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Hans-Dieter Kochs

# Dependability of Engineering Systems

A Markov Minimal Cut Approach

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# Preface

The dependability (defined in IEC 60050-192:2015) of engineering systems is strongly affected by stochastic dependencies (s-dependencies) between their components. Whenever a system crash occurs, it is always considered as a “concatenation of tragic circumstances”, but detailed analyses have mostly uncovered that system failures are caused by inadequate system design and s-dependencies between components, often triggered by common cause failures (*CCF*) or, in the worst case, by systematic failures. Also, the increasing interactions between components of complex structured systems as well as preventive and corrective maintenance (repair) strategies can cause s-dependencies and strongly affect system dependability. The recent severe crashes of two airplanes of the type Boeing 737 Max in 2018 and the failure of the Galileo satellite navigation system in 2019 underline the increasing importance of designing redundant structures including human interaction for systems with high dependability requirements. Therefore, a challenge is the accurate modeling and calculation of the dependability of system structures in order to identify weaknesses in the system design and to assess the impact of s-dependency on system dependability. A particular interest in this book focuses on the impact of s-dependency on system-relevant redundant structures.

*Dependability analysis method.* The *Markov minimal cut (MMC)* approach combined with the *probable Markov path (pMp)* approach is a powerful method for the evaluation of the dependability of large and complex engineering systems including s-dependent components. A *MMC* is a minimal cut (*MC*) which is modeled as a Markov process. The main task in the development process of *MMC* is to create Markov process models (Markov models), which are compatible to the *MC*. The mathematical basics and the procedure to create these models are described in (Kochs 2017).

*What is new?* The newly introduced systematics in the following chapters gives an in-depth understanding of *precise* and *approximate MMC* modeling and calculation techniques of engineering systems, which has not been addressed in previous works. The in-depth analysis demonstrates that it is only possible to *precisely* model and calculate the dependability of systems including s-dependent components with the knowledge of their (total) universe spaces, represented here by

Markov spaces. They provide the basis for developing and verifying *approximated MMC* models for engineering systems. Missing universe spaces can be a severe problem for dependability analyses (apart from the lack of data and data uncertainty). However, with the assumption of using realistic parameter values, *approximated MMC* models of systems can be developed without consideration of the total Markov spaces. Many examples of redundant structures show clearly the universal application of *MMC* modeling and calculation techniques with emphasis on model accuracy.

*Objective of this book.* The book pursues the following aims with focus on reliability, availability, and safety: (1) Determination of the minimal cuts (*MC*) of systems; (2) *New: Precise* modeling of the Markov minimal cuts (*MMC*), based on the equivalence between logical networks and Markov space models including s-dependencies; (3) Development of *approximated MMC* models and analytical calculation of their indices using the probable Markov path (*pMp*) approach; (4) Integration of *MMC* models into the framework of Fig. 1.2 and calculation of the system indices; (5) Application of the *MMC* approach to several examples, which represent basic redundant structures of engineering systems; (6) Estimation of the impact of s-dependencies on systems using the stochastic dependency impact (*sDI*) factor; (7) *New: Comparison* between *exact* and *approximate* results. Points 1 to 4 represent the main steps of the *MMC* approach. Special emphasis is placed on Points 1 and 7 in combination with Points 2, 3, and 4, which is new. With the mathematical steps, described and applied to several examples throughout this text, interested system developers and users can perform dependability analyses themselves. All examples are structured in precisely the same way.

*Prerequisite.* Mathematical interest, basic knowledge of dependability and Boolean algebra, probability theory, and theory of stochastic processes.

*Expression of thanks.* I particularly would like to thank Prof. Dr. K. Echte from the university of Duisburg-Essen and the community of the Fault Tolerant Discussion Panel (Diskussionskreis Fehlertoleranz, FG-FERS) as well as Dr. J. Petersen for their substantial contributions and discussions. Furthermore, I would like to cordially thank Dr. J. Nachtkamp for his profound comments and remarks. He is one of the former initiators of the probable Markov path approach and its integration into the minimal cut approach for power systems and substations at the Institute of Power Systems and Power Economics (IAEW) at the RWTH Aachen. I am very grateful to Ms. S. Heidtmann for many remarks and proof-reading of the manuscript.

Finally, but by no means least, I would like to particularly highlight and cordially thank my wife Anne for her support during the creation of this book.

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## About the Author

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# Symbols and Abbreviations

No distinction is made between singular and plural notation of the abbreviations. For example, *DBD*, *CCF*, *MC*, *MMC*, and *pMp* indicate the singular as well as the plural form.

$a_{j,k}$	Constant transition rate from $Z_j \rightarrow Z_k$
AND	Logical AND (conjunction, $\wedge$ )
$CCF, CCF_{i,k}$	Common cause failure, impact of component $i$ on $k$
$c_{i,k}$	Probability of $CCF_{i,k}$ , $0 \leq c_{i,k} \leq 1$
$D, D_i, D_S$	Down state due to failure, of component $i$ , of system $S$
<i>DBD</i>	Dependability block diagram $\equiv$ logical diagram or logical network which represents the up state mode (similar to a reliability block diagram <i>RBD</i> , but used in a more general context concerning the integration of the <i>MMC</i> models)
$\Delta$	Deviation, model inaccuracy (quantified in the tables)
DFG	German Research Foundation (Deutsche Forschungsgemeinschaft)
$Fr(Z)$	Frequency of state $Z$ (steady state)
<i>MC</i>	Minimal cut
$\overline{MC}$	$\equiv \neg MC$ (up state mode in relation to <i>MC</i> )
<i>MMC</i>	Markov minimal cut
$\overline{MMC}$	$\equiv \neg MMC$ (up state mode in relation to <i>MMC</i> )
<i>MTTSF</i>	Mean operating time to system failure
$n, n_{MMC}, n_Z$	Number, number of <i>MMC</i> , number of $Z$
$\Omega, \Omega_i, \Omega_S$	Universe space, of component $i$ , of system $S$ , $Pr(\Omega_{...}) = 1$ ( $\Omega_{...}$ represented by Markov spaces)
OR	Logical OR (disjunction, $\vee$ )
<i>pMp</i>	Probable Markov path
$Pr(Z), Pr(Z_{ind}), Pr(Z_{dep})$	Probability of state $Z$ (steady state), of <i>s-independent</i> $Z$ , of <i>s-dependent</i> $Z$

<i>RBD</i>	Reliability block diagram (replaced by <i>DBD</i> )
<i>r</i> oof	$r$ -out-of- $n$ , $1 \leq r \leq n$ , $n \geq 1$
<i>S</i>	Index for system
<i>sDI</i> (...)	s-Dependency impact of <i>MMC</i> and $D_S$
<i>sDI</i> (...)-factor	$= Pr(\dots_{dep})/Pr(\dots_{ind})$
SFB	German Collaborative Research Centre of the DFG (Sonderforschungsbereich 291 der DFG)
$T_i(Z)$	Mean time (duration) of state $Z$ (steady state)
$U, U_i, U_S$	Up state (operating state), of component $i$ , of system $S$
$x_C$	Limit value for $c$ , depending on the acceptable $\Delta$
$X, Z$	Markov states
$\lambda, \mu$	Constant failure rate, constant restoration ( $\equiv$ repair) rate, $\lambda < \mu$

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