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Chen Qiu · Jian S. Dai

# Analysis and Synthesis of Compliant Parallel Mechanisms—Screw Theory Approach

 Springer

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*The first author would like to dedicate this  
book to Zhen Li, Wei Qiu and Ye Pan*

# Foreword

At the dawn of the century's third decade, robotics is reaching an elevated level of maturity and continues to benefit from the advances and innovations in its enabling technologies. These all are contributing to an unprecedented effort to bringing robots to human environment in hospitals and homes, factories and schools; in the field for robots fighting fires, making goods and products, picking fruits and watering the farmland, saving time and lives. Robots today hold the promise for making a considerable impact in a wide range of real-world applications from industrial manufacturing to healthcare, transportation, and exploration of the deep space and sea. Tomorrow, robots will become pervasive and touch upon many aspects of modern life.

The *Springer Tracts in Advanced Robotics (STAR)* is devoted to bringing to the research community the latest advances in the robotics field on the basis of their significance and quality. Through a wide and timely dissemination of critical research developments in robotics, our objective with this series is to promote more exchanges and collaborations among the researchers in the community and contribute to further advancements in this rapidly growing field.

The monograph by Chen Qiu and Jian S. Dai is the outcome of the work accomplished by both authors on screw theory for compliant mechanisms and compliant parallel robots with their presentation, design, stiffness construction, parametrisation and optimisation. Several aspects of mechanics and robotics are mastered, ranging from screw theory and flexible systems to design and optimisation of compliant mechanisms. While conjugating theoretical methods with practical implementation, an excellent way of solving the numerous problems is presented throughout the chapters.

The volume will last long in the field, and future research on the topic will benefit from the impact of the results available with this work. A fine addition to the STAR series!

Naples, Italy  
March 2020

Bruno Siciliano  
STAR Editor

# Preface

As an emerging technology, compliant mechanisms have been used in research and many engineering applications, such as remote centre devices, micro and nano manipulators, continuum manipulators, etc. A compliant mechanism generates a motion based on deformation of flexible elements. As such, to successfully design a compliant mechanism requires a good understanding of both deformation of flexible elements and the combination pattern of them. Deformation evaluation of flexible elements is in the field of solid mechanics, while assembly of flexible elements is more related to a traditional mechanism design. In particular, the latter provides a potential of using the mechanism-equivalence principle for a design of compliant mechanisms.

As a well-established algebra approach in the study of kinematics and dynamics of traditional mechanisms, screw theory has become an increasingly popular tool in design of compliant mechanisms following the fundamental mechanism-equivalence principle. This monograph is the outcome of the work accomplished by both authors on screw theory for compliant mechanisms and compliant parallel robots with their presentation, design, stiffness construction and parametrization and optimization.

The book covers several aspects of mechanics and robotics, ranging from screw theory and flexible systems to design and optimisation of compliant mechanisms and compliant parallel mechanisms. In this book, a unified screw-theory based framework for design of both traditional and compliant mechanisms is proposed, including the description of flexible elements and the stiffness/compliance construction of the whole mechanisms and whole parallel mechanisms. A novel compliant parallel mechanism employing shape-memory-alloy spring based actuators is introduced using a constraint-based approach, and both stiffness analysis and synthesis design problems are tackled. This naturally goes to parameterisation and optimisation with respect to design parameters of ortho-planar springs and leads to development of a novel continuum manipulator with large bending capabilities, and to an original origami-inspired compliant mechanism with good flexibility and controllable motion in a large workspace.

The book presents a comprehensive study on screw theory and its application in the design of compliant mechanisms, with particular focuses on compliant parallel mechanisms. Chapter 2 introduces the theoretical background of screw theory, based on which both compliance characteristics of flexible elements and their integration designs are addressed in Chaps. 3 and 4. This paves a way of analysis and synthesis, where a number of common design topics are covered in Chaps. 5–8.

Chapter 5 looks into synthesis problems of compliant parallel mechanisms at the conceptual-design level, where screw theory is implemented in generating an arrangement of constraint flexible elements according to degrees of freedom of a compliant parallel mechanism. Chapter 6 extends Chap. 5 and addresses the stiffness synthesis problem, where algorithms are developed that are able to synthesize a compliant parallel mechanism not only according to the motion requirement but also according to a stiffness requirement. Chapter 7 looks into the dimensional design issue, where parameterization and optimization analysis of compliant parallel mechanisms are investigated using ortho-planar springs as examples. Finally, Chap. 8 addresses the large deformation of compliant parallel mechanisms using a repelling-screw based approach, where the force equilibrium is established when a compliant parallel mechanism is deformed. In this chapter, Origami-inspired compliant mechanisms are selected to demonstrate the proposed approach.

To help readers better understand each design topic, both simulations and physical experiments are provided in accordance with the mathematical models. The prerequisite for this book is a basic knowledge in linear algebra, kinematics and statics, and solid mechanics. The book is appropriate for researchers, developers, engineers and graduate students with interests in compliant mechanisms and robotics and screw theory.

The book would not have been possible without the help of many people. We would like to thank colleagues in the Group of Advanced Kinematics and Reconfigurable Robotics of King's College London, who offered generous supports in a number of research projects that resulted in the main contents of this book. We acknowledge the support of National Natural Science Foundation of China (NSFC) in the number of 51535008 and the support of Engineering and Physical Science Research Council (EPSRC) in the UK under the grant of EP/E012574/1 and EP/S019790/1. Special thanks to the STAR book series editors Prof. Bruno Siciliano and Prof. Oussama Khatib for valuable suggestions and comments, and Dr. Thomas Ditzinger for the kind help in the publication of this book.

London, UK  
March 2020

Chen Qiu  
Jian S. Dai

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# Acronyms

<b>S</b>	Screw
<b>s</b>	The primary part of a screw
<b>s<sub>0</sub></b>	The second part of a screw
<b>S<sub>axis</sub></b>	Screw in Plücker axis coordinate frame
<b>Δ</b>	Screw elliptical polar operator
<b>Ad</b>	Adjoint transformation matrix
<b>§</b>	Screw system
<b>S<sup>r</sup></b>	Reciprocal screw
<b>§<sup>r</sup></b>	Reciprocal screw system
<b>§<sup>p</sup></b>	Repelling screw system
<b>dim</b>	Dimension of a screw system
<b>t</b>	Twist representing an instantaneous velocity or an infinitesimal displacement
<b>ω</b>	The primary part of a twist representing the angular velocity when the twist represents an instantaneous velocity
<b>v</b>	The second part of a twist representing the linear velocity when the twist represents an instantaneous velocity
<b>θ</b>	The primary part of a twist representing the rotational displacement when the twist represents an infinitesimal displacement
<b>δ</b>	The second part of a twist representing the translational displacement when the twist represents an infinitesimal displacement
<b>T</b>	Twist in Plücker axis coordinate frame
<b>w</b>	Wrench representing a spatial force
<b>f</b>	The primary part of a wrench representing the linear component (pure force)
<b>m</b>	The second part of a wrench representing the angular component (pure moment)
<b>W</b>	Wrench in Plücker axis coordinate frame
<b>C</b>	Compliance matrix
<b>K</b>	Stiffness matrix
<b>k<sub>δ</sub></b>	Stiffness coefficient of a linear spring

$c_\delta$	Compliance coefficient of a linear spring
$k_\theta$	Stiffness coefficient of a torsional spring
$c_\theta$	Compliance coefficient of a torsional spring
$\mathbf{C}_\theta$	Compliance matrix of a revolute joint in the serial configuration
$\mathbf{C}_\delta$	Compliance matrix of a translational joint in the serial configuration
$\mathbf{K}_\theta$	Stiffness matrix of a revolute joint in the parallel configuration
$\mathbf{K}_\delta$	Stiffness matrix of a translational joint in the parallel configuration
$\mathbf{J}_S$	Jacobian matrix
$E$	Young's modulus
$G$	Shear modulus
$\nu$	Poisson's ratio

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