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Theory of Oscillations

Structural Mathematical Modeling
in Problems of Dynamics of Technical Objects

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Preface

Ensuring safety of the operation of the modern machines and their reliable performance in the conditions of intensive dynamic loading is a topical area of focus in theoretical researches and constructional and technological developments in the sphere of the mechanical engineering, theory of machines and mechanisms, theory of vibrations, dynamics and strength, theory of control, mathematical modelling that are all parts of the interdisciplinary space combining the efforts of many specialists. Dynamic quality of different machines and the possibilities of their effective operation are studied and considered in detail and are in the zone of special attention at all the stages of making modern technical objects. In most cases, dynamic interactions of elements of different machines that are manifested through vibrations evoked by the internal and external elements become important trends in estimating the possibilities of machines while providing working efficiency and operation safety of technical systems. The basis of the exploratory researches and applied developments are most often the dynamic processes in mechanical vibration systems of different complexity that are considered as computational schemes, with the problems of dynamics of technical objects being converted into the corresponding research areas of the applied vibration theory.

In the presented monography, the generalized approaches in the researches of the properties of the mechanical vibratory systems based on the structural methods using the principles of dynamic analogies that characteristic for movements of elements of mechanical vibrating systems, including automatic control systems are developed. The commonality of approaches is especially well manifested through the problems of vibration protection of machines, equipment, instruments and equipment and implementations of vibration technological processes.

In the monography, the results of the researches that have been carried out during the recent years in the Irkutsk State Transport University are presented.

The area of the research is connected to the problems of machine dynamics and reflects the interests that have been formed during long-standing contacts with scientific schools in the sphere of theoretical and applied mechanics supported.

The generalized idea of the suggested research is the structural mathematical modelling in the dynamics of the mechanical vibratory systems in general and vibration protection systems in particular. The structural model (or scheme) within the developed methodology, which is regarded, is a graphical analogue of the source mathematical model in the form of a system of linear ordinary differential equations in operator form. In this regard, structural mathematical modelling based on the analogy with automated control systems, the principle of feedback and the equivalent transformations can be compared with the theory of circuits and theory of graphs.

The primary focus in the monography is the study of the peculiarities of the formation of extra connections in the mechanical vibratory systems. These extra connections are brought into the dynamic interaction by the elements of a different nature. In particular, the motion translation devices, lever mechanisms and lever linkages are closely regarded. The first three chapters convey general information and contain necessary survey material that specifies certain tendencies of formation of understanding on the expansion of the element base of the mechanical vibratory systems in general and vibration insulation systems in particular. There is wide diversity in constructive and technical forms of the elements' interconnection into different structures. That generally approves the necessity of separation of not typical elements only but more complex constructs defined as compacts, quasisprings etc., having the reduced mass-and-inertia and elastic dissipational properties.

The fourth chapter of the monography is dedicated to the specification of understanding ways and technologies of the transformation of structural diagrams that reflect the properties of the mechanical vibratory systems with one and more degrees of freedom.

The present work includes the definitions of automated control theory that are connected to the transfer functions of systems, frequency properties of systems and their particular aspects in the dynamic responses to different external perturbations. The specific feature is the capability of obtaining the constructs from a few interconnected elementary units that have the properties of interacting with each other using the same rules of transformations as elementary units.

The fifth chapter of the monography represents the further development of the methodological positions regarding the transformation of structural diagrams of mechanical vibratory systems. The specific aspect of the approaches is the particularization of ways of simplifying the systems by introducing concatenations. Some variations in changing the systems' properties by concatenating the elements are suggested, either based on the «zeroing» the relative coordinated of motion, or as well as by selecting the limiting values of mass and inertial or elastic characteristics in the certain parts of the system.

Chapter 6 contains the results of the researches dedicated to the studies of possible equivalent transformations of structural diagrams of mechanical vibratory systems. It describes the ways of defining the reduced stiffnesses and the reduced masses with regard to the computational schemes of technical objects that contain different mechanisms or motion translation devices.

In Chapter 7, the ideas of distinguishing the lever linkages and taking their special aspects into consideration have been further developed, being applied to mechanical vibratory systems with one or more degrees of freedom. Some dynamic effects that take place with the lever linkages are implemented in mechanical structures with partial systems that have different types of motions (translational, rotational and helicoidal).

The way different mechanisms influence the dynamic properties of the vibration protection systems, and the researches on the special aspects of this influence are represented in Chapter 8. Some original construction and technical solutions are considered. The approaches that are developed in the present monography make it possible to considerably expand the concepts of capabilities of systems while implementing the modes of dynamic absorbing of vibrations.

In the authors' opinion, the researches that have been done broaden the concepts about the capabilities of structural methods of mathematical modelling, due to their deep connection with control theory and system analysis, and also provide new opportunities in the solutions of problems of dynamic synthesis being applied to the dynamics of wide class of technical objects that are subject to intensive vibration loadings.

The authors would like to express their gratitude to N. K. Kuznetsov (D.Sc. in Engineering, Prof.) and P. A. Lontsikh (D.Sc. in Engineering, Prof.) from the Irkutsk National Research Technical University for their support, kind attention and help in finding solutions of scientific and technical questions.

Irkutsk, Russia
March, 2019

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Introduction

Ensuring reliability and safety of operation of machines is a modern problem solution of which is connected with interdisciplinary concepts of the dynamic interactions of the numerous elements of technical systems. Creating new machines and equipment is usually preceded by the stage of representational scientific researches and developments, in the course of which the capabilities of machines during the implementation of specified processes with the defined accuracy and the effectiveness, as well as their adaptive properties in case of any changes in conditions of their functioning.

Vibrations of machines and equipment are characteristic of the operation of the most technological and transport machines. That is specified by regulations and is formalized in the corresponding regulatory technical documents. The problems of vibration protection of machines, equipment and instruments are referred as widespread and sufficiently studied that have been reflected in the works of indigenous Russian scientists [1–3]. In the capacity of computational schemes, machines and mechanisms, in many cases mechanical vibratory systems with one, two and more degrees of freedom are considered. The greatest development in the evaluation of the dynamic properties of the mechanical vibratory systems is the analytical methods that are based on the mathematical models in the form of a system of a regular differential equations with constant coefficients. This approach is oriented to the models with linear properties and lumped parameters. Theoretical basics of the evaluation and studying of the dynamic properties of such systems found their implementation in the works dedicated to the theory of oscillations [4–8], which is acceptable enough to assume that the object of protection executes little vibrations with a relation to the static equilibrium position. These assumptions have been made during the preliminary and exploratory researches.

In more detail, nonlinear systems are considered, with special methods being applied to estimate their properties [9–11]. In theory and practice of vibration protection, there is an experience widely applied while solving the problems connected to the dynamics of transport systems [12, 13].

The development of technical means of restriction of vibration processes initiated the development of wide class of special devices in the form of suspension

brackets, buffer springs, shock absorbers, dampers, vibration absorbers, etc. [14], which to the certain degree presupposed the interest towards the capabilities of expanding the set of typical elements of vibration protection systems and usage of different mechanisms to transform motion and implement the lever linkages ensuring the dynamic interactions in the certain sphere of interaction.

The ideas of active vibration control [15–19] had a considerable impact on the vibration protection theory. Within the framework of the active vibration control, the servo-actuators of different types, measuring devices and means of computer equipment became applicable within the structure of mechanical vibratory systems.

In their developed form, the vibration protection systems represent tailor-made automated control systems, which naturally presuppose the conditions of the generalized approach based on the concepts of the object state control and application of the feedback principles in the development of the corresponding mathematical models [1, 2, 20–22].

Regarding the problems of vibration protection in the linear setting satisfies only those research intentions that correspond to the preliminary evaluations, which is characteristic of the exploratory developments and approximate evaluations in case if there is not enough data on the structure and parameters of the technical object that is subject to the vibrations.

Structural mathematical modelling technology is based on the use of Laplace transformations in relation to the reference mathematical model in a form of a system of linear regular differential equations. Distinguishing the object of protection allows creating a structural diagram of a linear mechanical vibratory system which has the same form as an automated control system with an analytical mathematical model. Having functional analogues in relation to the elements of mechanical systems in the form of springs, dampers and mass-and-inertia units, it is easy to form a set of typical elements of the differential units of the first and second orders, and also integrating units of the first and second orders.

The obligatory condition of the inseparability of processes and continuity of interactions is the homogeneity of typical elements, when displacement is the input signal of each unit, and effort (or the force) is the output signal of a typical unit. The object of protection is a unit with a transfer function of the integration of the second order. The technology of these constructions and structural transformations has been reflected in [20, 23–24].

Transformation of structural diagrams is implemented on the basis of the rules of structural transformations of the automated control theory. Complication of computational schemes is related to the fact that different motion translation devices, as well as mechanisms expanding their functions of vibration protection, have been introduced into their structure.

Structural mathematical modelling has a certain convenience in distinguishing the constraints occurring during the introduction of elements, that is manifested while the structural transformations of the reference systems are being done, and also while taking into consideration the different factors arising during the additional constraints of a different nature being distinguished. In the physical signal, such additional constraints can be implemented in a form of special (lever, toothed,

non-locking screw, etc.) mechanisms and devices. These kinds of approaches are characteristics of not only vibration protection systems but also of robotic science, suspension transport systems and vibrational technological machines [26–29].

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