/ver was created by  $T_j$ . Further, we can see when  $T_j$  executed stm-tryC, we have that  $x[nextVer].vrt = tltl_j$  (from Line 99). From Lemma 36, we get that  $tltl_j \leq comTime_j$ . This implies that  $\alpha < comTime_j$ . Now, we have that  $T_j$  has already committed before the termination of  $T_i$ . Thus from Lemma 35, we get that  $comTime_j < comTime_i$ . Hence, we have that,

$$\alpha < comTime_i$$
 (5)

Now let us consider Line 62 executed by  $T_i$  which causes  $tutl_i$  to change. This line will get executed only after both Line 17 of read method, Line 58 of stm-tryC method. This is because every transaction executes stm-tryC method only after read method. Further within stm-tryC method, Line 62 follows Line 58.

There are two sub-cases depending on the value of  $tutl_i$  before the execution of Line 62: (i) If  $tutl_i$  was  $\infty$  and then get decremented to  $comTime_i$  upon executing this line, then we get  $comTime_i = tutl_i$ . From Eq.(5), we can ignore this case. (ii) Suppose the value of  $tutl_i$  before executing Line 62 was  $\alpha$ . Then from Eq.(5) we get that  $tutl_i$  remains at  $\alpha$  on execution of Line 62. This implies that a transaction  $T_j$  committed such that  $wts_j > wts_i$ . The following lemma connects the G\_tltl of a committed transaction  $T_j$  and comTime of a transaction  $T_i$  that commits later.

**Lemma 38** Consider a history H1 with transactions  $T_i, T_j$  in it. Suppose  $T_j$  is committed and  $T_i$  is live in H1. Then in any extension of H1, say H2,  $tltl_j$  is less than or equal to  $comTime_i$ . Formally,  $\langle H1, H2 \in gen(\textit{KSFTM}), \{T_i, T_j\} \subseteq H1, H2.txns : (H1 \sqsubseteq H2) \land (T_j \in H1.committed) \land (T_i \in H1.live) \Longrightarrow (H2.tltl_j < H2.comTime_i) \rangle.$ 

**Proof.** As observed in the previous proof of Lemma 36, if  $T_i$  is live or aborted in H2, then its comTime is  $\infty$ . In both these cases, the result follows.

If  $T_i$  is committed in H2 then, one can see that comTime of  $T_i$  is not  $\infty$ . In this case, it can be seen that  $T_j$  committed before  $T_i$ . Hence, we have that  $comTime_j < comTime_i$ . From Lemma 36, we get that  $tltl_j \leq comTime_j$ . This implies that  $tltl_j < comTime_i$ .

In the following sequence of lemmas, we identify the condition by when a transaction will commit.

**Lemma 39** Consider two histories H1, H3 such that H3 is a strict extension of H1. Let  $T_i$  be a transaction in H1.live such that  $T_i$  itsEnabled in H1 and  $G\_valid_i$  flag is true in H1. Suppose  $T_i$  is aborted in H3. Then there is a history H2 which is an extension of H1 (and could be same as H1) such that (1) Transaction  $T_i$  is live in H2; (2) there is a transaction  $T_j$  that is live in H2; (3)  $H2.wts_j$  is greater than  $H2.wts_i$ ; (4)  $T_j$  is committed in H3. Formally,  $\langle H1, H3, T_i : (H1 \sqsubset H3) \land (T_i \in H1.live) \land (H1.valid_i = True) \land (H1.itsEnabled(T_i)) \land (T_i \in H3.aborted)) \implies (\exists H2, T_j : (H1 \sqsubseteq H2 \sqsubset H3) \land (T_i \in H2.live) \land (T_j \in H2.txns) \land (H2.wts_i < H2.wts_j) \land (T_j \in H3.committed)) \rangle.$ 

**Proof.** To show this lemma, w.l.o.g we assume that  $T_i$  on executing either read or stm-tryC in H2 (which could be same as H1) gets aborted resulting in H3. Thus, we have that  $T_i$  is live in H2. Here  $T_i$  is itsEnabled in H1. From Lemma 34, we get that  $T_i$  is itsEnabled in H2 as well.

Let us sequentially consider all the lines where a  $T_i$  could abort. In H2,  $T_i$  executes one of the following lines and is aborted in H3. We start with stm-tryC method.

### 1. STM stm-tryC:

- (a) Line 3: This line invokes abort() method on  $T_i$  which releases all the locks and returns  $\mathscr{A}$  to the invoking thread. Here  $T_i$  is aborted because its valid flag, is set to false by some other transaction, say  $T_j$ , in its stm-tryC algorithm. This can occur in Lines: 45, 74 where  $T_i$  is added to  $T_j$ 's abortRL set. Later in Line 94,  $T_i$ 's valid flag is set to false. Note that  $T_i$ 's valid is true (after the execution of the last event) in H1. Thus,  $T_i$ 's valid flag must have been set to false in an extension of H1, which we again denote as H2.
  - This can happen only if in both the above cases,  $T_j$  is live in H2 and its ITS is less than  $T_i$ 's ITS. But we have that  $T_i$ 's itsEnabled in H2. As a result, it has the smallest among all live and aborted transactions of H2. Hence, there cannot exist such a  $T_j$  which is live and  $H2.its_j < H2.its_i$ . Thus, this case is not possible.
- (b) Line 15: This line is executed in H2 if there exists no version of x whose ts is less than  $T_i$ 's WTS. This implies that all the versions of x have tss greater than  $wts_i$ . Thus the transactions that created these versions have WTS greater than  $wts_i$  and have already committed in H2. Let  $T_j$  create one such version. Hence, we have that  $\langle (T_j \in H2.committed) \implies (T_j \in H3.committed) \rangle$  since H3 is an extension of H2.
- (c) Line 34: This case is similar to Case 1a, i.e., Line 3.
- (d) Line 47: In this line,  $T_i$  is aborted as some other transaction  $T_j$  in  $T_i$ 's largeRL has committed. Any transaction in  $T_i$ 's largeRL has WTS greater than  $T_i$ 's WTS. This implies that  $T_j$  is already committed in H2 and hence committed in H3 as well.
- (e) Line 64: In this line,  $T_i$  is aborted because its lower limit has crossed its upper limit. First, let us consider  $tutl_i$ . It is initialized in stm-begin method to  $\infty$ . As long as it is  $\infty$ , these limits cannot cross each other. Later,  $tutl_i$  is updated in Line 17 of read method, Line 58 & Line 62 of stm-tryC method. Suppose  $tutl_i$  gets decremented to some value  $\alpha$  by one of these lines.
  - Now there are two cases here: (1) Suppose  $tutl_i$  gets decremented to  $comTime_i$  due to Line 62 of stm-tryC method. Then from Lemma 36, we have  $tltl_i \leq comTime_i = tutl_i$ . Thus in this case,  $T_i$  will not abort. (2)  $tutl_i$  gets decremented to  $\alpha$  which is less than  $comTime_i$ . Then from Lemma 37, we get that there is a committed transaction  $T_j$  in H2.committed such that  $wts_j > wts_i$ . This implies that  $T_j$  is in H3.committed.
- (f) Line 76: This case is similar to Case 1a, i.e., Line 3.
- (g) Line 79: In this case,  $T_k$  is in  $T_i$ 's smallRL and is committed in H1. And, from this case, we have that

$$H2.tutl_i \le H2.tltl_k$$
 (6)

From the assumption of this case, we have that  $T_k$  commits before  $T_i$ . Thus, from Lemma 38, we get that  $comTime_k < comTime_i$ . From Lemma 36, we have that  $tltl_k \leq comTime_k$ . Thus, we get that  $tltl_k < comTime_i$ . Combining this with the inequality of this case Eq.(6), we get that  $tutl_i < comTime_i$ .

Combining this inequality with Lemma 37, we get that there is a transaction  $T_j$  in H2.committed and  $H2.wts_j > H2.wts_i$ . This implies that  $T_j$  is in H3.committed as well.

#### 2. STM read:

- (a) Line 7: This case is similar to Case 1a, i.e., Line 3
- (b) Line 22: The reasoning here is similar to Case 1e, i.e., Line 64.

The interesting aspect of the above lemma is that it gives us a insight as to when a  $T_i$  will get commit. If an itsEnabled transaction  $T_i$  aborts then it is because of another transaction  $T_j$  with WTS higher than  $T_i$  has committed. To precisely capture this, we define two more notions of a transaction being enabled cdsEnabled and finEnabled. To define these notions of enabled, we in turn define a few other auxiliary notions. We start with affectSet,

$$H.affectSet(T_i) = \{T_j | (T_j \in H.txns) \land (H.its_j < H.its_i + 2 * L)\}$$

From the description of KSFTM algorithm and Lemma 33, it can be seen that a transaction  $T_i$ 's commit can depend on committing of transactions (or their incarnations) which have their ITS less than ITS of  $T_i + 2*L$ , which is  $T_i$ 's affectSet. We capture this notion of dependency for a transaction  $T_i$  in a history H as commit dependent set or cds as: the set of all transactions  $T_j$  in  $T_i$ 's affectSet that do not any incarnation that is committed yet, i.e., not yet have their incarCt flag set as true. Formally,

$$H.cds(T_i) = \{T_i | (T_i \in H.affectSet(T_i)) \land (\neg H.incarCt(T_i)) \}$$

Based on this definition of cds, we next define the notion of cdsEnabled.

**Definition 3** We say that transaction  $T_i$  is cdsEnabled if the following conditions hold true (1)  $T_i$  is live in H; (2) CTS of  $T_i$  is greater than or equal to ITS of  $T_i + 2 * L$ ; (3) cds of  $T_i$  is empty, i.e., for all transactions  $T_j$  in H with ITS lower than ITS of  $T_i + 2 * L$  in H have their incarCt to be true. Formally,

$$H.cdsEnabled(T_i) = \begin{cases} True & (T_i \in H.live) \land (H.cts_i \ge H.its_i + 2 * L) \land (H.cds(T_i) = \phi) \\ False & otherwise \end{cases}$$

The meaning and usefulness of these definitions will become clear in the course of the proof. In fact, we later show that once the transaction  $T_i$  is cdsEnabled, it will eventually commit. We will start with a few lemmas about these definitions.

**Lemma 40** Consider a transaction  $T_i$  in a history H. If  $T_i$  is cdsEnabled then  $T_i$  is also itsEnabled. Formally,  $\langle H, T_i : (T_i \in H.txns) \land (H.cdsEnabled(T_i)) \implies (H.itsEnabled(T_i)) \rangle$ .

**Proof.** If  $T_i$  is cdsEnabled in H then it implies that  $T_i$  is live in H. From the definition of cdsEnabled, we get that  $H.cds(T_i)$  is  $\phi$  implying that any transaction  $T_j$  with  $its_k$  less than  $its_i + 2 * L$  has its incarCt flag as true in H. Hence, for any transaction  $T_k$  having  $its_k$  less than  $its_i$ ,  $H.incarCt(T_k)$  is also true. This shows that  $T_i$  is itsEnabled in H.

**Lemma 41** Consider a transaction  $T_i$  which is cdsEnabled in a history H1. Consider an extension of H1, H2 with a transaction  $T_j$  in it such that  $T_i$  is an incarnation of  $T_j$ . Let  $T_k$  be a transaction in the affectSet of  $T_j$  in H2 Then  $T_k$  is also in the set of transaction of H1. Formally,  $\langle H1, H2, T_i, T_j, T_k : (H1 \sqsubseteq H2) \land (H1.cdsEnabled(T_i)) \land (T_i \in H2.incarSet(T_j)) \land (T_k \in H2.affectSet(T_j)) \Longrightarrow (T_k \in H1.txns) \rangle$ 

**Proof.** Since  $T_i$  is cdsEnabled in H1, we get (from the definition of cdsEnabled) that

$$H1.cts_i \ge H1.its_i + 2 * L \tag{7}$$

Here, we have that  $T_k$  is in  $H2.affectSet(T_i)$ . Thus from the definition of affectSet, we get that

$$H2.its_k < H2.its_i + 2 * L \tag{8}$$

Since  $T_i$  and  $T_j$  are incarnations of each other, their ITS are the same. Combining this with Eq.(8), we get that

$$H2.its_k < H1.its_i + 2 * L \tag{9}$$

We now show this proof through contradiction. Suppose  $T_k$  is not in H1.txns. Then there are two cases:

• No incarnation of  $T_k$  is in H1: This implies that  $T_k$  starts afresh after H1. Since  $T_k$  is not in H1, from Corollary 29 we get that

$$\begin{array}{lll} H2.cts_k > H1.sys\text{-}time & \xrightarrow{T_k \text{ starts afresh}} & H2.its_k > H1.sys\text{-}time & \xrightarrow{(T_i \in H1) \land Lemma \ 28} \\ & H1.cts_i & \xrightarrow{Eq.(7)} & H2.its_k > H1.its_i + 2*L & \xrightarrow{H1.its_i = H2.its_j} & H2.its_k > H2.its_j + 2*L \end{array}$$

But this result contradicts with Eq.(8). Hence, this case is not possible.

• There is an incarnation of  $T_k$ ,  $T_l$  in H1: In this case, we have that

$$H1.its_l = H2.its_k \tag{10}$$

Now combing this result with Eq.(9), we get that  $H1.its_l < H1.its_i + 2 * L$ . This implies that  $T_l$  is in affectSet of  $T_i$  in H1. Since  $T_i$  is cdsEnabled, we get that  $T_l$ 's incarCt must be true.

We also have that  $T_k$  is not in H1 but in H2 where H2 is an extension of H1. Since H2 has some events more than H1, we get that H2 is a strict extension of H1.

Thus, we have that,  $(H1 \sqsubset H2) \land (H1.incarCt(T_l)) \land (T_k \in H2.txns) \land (T_k \notin H1.txns)$ . Combining these with Lemma 21, we get that  $(H1.its_l \neq H2.its_k)$ . But this result contradicts Eq.(10). Hence, this case is also not possible.

Thus from both the cases we get that  $T_k$  should be in H1. Hence proved.

**Lemma 42** Consider two histories H1, H2 where H2 is an extension of H1. Let  $T_i, T_j, T_k$  be three transactions such that  $T_i$  is in H1.txns while  $T_j, T_k$  are in H2.txns. Suppose we have that (1) cts\_i is greater than its\_i + 2 \* L in H1; (2)  $T_i$  is an incarnation of  $T_j$ ; (3)  $T_k$  is in affectSet of  $T_j$  in H2. Then an incarnation of  $T_k$ , say  $T_l$  (which could be same as  $T_k$ ) is in H1.txns. Formally,  $\langle H1, H2, T_i, T_j, T_k : (H1 \sqsubseteq H2) \land (T_i \in H1.txns) \land (\{T_j, T_k\} \in H2.txns) \land (H1.cts_i > H1.its_i + 2 * L) \land (T_i \in H2.incarSet(T_j)) \land (T_k \in H2.affectSet(T_j)) \implies (\exists T_l : (T_l \in H2.incarSet(T_k)) \land (T_l \in H1.txns)) \rangle$ 

#### Proof

This proof is similar to the proof of Lemma 41. We are given that

$$H1.cts_i \ge H1.its_i + 2 * L \tag{11}$$

We now show this proof through contradiction. Suppose no incarnation of  $T_k$  is in H1.txns. This implies that  $T_k$  must have started afresh in some history H3 which is an extension of H1. Also note that H3 could be same as H2 or a prefix of it, i.e.,  $H3 \sqsubseteq H2$ . Thus, we have that

$$H3.its_k > H1.sys\text{-}time \xrightarrow{Lemma~28} H3.its_k > H1.cts_i \xrightarrow{Eq.(11)} H3.its_k > H1.its_i + 2 * L \xrightarrow{H1.its_i = H2.its_j} H3.its_k > H2.its_j + 2 * L \xrightarrow{H3 \sqsubseteq H2} H2.its_j > H2.its_j + 2 * L \xrightarrow{affectSet} T_k \notin H2.affectSet(T_j)$$

But we are given that  $T_k$  is in affectSet of  $T_j$  in H2. Hence, it is not possible that  $T_k$  started afresh after H1. Thus,  $T_k$  must have a incarnation in H1.

**Lemma 43** Consider a transaction  $T_i$  which is cdsEnabled in a history H1. Consider an extension of H1, H2 with a transaction  $T_j$  in it such that  $T_j$  is an incarnation of  $T_i$  in H2. Then affectSet of  $T_i$  in H1 is same as the affectSet of  $T_j$  in H2. Formally,  $\langle H1, H2, T_i, T_j : (H1 \sqsubseteq H2) \land (H1.cdsEnabled(T_i)) \land (T_j \in H2.txns) \land (T_i \in H2.incarSet(T_j)) \implies ((H1.affectSet(T_i) = H2.affectSet(T_j))) \rangle$ 

**Proof.** From the definition of cdsEnabled, we get that  $T_i$  is in H1.txns. Now to prove that affectSets are the same, we have to show that  $(H1.affectSet(T_i) \subseteq H2.affectSet(T_j))$  and  $(H1.affectSet(T_j) \subseteq H2.affectSet(T_i))$ . We show them one by one:

 $(H1.affectSet(T_i) \subseteq H2.affectSet(T_j))$ : Consider a transaction  $T_k$  in  $H1.affectSet(T_i)$ . We have to show that  $T_k$  is also in  $H2.affectSet(T_j)$ . From the definition of affectSet, we get that

$$T_k \in H1.txns \tag{12}$$

Combining Eq.(12) with Observation 23, we get that

$$T_k \in H2.txns \tag{13}$$

From the definition of ITS, we get that

$$H1.its_k = H2.its_k \tag{14}$$

Since  $T_i, T_j$  are incarnations we have that .

$$H1.its_i = H2.its_j \tag{15}$$

From the definition of affectSet, we get that,

 $H1.its_k < H1.its_i + 2*L \xrightarrow{Eq.(14)} H2.its_k < H1.its_i + 2*L \xrightarrow{Eq.(15)} H2.its_k < H2.its_j + 2*L$  Combining this result with Eq.(13), we get that  $T_k \in H2.affectSet(T_j)$ .

 $(H1.affectSet(T_i) \subseteq H2.affectSet(T_j))$ : Consider a transaction  $T_k$  in  $H2.affectSet(T_j)$ . We have to show that  $T_k$  is also in  $H1.affectSet(T_i)$ . From the definition of affectSet, we get that  $T_k \in H2.txns$ .

Here, we have that  $(H1 \sqsubseteq H2) \land (H1.cdsEnabled(T_i)) \land (T_i \in H2.incarSet(T_j)) \land (T_k \in H2.affectSet(T_j))$ . Thus from Lemma 41, we get that  $T_k \in H1.txns$ . Now, this case is similar to the above case. It can be seen that Equations 12, 13, 14, 15 hold good in this case as well.

Since  $T_k$  is in  $H2.affectSet(T_j)$ , we get that

 $H2.its_k < H2.its_i + 2*L \xrightarrow{Eq.(14)} H1.its_k < H2.its_j + 2*L \xrightarrow{Eq.(15)} H1.its_k < H1.its_i + 2*L$  Combining this result with Eq.(12), we get that  $T_k \in H1.affectSet(T_i)$ .

Next we explore how a cdsEnabled transaction remains cdsEnabled in the future histories once it becomes true.

**Lemma 44** Consider two histories H1 and H2 with H2 being an extension of H1. Let  $T_i$  and  $T_j$  be two transactions which are live in H1 and H2 respectively. Let  $T_i$  be an incarnation of  $T_j$  and  $cts_i$  is less than  $cts_j$ . Suppose  $T_i$  is cdsEnabled in H1. Then  $T_j$  is cdsEnabled in H2 as well. Formally,  $\langle H1, H2, T_i, T_j : (H1 \sqsubseteq H2) \land (T_i \in H1.live) \land (T_j \in H2.live) \land (T_i \in H2.incarSet(T_j)) \land (H1.cts_i < H2.cts_j) \land (H1.cdsEnabled(T_i)) \Longrightarrow (H2.cdsEnabled(T_j)) \rangle$ .

**Proof.** We have that  $T_i$  is live in H1 and  $T_j$  is live in H2. Since  $T_i$  is cdsEnabled in H1, we get (from the definition of cdsEnabled) that

$$H1.cts_i \ge H2.its_i + 2 * L \tag{16}$$

We are given that  $cts_i$  is less than  $cts_j$  and  $T_i, T_j$  are incarnations of each other. Hence, we have that

$$H2.cts_j > H1.cts_i$$
  
 $> H1.its_i + 2 * L$  [From Eq.(16)]  
 $> H2.its_j + 2 * L$  [ $its_i = its_j$ ]

Thus we get that  $cts_j > its_j + 2 * L$ . We have that  $T_j$  is live in H2. In order to show that  $T_j$  is cdsEnabled in H2, it only remains to show that cds of  $T_j$  in H2 is empty, i.e.,  $H2.cds(T_j) = \phi$ . The cds becomes empty when all the transactions of  $T_j$ 's affectSet in H2 have their incarCt as true in H2.

Since  $T_j$  is live in H2, we get that  $T_j$  is in H2.txns. Here, we have that  $(H1 \sqsubseteq H2) \land (T_j \in H2.txns) \land (T_i \in H2.incarSet(T_j)) \land (H1.cdsEnabled(T_i))$ . Combining this with Lemma 43, we get that  $H1.affectSet(T_i) = H2.affectSet(T_j)$ .

Now, consider a transaction  $T_k$  in  $H2.affectSet(T_j)$ . From the above result, we get that  $T_k$  is also in  $H1.affectSet(T_i)$ . Since  $T_i$  is cdsEnabled in H1, i.e.,  $H1.cdsEnabled(T_i)$  is true, we get that  $H1.incarCt(T_k)$  is true. Combining this with Observation 20, we get that  $T_k$  must have its incarCt as true in H2 as well, i.e.  $H2.incarCt(T_k)$ . This implies that all the transactions in  $T_j$ 's affectSet have their incarCt flags as true in H2. Hence the  $H2.cds(T_i)$  is empty. As a result,  $T_i$  is cdsEnabled in H2, i.e.,  $H2.cdsEnabled(T_i)$ .

Having defined the properties related to cdsEnabled, we start defining notions for finEnabled. Next, we define maxWTS for a transaction  $T_i$  in H which is the transaction  $T_j$  with the largest WTS in  $T_i$ 's incarSet. Formally,

$$H.maxWTS(T_i) = max\{H.wts_j | (T_j \in H.incarSet(T_i))\}$$

From this definition of maxWTS, we get the following simple observation.

**Observation 45** For any transaction  $T_i$  in H, we have that  $wts_i$  is less than or equal to  $H.maxWTS(T_i)$ . Formally,  $H.wts_i \leq H.maxWTS(T_i)$ .

Next, we combine the notions of affectSet and maxWTS to define *affWTS*. It is the maximum of maxWTS of all the transactions in its affectSet. Formally,

$$H.affWTS(T_i) = max\{H.maxWTS(T_i) | (T_i \in H.affectSet(T_i))\}$$

Having defined the notion of affWTS, we get the following lemma relating the affectSet and affWTS of two transactions.

**Lemma 46** Consider two histories H1 and H2 with H2 being an extension of H1. Let  $T_i$  and  $T_j$  be two transactions which are live in H1 and H2 respectively. Suppose the affectSet of  $T_i$  in H1 is same as affectSet of  $T_j$  in H2. Then the affWTS of  $T_i$  in H1 is same as affWTS of  $T_j$  in H2. Formally,  $\langle H1, H2, T_i, T_j : (H1 \sqsubseteq H2) \land (T_i \in H1.txns) \land (T_j \in H2.txns) \land (H1.affectSet(T_i) = H2.affectSet(T_j)) \Longrightarrow (H1.affWTS(T_i) = H2.affWTS(T_j)) \rangle$ .

#### Proof.

From the definition of affWTS, we get the following equations

$$H.affWTS(T_i) = max\{H.maxWTS(T_k) | (T_k \in H1.affectSet(T_i))\}$$
(17)

$$H.affWTS(T_i) = max\{H.maxWTS(T_i) | (T_i \in H2.affectSet(T_i))\}$$
(18)

From these definitions, let us suppose that  $H1.affWTS(T_i)$  is  $H1.maxWTS(T_p)$  for some transaction  $T_p$  in  $H1.affectSet(T_i)$ . Similarly, suppose that  $H2.affWTS(T_j)$  is  $H2.maxWTS(T_q)$  for some transaction  $T_q$  in  $H2.affectSet(T_i)$ .

Here, we are given that  $H1.affectSet(T_i) = H2.affectSet(T_j)$ ). Hence, we get that  $T_p$  is also in  $H1.affectSet(T_i)$ . Similarly,  $T_q$  is in  $H2.affectSet(T_j)$  as well. Thus from Equations (17) & (18), we get that

$$H1.maxWTS(T_p) \ge H2.maxWTS(T_q)$$
 (19)

$$H2.maxWTS(T_q) \ge H1.maxWTS(T_p)$$
 (20)

Combining these both equations, we get that  $H1.maxWTS(T_p) = H2.maxWTS(T_q)$  which in turn implies that  $H1.affWTS(T_i) = H2.affWTS(T_i)$ .

Finally, using the notion of affWTS and cdsEnabled, we define the notion of finEnabled

**Definition 4** We say that transaction  $T_i$  is finEnabled if the following conditions hold true (1)  $T_i$  is live in H; (2)  $T_i$  is cdsEnabled is H; (3)  $H.wts_i$  is greater than  $H.affWTS(T_i)$ . Formally,

$$H.finEnabled(T_i) = \begin{cases} True & (T_i \in H.live) \land (H.cdsEnabled(T_i)) \land (H.wts_j > H.affWTS(T_i)) \\ False & \textit{otherwise} \end{cases}$$

It can be seen from this definition, a transaction that is finEnabled is also cdsEnabled. We now show that just like itsEnabled and cdsEnabled, once a transaction is finEnabled, it remains finEnabled until it terminates. The following lemma captures it.

**Lemma 47** Consider two histories H1 and H2 with H2 being an extension of H1. Let  $T_i$  and  $T_j$  be two transactions which are live in H1 and H2 respectively. Suppose  $T_i$  is finEnabled in H1. Let  $T_i$  be an incarnation of  $T_j$  and  $cts_i$  is less than  $cts_j$ . Then  $T_j$  is finEnabled in H2 as well. Formally,  $\langle H1, H2, T_i, T_j : (H1 \sqsubseteq H2) \land (T_i \in H1.live) \land (T_j \in H2.live) \land (T_i \in H2.incarSet(T_j)) \land (H1.cts_i < H2.cts_j) \land (H1.finEnabled(T_i)) \Longrightarrow (H2.finEnabled(T_j)) \rangle$ .

**Proof.** Here we are given that  $T_j$  is live in H2. Since  $T_i$  is finEnabled in H1, we get that it is cdsEnabled in H1 as well. Combining this with the conditions given in the lemma statement, we have that,

$$\langle (H1 \sqsubseteq H2) \land (T_i \in H1.live) \land (T_j \in H2.live) \land (T_i \in H2.incarSet(T_j)) \land (H1.cts_i < H2.cts_j) \land (H1.cdsEnabled(T_i)) \rangle$$

$$(21)$$

Combining Eq.(21) with Lemma 44, we get that  $T_j$  is cdsEnabled in H2, i.e.,  $H2.cdsEnabled(T_j)$ . Now, in order to show that  $T_j$  is finEnabled in H2 it remains for us to show that  $H2.wts_j > H2.affWTS(T_j)$ .

We are given that  $T_j$  is live in H2 which in turn implies that  $T_j$  is in H2.txns. Thus changing this in Eq.(21), we get the following

$$\langle (H1 \sqsubseteq H2) \land (T_j \in H2.txns) \land (T_i \in H2.incarSet(T_j)) \land (H1.cts_i < H2.cts_j) \\ \land (H1.cdsEnabled(T_i)) \rangle$$
(22)

Combining Eq.(22) with Lemma 43 we get that

$$H1.affWTS(T_i) = H2.affWTS(T_i)$$
(23)

We are given that  $H1.cts_i < H2.cts_j$ . Combining this with the definition of WTS, we get

$$H1.wts_i < H2.wts_i \tag{24}$$

Since  $T_i$  is finEnabled in H1, we have that

H1.
$$wts_i > H1.affWTS(T_i) \xrightarrow{Eq.(24)} H2.wts_j > H1.affWTS(T_i) \xrightarrow{Eq.(23)} H2.wts_j > H2.affWTS(T_i)$$

Now, we show that a transaction that is finEnabled will eventually commit.

**Lemma 48** Consider a live transaction  $T_i$  in a history H1. Suppose  $T_i$  is finEnabled in H1 and  $valid_i$  is true in H1. Then there exists an extension of H1, H3 in which  $T_i$  is committed. Formally,  $\langle H1, T_i : (T_i \in H1.live) \land (H1.valid_i) \land (H1.finEnabled(T_i)) \implies (\exists H3 : (H1 \sqsubset H3) \land (T_i \in H3.committed)) \rangle$ .

**Proof.** Consider a history H3 such that its sys-time being greater than  $cts_i + L$ . We will prove this lemma using contradiction. Suppose  $T_i$  is aborted in H3.

Now consider  $T_i$  in H1:  $T_i$  is live; its valid flag is true; and is finEnabled. From the definition of finEnabled, we get that it is also cdsEnabled. From Lemma 40, we get that  $T_i$  is itsEnabled in H1. Thus from Lemma 39, we get that there exists an extension of H1, H2 such that (1) Transaction  $T_i$  is live in H2; (2) there is a transaction  $T_j$  in H2; (3)  $H2.wts_j$  is greater than  $H2.wts_i$ ; (4)  $T_j$  is committed in H3. Formally,

$$\langle (\exists H2, T_j : (H1 \sqsubseteq H2 \sqsubset H3) \land (T_i \in H2.live) \land (T_j \in H2.txns) \land (H2.wts_i < H2.wts_j) \\ \land (T_i \in H3.committed)) \rangle$$
(25)

Here, we have that H2 is an extension of H1 with  $T_i$  being live in both of them and  $T_i$  is finEnabled in H1. Thus from Lemma 47, we get that  $T_i$  is finEnabled in H2 as well. Now, let us consider  $T_j$  in H2. From Eq.(25), we get that  $(H2.wts_i < H2.wts_j)$ . Combining this with the observation that  $T_i$  being live in H2, Lemma 33 we get that  $(H2.its_j \le H2.its_i + 2*L)$ .

This implies that  $T_j$  is in affectSet of  $T_i$  in H2, i.e.,  $(T_j \in H2.affectSet(T_i))$ . From the definition of affWTS, we get that

$$(H2.affWTS(T_i) \ge H2.maxWTS(T_i)) \tag{26}$$

Since  $T_i$  is finEnabled in H2, we get that  $wts_i$  is greater than affWTS of  $T_i$  in H2.

$$(H2.wts_i > H2.affWTS(T_i)) \tag{27}$$

Now combining Equations 26, 27 we get,

$$\begin{split} H2.wts_i > & H2.affWTS(T_i) \geq H2.maxWTS(T_j) \\ > & H2.affWTS(T_i) \geq H2.maxWTS(T_j) \geq H2.wts_j \\ > & H2.wts_j \end{split}$$
 [From Observation 45]

But this equation contradicts with Eq.(25). Hence our assumption that  $T_i$  will get aborted in H3 after getting finEnabled is not possible. Thus  $T_i$  has to commit in H3.

Next we show that once a transaction  $T_i$  becomes its Enabled, it will eventually become fine nabled as well and then committed. We show this change happens in a sequence of steps. We first show that Transaction  $T_i$  which is its Enabled first becomes cds Enabled (or gets committed). We next show that  $T_i$  which is cds Enabled becomes fine nabled or get committed. On becoming fine nabled, we have already shown that  $T_i$  will eventually commit.

Now, we show that a transaction that is its Enabled will become cds Enabled or committed. To show this, we introduce a few more notations and definitions. We start with the notion of *depIts* (*dependent-its*) which is the set of ITSs that a transaction  $T_i$  depends on to commit. It is the set of ITS of all the transactions in  $T_i$ 's cds in a history H. Formally,

$$H.depIts(T_i) = \{H.its_j | T_j \in H.cds(T_i)\}$$

We have the following lemma on the depIts of a transaction  $T_i$  and its future incarnation  $T_j$  which states that depIts of a  $T_i$  either reduces or remains the same.

**Lemma 49** Consider two histories H1 and H2 with H2 being an extension of H1. Let  $T_i$  and  $T_j$  be two transactions which are live in H1 and H2 respectively and  $T_i$  is an incarnation of  $T_j$ . In addition, we also have that  $cts_i$  is greater than  $its_i + 2 * L$  in H1. Then, we get that  $H2.depIts(T_j)$  is a subset of  $H1.depIts(T_i)$ . Formally,  $\langle H1, H2, T_i, T_j : (H1 \sqsubseteq H2) \land (T_i \in H1.live) \land (T_j \in H2.live) \land (T_i \in H2.incarSet(T_j)) \land (H1.cts_i \ge H1.its_i + 2 * L) \implies (H2.depIts(T_j) \subseteq H1.depIts(T_i)) \rangle$ .

**Proof.** Suppose  $H2.depIts(T_j)$  is not a subset of  $H1.depIts(T_i)$ . This implies that there is a transaction  $T_k$  such that  $H2.its_k \in H2.depIts(T_j)$  but  $H1.its_k \notin H1.depIts(T_j)$ . This implies that  $T_k$  starts afresh after H1 in some history say H3 such that  $H1 \sqsubset H3 \sqsubseteq H2$ . Hence, from Corollary 29 we get the following

some history say 
$$H3$$
 such that  $H1 \sqsubset H3 \sqsubseteq H2$ . Hence, from Corollary 29 we get the following  $H3.its_k > H1.sys\text{-}time \xrightarrow{Lemma~28} H3.its_k > H1.cts_i \implies H3.its_k > H1.its_i + 2*L \xrightarrow{H1.its_i = H2.its_j} H3.its_k > H2.its_j + 2*L \xrightarrow{affectSet, depIts} H2.its_k \notin H2.depIts(T_j)$ 

We started with its\_sin  $H2$  does  $H_2(T)$  and ended with its\_snot in  $H2$  does  $H_2(T)$ . Thus, we have a contraction of the  $H3$  does  $H4$  and  $H4$  are  $H4$  and  $H4$  and  $H4$  and  $H4$  are  $H4$  and  $H4$  and  $H4$  are  $H4$  are  $H4$  are  $H4$  are  $H4$  and  $H4$  are  $H4$  are  $H4$  are  $H4$  and  $H4$  are  $H$ 

We started with  $its_k$  in  $H2.depIts(T_j)$  and ended with  $its_k$  not in  $H2.depIts(T_j)$ . Thus, we have a contradiction. Hence, the lemma follows.

Next we denote the set of committed transactions in  $T_i$ 's affectSet in H as cis (commit independent set). Formally,

$$H.cis(T_i) = \{T_i | (T_i \in H.affectSet(T_i)) \land (H.incarCt(T_i)) \}$$

In other words, we have that  $H.cis(T_i) = H.affectSet(T_i) - H.cds(T_i)$ . Finally, using the notion of cis we denote the maximum of maxWTS of all the transactions in  $T_i$ 's cis as partAffWTS (partly affecting WTS). It turns out that the value of partAffWTS affects the commit of  $T_i$  which we show in the course of the proof. Formally, partAffWTS is defined as

$$H.partAffWTS(T_i) = max\{H.maxWTS(T_i)|(T_i \in H.cis(T_i))\}$$

Having defined the required notations, we are now ready to show that a itsEnabled transaction will eventually become cdsEnabled.

**Lemma 50** Consider a transaction  $T_i$  which is live in a history H1 and  $cts_i$  is greater than or equal to  $its_i+2*L$ . If  $T_i$  is itsEnabled in H1 then there is an extension of H1, H2 in which an incarnation  $T_i$ ,  $T_j$  (which could be same as  $T_i$ ), is either committed or cdsEnabled. Formally,  $\langle H1, T_i : (T_i \in H1.live) \land (H1.cts_i \geq H1.its_i + 2*L) \land (H1.itsEnabled(T_i)) \implies (\exists H2, T_j : (H1 \sqsubseteq H2) \land (T_j \in H2.incarSet(T_i)) \land ((T_j \in H2.committed) \lor (H2.cdsEnabled(T_j))))$ .

**Proof.** We prove this by inducting on the size of  $H1.depIts(T_i)$ , n. For showing this, we define a boolean function P(k) as follows:

$$P(k) = \begin{cases} True & \langle H1, T_i : (T_i \in H1.live) \land (H1.cts_i \geq H1.its_i + 2 * L) \land (H1.itsEnabled(T_i)) \land \\ & (k \geq |H1.depIts(T_i)|) \implies (\exists H2, T_j : (H1 \sqsubset H2) \land (T_j \in H2.incarSet(T_i)) \land \\ & ((T_j \in H2.committed) \lor (H2.cdsEnabled(T_j)))) \rangle \\ & False & \text{otherwise} \end{cases}$$

As can be seen, here P(k) means that if (1)  $T_i$  is live in H1; (2)  $cts_i$  is greater than or equal to  $its_i + 2 * L$ ; (3)  $T_i$  is itsEnabled in H1 (4) the size of  $H1.depIts(T_i)$  is less than or equal to k; then there exists a history H2 with a transaction  $T_j$  in it which is an incarnation of  $T_i$  such that  $T_j$  is either committed or cdsEnabled in H2. We show P(k) is true for all (integer) values of k using induction.

**Base Case** - P(0): Here, from the definition of P(0), we get that  $|H1.depIts(T_i)| = 0$ . This in turn implies that  $H1.cds(T_i)$  is null. Further, we are already given that  $T_i$  is live in H1 and  $H1.cts_i \geq H1.its_i + 2*L$ . Hence, all these imply that  $T_i$  is cdsEnabled in H1.

Induction case - To prove P(k+1) given that P(k) is true: If  $|H1.depIts(T_i)| \le k$ , from the induction hypothesis P(k), we get that  $T_i$  is either committed or cdsEnabled in H2. Hence, we consider the case when

$$|H1.depIts(T_i)| = k+1 \tag{28}$$

Let  $\alpha$  be  $H1.partAffWTS(T_i)$ . Suppose  $H1.wts_i < \alpha$ . Then from Lemma 27, we get that there is an extension of H1, say H3 in which an incarnation of  $T_i$ ,  $T_l$  (which could be same as  $T_i$ ) is committed or is live in H3 and has WTS greater than  $\alpha$ . If  $T_l$  is committed then P(k+1) is trivially true. So we consider the latter case in which  $T_l$  is live in H3. In case  $H1.wts_i \geq \alpha$ , then in the analysis below follow where we can replace  $T_l$  with  $T_i$ .

Next, suppose  $T_l$  is aborted in an extension of H3, H5. Then from Lemma 39, we get that there exists an extension of H3, H4 in which (1)  $T_l$  is live; (2) there is a transaction  $T_m$  in H4.txns; (3)  $H4.wts_m > H4.wts_l$  (4)  $T_m$  is committed in H5.

Combining the above derived conditions (1), (2), (3) with Lemma 36 we get that in H4,

$$H4.its_m \le H4.its_l + 2 * L \tag{29}$$

Eq.(29) implies that  $T_m$  is in  $T_l$ 's affectSet. Here, we have that  $T_l$  is an incarnation of  $T_i$  and we are given that  $H1.cts_i \ge H1.its_i + 2 * L$ . Thus from Lemma 42, we get that there exists an incarnation of  $T_m$ ,  $T_n$  in H1.

Combining Eq.(29) with the observations (a)  $T_n, T_m$  are incarnations; (b)  $T_l, T_i$  are incarnations; (c)  $T_i, T_n$  are in H1.txns, we get that  $H1.its_n \leq H1.its_i + 2 * L$ . This implies that  $T_n$  is in  $H1.affectSet(T_i)$ . Since  $T_n$  is not committed in H1 (otherwise, it is not possible for  $T_m$  to be an incarnation of  $T_n$ ), we get that  $H1.cds(T_i)$ . Hence, we get that  $H4.its_m = H1.its_n$  is in  $H1.depIts(T_i)$ .

From Eq.(28), we have that  $H1.depIts(T_i)$  is k+1. From Lemma 49, we get that  $H4.depIts(T_i)$  is a subset of  $H1.depIts(T_i)$ . Further, we have that transaction  $T_m$  has committed. Thus  $H4.its_m$  which was in  $H1.depIts(T_i)$  is no longer in  $H4.depIts(T_i)$ . This implies that  $H4.depIts(T_i)$  is a strict subset of  $H1.depIts(T_i)$  and hence  $|H4.depIts(T_i)| \le k$ .

Since  $T_i$  and  $T_l$  are incarnations, we get that  $H4.depIts(T_i) = H1.depIts(T_l)$ . Thus, we get that

$$|H4.depIts(T_i)| \le k \implies |H4.depIts(T_l)| \le k$$
 (30)

Further, we have that  $T_l$  is a later incarnation of  $T_i$ . So, we get that

$$H4.cts_l > H4.cts_i \xrightarrow{given} H4.cts_l > H4.its_i + 2 * L \xrightarrow{H4.its_i = H4.its_l} H4.cts_l > H4.its_l + 2 * L \tag{31}$$

We also have that  $T_l$  is live in H4. Combining this with Equations 30, 31 and given the induction hypothesis that P(k) is true, we get that there exists a history extension of H4, H6 in which an incarnation of  $T_l$  (also  $T_i$ ),  $T_p$  is either committed or cdsEnabled. This proves the lemma.

**Lemma 51** Consider a transaction  $T_i$  in a history H1. If  $T_i$  is cdsEnabled in H1 then there is an extension of H1, H2 in which an incarnation  $T_i$ ,  $T_j$  (which could be same as  $T_i$ ), is either committed or finEnabled. Formally,  $\langle H1, T_i: (T_i \in H.live) \land (H1.cdsEnabled(T_i)) \implies (\exists H2, T_j: (H1 \sqsubset H2) \land (T_j \in H2.incarSet(T_i)) \land ((T_j \in H2.committed) \lor (H2.finEnabled(T_j))) \rangle.$ 

**Proof.** In H1, suppose  $H1.affWTS(T_i)$  is  $\alpha$ . From Lemma 27, we get that there is a extension of H1, H2 with a transaction  $T_j$  which is an incarnation of  $T_i$ . Here there are two cases: (1) Either  $T_j$  is committed in H2. This trivially proves the lemma; (2) Otherwise,  $wts_j$  is greater than  $\alpha$ . In the second case, we get that

$$(T_i \in H1.live) \land (T_j \in H2.live) \land (H.cdsEnabled(T_i)) \land (T_j \in H2.incarSet(T_i)) \land (H1.wts_i < H2.wts_j)$$

$$(32)$$

Combining the above result with Lemma 26, we get that  $H1.cts_i < H2.cts_j$ . Thus the modified equation is

$$(T_i \in H1.live) \land (T_j \in H2.live) \land (H1.cdsEnabled(T_i)) \land (T_j \in H2.incarSet(T_i)) \land (H1.cts_i < H2.cts_j)$$

$$(33)$$

Next combining Eq.(33) with Lemma 43, we get that

$$H1.affectSet(T_i) = H2.affectSet(T_i)$$
 (34)

Similarly, combining Eq.(33) with Lemma 44 we get that  $T_i$  is cdsEnabled in H2 as well. Formally,

$$H2.cdsEnabled(T_i)$$
 (35)

Now combining Eq.(34) with Lemma 46, we get that

$$H1.affWTS(T_i) = H2.affWTS(T_i)$$
(36)

From our initial assumption we have that  $H1.affWTS(T_i)$  is  $\alpha$ . From Eq.(36), we get that  $H2.affWTS(T_j) = \alpha$ . Further, we had earlier also seen that  $H2.wts_j$  is greater than  $\alpha$ . Hence, we have that  $H2.wts_j > H2.affWTS(T_j)$ . Combining the above result with Eq.(35),  $H2.cdsEnabled(T_j)$ , we get that  $T_j$  is finEnabled, i.e.,  $H2.finEnabled(T_j)$ .

Next, we show that every live transaction eventually become itsEnabled.

**Lemma 52** Consider a history H1 with  $T_i$  be a transaction in H1.live. Then there is an extension of H1, H2 in which an incarnation of  $T_i$ ,  $T_j$  (which could be same as  $T_i$ ) is either committed or is itsEnabled. Formally,  $\langle H1, T_i : (T_i \in H.live) \implies (\exists T_j, H2 : (H1 \sqsubset H2) \land (T_j \in H2.incarSet(T_i)) \land (T_j \in H2.committed) \lor (H.itsEnabled(T_i))) \rangle$ .

**Proof.** We prove this lemma by inducting on ITS.

**Base Case -**  $its_i = 1$ : In this case,  $T_i$  is the first transaction to be created. There are no transactions with smaller ITS. Thus  $T_i$  is trivially itsEnabled.

**Induction Case:** Here we assume that for any transaction  $its_i \leq k$  the lemma is true.

Combining these lemmas gives us the result that for every live transaction  $T_i$  there is an incarnation  $T_j$  (which could be the same as  $T_i$ ) that will commit. This implies that every application-transaction eventually commits. The follow lemma captures this notion.

**Theorem 53** Consider a history H1 with  $T_i$  be a transaction in H1.live. Then there is an extension of H1, H2 in which an incarnation of  $T_i$ ,  $T_j$  is committed. Formally,  $\langle H1, T_i : (T_i \in H.live) \implies (\exists T_j, H2 : (H1 \sqsubset H2) \land (T_j \in H2.incarSet(T_i)) \land (T_j \in H2.committed)) \rangle$ .

**Proof.** Here we show the states that a transaction  $T_i$  (or one of it its incarnations) undergoes before it commits. In all these transitions, it is possible that an incarnation of  $T_i$  can commit. But to show the worst case, we assume that no incarnation of  $T_i$  commits. Continuing with this argument, we show that finally an incarnation of  $T_i$  commits.

Consider a live transaction  $T_i$  in H1. Then from Lemma 52, we get that there is a history H2, which is an extension of H1, in which  $T_j$  an incarnation of  $T_i$  is either committed or itsEnabled. If  $T_j$  is itsEnabled in H2, then from Lemma 50, we get that  $T_k$ , an incarnation of  $T_j$ , will be cdsEnabled in a extension of H2, H3 (assuming that  $T_k$  is not committed in H3).

From Lemma 51, we get that there is an extension of H3, H4 in which an incarnation of  $T_k$ ,  $T_l$  will be finEnabled assuming that it is not committed in H4. Finally, from Lemma 48, we get that there is an extension of H4 in which  $T_m$ , an incarnation of  $T_l$ , will be committed. This proves our theorem.

# 7 Discussion and Conclusion

In this paper, we propose a K version *starvation-free* STM system, KSFTM. The algorithm ensures that if an *aborted* transaction is retried successively, then it will eventually commit. The algorithm maintains K versions where K can range from between one to infinity. For correctness, we show KSFTM satisfies strict-serializability [22] and local opacity [18, 19]. To the best of our knowledge, this is the first work to explore *starvation-freedom* with MVSTMs.

Our experiments show that *KSFTM* performs better than single-version STMs (ESTM, Norec STM) under high contention and also single-version *starvation-free* STM *SV-SFTM* developed based on the principle of priority. On the other hand, its performance is comparable or slightly worse than multi-version STM, *PKTO* (around 2%). This is the cost of the overhead required to achieve *starvation-freedom* which we believe is a marginal price.

In this document, we have not considered a transactional solution based on two-phase locking (2PL) and its multi-version variants [28]. With the carefully designed 2PL solution, one can ensure that none of the transactions abort [28]. But this will require advance knowledge of the code of the transactions which may not always be available with the STM library. Without such knowledge, it is possible that a 2PL solution can deadlock and cause further aborts which will, raise the issue of *starvation-freedom* again.

Since we have considered strict-serializable as one of the *correctness-criteria*, this algorithm can be extended to databases as well. In fact, to the best of our knowledge, there has been no prior work on *starvation-freedom* in the context of database concurrency control.

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