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Intelligent Analysis: Fractional Inequalities and Approximations Expanded

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Dedicated to My Family

Preface

Computational and fractional analysis plays a very important role nowadays either by themselves because of their rapid development, which is based on very old foundations, or because they cover a great variety of applications in the real world. In this monograph all presented is original work by the author. We start with the well-known Iyengar type inequalities for the first 12 chapters working in all possible directions, especially along the fractional aspect. Our results are optimal or nearly optimal. Iyengar inequalities estimate the distance of the average of a function from the average of its values at the endpoints of their interval of definition, with a great impact to numerical analysis. We continue with Choquet integral analytical inequalities, related to great applications in economics. We deal with the local fractional derivatives of Riemann–Liouville type and related results including inequalities. We encounter the case of low order Riemann–Liouville fractional derivatives and inequalities without initial conditions at the univariate and multivariate level, we study also related approximations. We continue with the quantitative complex approximation theory by operators and various important complex fractional inequalities. We also deal with the conformable fractional approximation of well-known Csiszar’s f -divergence, which is the most essential and general measure for the comparison between two probability measures. We also present conformable fractional self-adjoint operator inequalities. We continue with the study of new local fractional M -derivatives which carry all the basic properties of ordinary derivatives. We finish with the new complex multivariate Taylor formula with integral remainder.

This monograph is the natural evolution of the recent author’s research work put in a book form for the first time. The presented approaches are original, and chapters are self-contained and can be read independently. This monograph is suitable to be used in related graduate classes and research projects. We exhibit to the maximum our computational and approximation methods to all possible directions.

The motivation to write this monograph came by the following: various issues related to the modeling and analysis of ordinary- and fractional-order systems have gained an increased popularity, as witnesses by many books and volumes in

Springer's program: <http://www.springer.com/gp/search?query=fractional&submit=Prze%C5%9Blij> and the purpose of our book is to provide a deeper formal analysis on some issues that are relevant to many areas for instance: decision-making, complex processes, systems modeling and control, and related areas.

The above are deeply embedded in the fields of mathematics, engineering, computer science, physics, economics, social and life sciences.

The complete list of presented topics follows:

General Iyengar type inequalities,
 Caputo fractional Iyengar type inequalities,
 Canavati fractional Iyengar type inequalities,
 General Multivariate Iyengar type inequalities,
 Multivariate Iyengar type inequalities for radial functions,
 Multidimensional Fractional Iyengar type inequalities for radial functions,
 General Multidimensional Fractional Iyengar type inequalities,
 Time Scales Delta Iyengar type inequalities,
 Nabla Time Scales Iyengar type inequalities,
 Choquet–Iyengar type advanced inequalities,
 Conformable fractional Iyengar type inequalities,
 Fuzzy Iyengar type inequalities,
 Choquet integral analytic inequalities,
 Local Fractional Taylor Formula,
 Negative Domain local fractional inequalities,
 Approximation with Riemann–Liouville fractional derivatives,
 Riemann–Liouville fractional fundamental theorem of Calculus and Riemann–Liouville Fractional Polya type integral inequality and its extension to Choquet integral setting,
 Low order Riemann–Liouville fractional inequalities without initial conditions,
 Low order Riemann–Liouville fractional inequalities on a spherical shell,
 Complex Korovkin Theory,
 Complex left Caputo fractional inequalities,
 Complex Opial type inequalities,
 Complex Multivariate Montgomery type identity leading to Complex Multivariate Ostrowski and Grüss Inequalities,
 Complex Multivariate Fink type identity applied to Complex Multivariate Ostrowski and Grüss inequalities,
 Conformable Fractional Approximation of Csiszar's f -Divergence,
 Conformable fractional self adjoint operator analytic inequalities,
 On the left fractional local general M -derivative,
 About the right fractional local general M -derivative,
 Complex Multivariate Taylor's formula.
 An extensive list of references is given per chapter.

The book's results are expected to find applications in many areas of pure and applied mathematics, especially in approximation theory and inequalities in both ordinary and fractional sense. As such this monograph is suitable for researchers, graduate students, and seminars of the above disciplines, also to be in all science and engineering libraries.

The preparation of the book took place during 2018–2019 at the University of Memphis.

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Memphis, TN, USA
September 2019

George A. Anastassiou

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