

Studies in Systems, Decision and Control

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Uncertain Information and Linear Systems

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To My Father

And

My Late Teacher, Prof. G. R. Jahanshahloo

Preface

In this book, I tried to introduce and apply the uncertain information or data in several types to analyze the linear systems. These versions of information are very applicable in our applied science. The initial subjects of this book point out the important uncertainties to use in real-life problem modeling.

Having information about several types of ambiguities, vagueness, and uncertainties is important in modeling the problems that involve linguistic variables, parameters, and word computing. Nowadays, most of our real-life problems are related to decision making at the right time, and therefore, we should use intelligent decision science. Clearly, every intelligent system needs real data in our environment to have an appropriate and flexible mathematical model. Most of the mentioned problems can be modeled by mathematical models, and a system of linear equations is their final status that must be solved. The newest versions of uncertain information have been discussed in this book.

This book has been prepared for all undergraduate students in mathematics, computer science, and engineering involved with fuzzy and uncertainty. Especially in industrial engineering and applied mathematics in the field of optimization, one of the most important subjects is the linear systems with uncertainty.

Istanbul, Turkey
September, 2019

Tofiqh Allahviranloo

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