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Fuzzy Relational Mathematical Programming

Linear, Nonlinear and Geometric Programming Models

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Preface

In 1987, I began to pay attention to the study of fuzzy relational equations, then, in 1998, I tried to introduce fuzzy relation to fuzzy geometric programming, and in 2004, while working in Shantou University, I enrolled my first Ph.D. (Ji-Hui Yang), who was the first student of mine researching for fuzzy geometric Programming. He began to study the programming, and in particular, the relationship between fuzzy relational inequalities and geometric programming, and put it as topic of his doctoral work. In 2005, we proposed for the first time fuzzy relational geometric programming at the Fuzzy Systems, IEEE International Conference held in the United States. Since then, my Ph.D. and postdoctoral students have been studying fuzzy relational geometric programming as their major topics. We have combined our research on fuzzy geometric programming with researches on fuzzy relational linear programming, achieving interesting results. In this book, we describe our own research work, and refer to papers by other scholars such as S. C. Fang, Y. K. Wu, etc.

The writing tasks have been distributed as follows:

1. Basic Theory of Fuzzy Set;
2. Fuzzy Relation;
3. Fuzzy Relational Equations/Inequalities;
4. Fuzzy Relational Linear Programming;
5. Fuzzy Relational Geometric Programming;
6. Relational Geometric Programming with Fuzzy Coefficient;
7. Fuzzy Relational Non-linear Programming;
8. $(+, \wedge)$ Fuzzy Relational Inequality and Its Network Optimization;
9. Research Progress of Fuzzy Relational Geometric Programming.

Professor Bing-Yuan Cao from Foshan University, Guangzhou University and Guangzhou Vocational and Technical University of Science and Technology wrote Sects. 1.1–1.5, 3.2, 3.3, 4.3, 8.1, 8.2, Chap. 9 and was also responsible for compiling the book; Dr. Ji-Hui Yang, associate professor of Shenyang Agricultural University, took care of Chap. 2, and Sects. 5.3, 5.4, 6.1, 6.2.; Dr. Xue-Guang Zhou, associate professor of Guangdong University of Finance, wrote Sects. 3.1, 5.1, 5.2 and

Chap. 7. F. Zahmatkesh, China Ph.D. candidates, at Guangzhou University wrote Sects. 4.1, 4.2; Z. Kheiri, China Ph.D. Candidates at Guangzhou University took care of Sects. 1.6, 4.4, 6.3 and Dr. Xiao-Peng Yang, associate prof. of Hanshan Teachers College, completed Sects. 4.5, 8.1, 8.3.

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At present, research on fuzzy relational programming, especially the theory and applications of fuzzy relational geometric programming, is rising worldwide, achieving gratifying results. However three issues of fuzzy geometric programming, remains to be resolved. I hope that its publication enables researchers in fuzzy relational programming to move on in the conjecture of solutions to the three guesses.

Guangzhou, China Bing-Yuan Cao
2018 Chinese New Year

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