Nash Stability in a Multi-objective Graph Model with Interval Preference Weights: Application to a US-China Trade Dispute

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Abstract. In many real-world conflict situations, decision-makers (DMs) integrate multiple objectives rather than considering just one objective or dimension. A multi-objective graph model (MOGM) is proposed to balance each DM's objectives in both two-DM and multi-DM conflicts. To identify Nash stability in MOGMs, a comprehensive preference matrix with weight parameters on objectives is developed for each DM, along with a unilateral move matrix including preference weights (UMP). Then, considering the subjective uncertainty of DMs, interval numbers are used to represent the degree of uncertainty of preference. Subsequently, Nash equilibria and interval Nash equilibria are developed for MOGMs, and the dependence of these equilibria on weights is shown. To illustrate how MOGM can be applied in practice and provide valuable strategic insights, it is used to investigate a US-China trade dispute model. The stability results suggest potential strategic resolutions of bilateral trade disputes, and how DMs can attain them. The case analysis process suggests that a peaceful settlement of the dispute may be achievable.

Keywords: Multi-objective graph model (MOGM) · Preference weight · Interval preference · Nash equilibrium · US-China trade dispute

1 Introduction

Conflicting interests and objectives are a perpetual concern of economics and other social sciences. There are essentially three kinds of conflict: conflicts among several decision makers (DMs), conflicts within an individual, and conflicts within and among individuals. The first kind of conflict is studied in conventional game theory, where each DM tries to maximize a scalar payoff. The second kind can be seen as the subject of individual decision theory. The third kind, involving several DMs as well as within

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each DM, is modeled by a multi-objective game, which reflects that DMs in the game may have several objectives, possibly conflicting.

In the study of multi-objective non-cooperative games (MONCGs), Blackwell [1] first considered vector payoffs and formalized the two-person zero-sum vector matrix game, but addressed only the minimax property. Shapley [2] showed the existence of strategic equilibria assuming each DM will choose a weakly efficient or efficient solution given the choices of the rivals, while Zeleny [3] dealt with the same problem by using linear multi-objective mathematical programming. Borm et al. [4] considered the general two-person bi-matrix game and studied its comparative statics. Charnes et al. [5] proposed the more general n-person MONCG, but limited all DMs' choices to a cross-constrained set. Zhao [6] defined cooperative, non-cooperative, hybrid and quasi-hybrid solution concepts for multi-objective games and proved their existence. Yu [7] also studied the existence of Nash equilibrium and Pareto equilibrium for MONCG. Most of these studies define and prove existence for various solutions, but there are often no effective computational techniques to obtain the equilibrium solution.

Imprecision or fuzziness is inherent in human judgment, and some literature on MONCGs under uncertainty incorporates the concepts of fuzzy set, grey number and probability. Assuming that a DM has a fuzzy goal for each objective which can also be interpreted as a DM's degree of satisfaction for a payoff, Nishizaki and Sakawa [8] studied multi-objective fuzzy two-person zero-sum and non-zero-sum games. They considered the relation of equilibrium solutions for multi-objective two-person games combining fuzzy goals with the Pareto optimal equilibrium solutions defined in Borm [4]. However, in existing research on MONCGs, the payoff in each state is interpreted as a utility value. When game theory is applied to real world problems, it is often difficult to assess utilities exactly, but it is easier to determine the relative payoffs, or order of preference, of the states.

The graph model for conflict resolution (GMCR) is a flexible and comprehensive methodology for systematically investigating strategic conflicts, in which multiple DMs dynamically interact with each other in terms of potential moves and counter moves, in order to fare as well as possible [9–11]. Xu et al. [12, 13] devised matrix representations for calculating individual stability and equilibria for GMCR. Explicit algebraic formulations allow users to develop algorithms conveniently in order to assess the stabilities of states and permitted new solution concepts to be integrated into the decision support system GMCR II [14, 15]. Li et al. [16] proposed a new preference structure for the graph model to handle uncertainty in DMs' preferences and redefined several solution concepts with preference uncertainty. Bashar et al. [17] and Hipel et al. [18] developed a methodology to model and analyze a conflict with fuzzy preferences. Ke [19, 20] designed a multiple criteria decision analysis approach and incorporated an analytic hierarchy process to capture the relative preference information of a DM involved in a conflict through defining fuzzy preference relation. The ideas of grey and probabilistic preferences were also incorporated into the graph model methodology from different viewpoints by Kuang et al. [21] and Rego and dos Santos [22]. However, MONCG with fuzzy preference has not been studied until now.

In this paper, a multi-objective graph model (MOGM) is proposed to balance each DM's objectives in both two-DM and multi-DM games. Two types of Nash equilibria are developed for MOGMs; the dependence of equilibria on weights of objectives is shown. Our paper differs from the literature in the following three aspects.

We mainly focus on investigating the MONCG. This is the first use of GMCR to study this kind of conflict model. A multi-objective graph model (MOGM) is proposed and the Nash equilibrium solution method is given.

Compared with the existing matrix representation of preference, which requires three binary relations, the proposed matrix representation of crisp preference is more intuitive and requires only three values. We use the values 1, 0, and −1 to represent preference by a DM: positive preference, indifference, and negative preference.

Considering the subjective uncertainty of DMs, interval numbers are used to measure the degree of uncertainty of preference. In other words, each cell in the preference matrix is made up of interval numbers, which convey the uncertainty, subjectivity, and linguistic nature of DMs' judgments.

The US-China trade dispute is analyzed from the perspective of the graph model. Three scenarios are used to describe for the bilateral trade dispute and evaluated in terms of the preference of the two sides. In each scenario, a comprehensive analysis considering both short-term and long-term objectives is conducted.

The remainder of the paper is as follows. In Sect. 2, the basic structure of a graph model is reviewed and the multi-objective graph model is proposed. A comprehensive preference matrix with weight parameters on objectives is developed for each DM, along with a unilateral move matrix including preference weights (UMP). Furthermore, the Nash equilibrium solution method of MONCG, both two-DM and multi-DM, is given. In Sect. 3, the US-China trade dispute is introduced briefly and the MOGM methodology is used to analyze it. In Sect. 4, MOGM with interval preference weights is established to determine the interval Nash equilibrium solution. In Sect. 5, the models and methods of this paper are illustrated with the US-China trade dispute. Finally, some conclusions and ideas for future work are provided in Sect. 6.

2 Graph Model

2.1 Graph Model with Simple Preference

The key ingredients in a classical graph model are the DMs, states or scenarios that could take place, and the preferences of each DM [9, 10]. These ingredients are explained in detail followed by the definitions of reachable lists for a DM. Moreover, Nash stability is formally defined, which determines whether a state is stable for a DM.

A n-DM graph model is a structure $G = <N, S, \{A_t, \succeq_t, t \in N\}>$, where

(1) $N = \{1, 2, \cdots, n\}$ is the set of DMs.
(2) $S = \{s_1, s_2, \cdots, s_m\}$ is a nonempty, finite set, called the set of feasible states or situations.
(3) For each DM t, $(s_i, s_j) \in A_t$, $i, j = 1, 2, \cdots, m$ means that DM t can move from state s_i to s_j in one step, where A_t is DM t's set of all oriented arcs.

(4) For each DM t, $\succeq_t$ is a relation on S that indicates the preference between states of DM t. $\succeq_t$ is assumed to be irreflexive, transitive and complete. $s_i \succeq_t s_j$ means that DM t be indifferent or prefer state s_i than state s_j.

For $t \in N$, DM t's unilateral moves (UMs) and unilateral improvements (UIs) are sets $R_t(s_i) = \{s_j \in S : (s_i, s_j) \in A_t\}$ and $R_t^+(s_i) = \{s_j \in S : (s_i, s_j) \in A_t,\ s_j \succ_t s_i\}$. For matrix representation of UMs, DM t's UM matrix is an $m \times m$ matrix, $\mathbf{J}_t$, with $(s_i,\ s_j)$ entries

$$\mathbf{J}_t(s_i,\ s_j) = \begin{cases} 1, & \mathit{if}\,(s_i,\ s_j) \in A_t, \\ 0, & \mathit{otherwise}. \end{cases}$$

Note that $\mathbf{J}_t(s_i, s_j) = 1$ if and only if DM t can move from state s_i to s_j in one step. In other words, $(s_i, s_j) \in A_t$ [13].

Several preference relation matrices $\mathbf{P}_t^+$, $\mathbf{P}_t^-$, and $\mathbf{P}_t^=$ are defined as

$$\mathbf{P}_t^+(s_i, s_j) = \begin{cases} 1, & \mathit{if}\ s_j \succ_t s_i, \\ 0, & \mathit{otherwise}, \end{cases}$$

$$\mathbf{P}_t^-(s_i, s_j) = \begin{cases} 1, & \mathit{if}\ s_j \prec_t s_i, \\ 0, & \mathit{otherwise}, \end{cases}$$

and

$$\mathbf{P}_t^=(s_i, s_j) = \begin{cases} 1, & \mathit{if}\ s_j \sim_t s_i, \\ 0, & \mathit{otherwise}. \end{cases}$$

where $\mathbf{P}_t^+(s_i, s_j) = 1$ in the preference matrix indicates that DM t prefers state s_j to state s_i, while zero entry $\mathbf{P}_t^+(s_i, s_j) = 0$ indicates that DM t either prefers s_i to s_j or is indifferent between s_i and s_j. $\mathbf{P}_t^-(s_i, s_j)$ and $\mathbf{P}_t^=(s_i, s_j)$ can be interpreted similarly [13].

In contrast to Xu's matrix approach to preference [13], which requires three binary relations, we use only three values, 1, 0, and −1, to express a more intuitive preference matrix. We will use this matrix to represent Nash stability in n-DM graph model.

Definition 1. For a graph model G, the preference matrix for DM t is an $m \times m$ matrix, $\mathbf{P}_t$ with entries

$$\mathbf{P}_t(s_i, s_j) = \begin{cases} 1, & \mathit{if}\ s_j \succ_t s_i, \\ 0, & \mathit{if}\ s_j \sim_t s_i, \\ -1, & \mathit{if}\ s_j \prec_t s_i. \end{cases}$$

In the preference matrix, $\mathbf{P}_t(s_i, s_j) = 1$ indicates that DM t prefers state s_j to state s_i, $\mathbf{P}_t(s_i, s_j) = 0$ indicates that DM t is indifferent between s_i and s_j, while $\mathbf{P}_t(s_i, s_j) = -1$ implies that DM t prefers state s_i to state s_j.

For a graph model G, the UM matrix including preference information (UMP) for DM t can be calculated by

$$\mathbf{H}_t = \mathbf{J}_t \circ \mathbf{P}_t, \tag{1}$$

where "$\circ$" denotes the Hadamard product. Note that $\mathbf{H}_t$ is an $m \times m$ matrix. The (i,j) entry in the matrix $\mathbf{H}_t$ is $\mathbf{H}_t(i,j) = \mathbf{J}_t(i,j) \circ \mathbf{P}_t(i,j)$.

The logical definition of Nash stability of the graph model for conflict resolution is given as follows.

Definition 2 [13]. Let $t \in N$ and $s_i \in S$. s_i is Nash stable for DM t iff $R_t^+(s_i) = \varnothing$.

Theorem 1. State $s_i \in S$ is Nash stable for DM t iff $\mathbf{H}_t(i,j) \leq 0$ for all $j \neq i, j = 1, 2, \cdots, m$.

Proof: If state $s_i \in S$ is Nash stable for DM t, then according to Definition 2, there is no UI for DM t to any other state. From Eq. (1), we derive that each value in the i th row of the matrix $\mathbf{H}_t$ is less than or equal to zero. And vice versa. □

If each DM finds that he or she cannot do better than to stay in the current state, it is a Nash equilibrium [23]. According to Theorem 1, if the state $s_i \in S$ is Nash stable for all DMs, $R_t^+(s_i) = \varnothing$ for all $t \in N$. This indicates that starting in state s_i no DM will change their current strategy, making stable for all DMs.

Note that if a state $s_i \in S$ is Nash stable for all DMs $t \in N$, then s_i is a Nash equilibrium.

2.2 Graph Model with Multiple Objectives

In many real conflict problems, there are not only conflicts among DMs but also multiple conflicting objectives within individuals. For example, the orders of preference of states may be different according to different objectives. Alternatively, each DM may be a "team". This is a multi-objective game, which happens whenever DMs in a game have multiple objectives. The structure of the multi-objective graph model (MOGM) requires a graph model satisfying conditions (1), (2), and (3), with (4), replaced by a multi-objective structure in which $O = \{o_1, o_2, \cdots, o_K\}$ is the set of objectives that all DMs might choose in the n-DM graph model.

Assume that the weight on the objectives of DM t is $\boldsymbol{\omega}_t = (\omega_{t1}, \omega_{t2}, \cdots, \omega_{tK})$. In particular, if DM t does not have objective o_k, then weight $\omega_{tk} = 0$. The structure of the n-DM graph model with multiple objectives, which is denoted by $MOGM = <N, S, O, \{A_t, t \in N\}, \{\succeq_t^{o_k}, t \in N, o_k \in O\}>$, where $\succeq_t^{o_k}$ is a preference for DM t reflecting objective o_k.

The preference matrix of DM t on objective $o_k, k = 1, 2, \cdots, K$ is denoted by an $m \times m$ matrix $\mathbf{P}_{tk}$. In order to incorporate all objectives, the comprehensive preference matrix $\mathbf{P}_t$ with parameter $\boldsymbol{\omega}_t$ of DM t is determined by

$$\mathbf{P}_t(\boldsymbol{\omega}_t) = \sum\nolimits_{k=1}^{K} \omega_{tk} \mathbf{P}_{tk}. \tag{2}$$

Then, the UM matrix including preference weights of DM t can be calculated by

$$\mathbf{H}_t(\boldsymbol{\omega}_t) = \mathbf{J}_t \circ \mathbf{P}_t(\boldsymbol{\omega}_t), \tag{3}$$

where "$\circ$" denotes the Hadamard product. Note that $\mathbf{H}_t(\boldsymbol{\omega}_t)$ is an $m \times m$ matrix with parameter $\boldsymbol{\omega}_t$. The (i,j) entry in the matrix $\mathbf{H}_t(\boldsymbol{\omega}_t)$ is denoted as $\mathbf{H}_{t,\boldsymbol{\omega}_t}(i,j)$.

Analogous to Theorem 1, the Nash stability of DM t is determined by Definition 3.

Definition 3. For an MOGM, state $s_i \in S$ is Nash stable for DM t iff $\boldsymbol{\omega}_t$ satisfies $\mathbf{H}_{t,\boldsymbol{\omega}_t}(i,j) \leq 0$ for all $j \neq i, j = 1, 2, \cdots, m$.

Definition 4. Fix $s_i \in S$. If s_i is Nash stable for DM t, then $\boldsymbol{\omega}_t$ satisfies $\mathbf{H}_{t,\omega_t}(i,j) \leq 0$ for all $j \neq i, j = 1, 2, \cdots, m$. Moreover, for all $\boldsymbol{\omega}_t, t \in N$, the area defined by the intersection $\Delta = \bigcap_{\substack{t \in N \\ j \neq i}} \{\boldsymbol{\omega}_t : \mathbf{H}_{t,\boldsymbol{\omega}_t}(i,j) \leq 0\}$ is the location of the Nash equilibrium.

Note that for a MOGM, a state is Nash stable for a DM if that DM would not choose to move away from it. A Nash equilibrium of the MOGM is a state that is Nash stable for all DMs.

3 Application: US-China Trade Dispute

3.1 Background of US-China Trade Dispute

The trade dispute between the US and China, also known as the US-China trade dispute, is an ongoing economic conflict between the world's two largest national economies. It began on March 23, 2018, when the US imposed a tax on $60 billion of Chinese imports.

China's large trade surplus with the US, China's non-compliance with WTO commitments, and China's tendency to disputed use of US technology have been suggested as reasons for the dispute. The underlying cause of the dispute is undoubtedly related to the intensification of domestic conflicts over the distribution of wealth in the US, the gradual decline of American hegemony, and China's rapid rise that seems to seriously threaten US interests. In addition, changes in the international situation will also have a huge impact on the trend of the trade dispute. Therefore, it would be best if the dispute could be resolved peacefully, both for the benefit of both sides and for global economic stability.

Recently, a high-level US and China government trade delegation reached a framework agreement to resolve the dispute. However, the implementation of the framework agreement requires structural adjustments to the bilateral economy, especially the Chinese economy, and this is difficult to achieve quickly. As a consequence, the US claimed that China did not fully implement the agreement, and the US began to impose more tariffs, causing China to retaliate. In this way, US and China cycled back and forth between imposing tariffs, reducing them, and then imposing them again. This process is called a "Thucydides trap" [24].

In view of the reasons for the trade dispute, and consistent with the preferences of the US and China, three scenarios are plausible for the dispute. They will be analyzed below, considering both short-term and long-term objectives. Based on the MOGM of Sect. 2, a scenario demonstration of the development path of the dispute can be conducted.

3.2 Multi-objective Graph Model of US-China Trade Dispute

The DMs in the US-China trade dispute are US and China. The US's goals are to obtain more favorable conditions for the US, and to reach a new agreement that the US President Trump called "don't lose money", rather than to fully implement trade controls and raise trade barriers. China's main goals are that the US recognize China's market economy status, establish an equal basis for negotiation, and gain a greater voice in the global economic and trade system. To secure their goals, each DM has two strategies - to impose tariffs or not. All possible combinations of DM's strategies are then examined to identify the states or situations in the dispute. Two DMs, their strategies, and states of the conflict are shown in Table 1.

Table 1. DMs, strategies and states for the US-China trade dispute

		China	
		Impose	Don't impose
US	Impose	s_1	s_2
	Don't impose	s_3	s_4

As in Table 1, the US is the row DM which controls the two row strategies of "Impose" by continually imposing tariffs on imports from China, or "Don't impose" by making concessions and stopping imposing tariffs. China is the column DM which also has two strategies "Impose" and "Don't impose". When each DM selects a strategy, a state is represented by a cell in the matrix. Each cell is assigned a state number as shown in Table 1. For instance, when the US chooses "Impose" and China selects "Don't impose", then state s_2 is formed as shown in the upper right-hand cell.

The short term o_1 and long term o_2 impact of the dispute on each country are considered as two objectives or dimensions. Let the weights of US and China on the two objectives be $\boldsymbol{\omega}_{US} = (1 - \omega, \omega)$ and $\boldsymbol{\omega}_C = (1 - \theta, \theta)$, respectively. The MOGM is used to analyze the bilateral trade dispute. Three scenarios for the dispute, depending on the preferences of two sides, are assumed as follows.

(1) In Scenario 1, either in the short or long term, both DMs prefer a state in which they impose tariffs and their opponent does not. The least preferred state for a DM is not to impose tariffs while the other DM does. In particular, the US prefers state s_2 to

state s_3 ($s_2 \succ s_3$) and China prefers state s_3 to state s_2 ($s_3 \succ s_2$) both in the short and long term. In the short term, no matter whether it is the US or China, the DM will prefers to impose tariffs rather than end the dispute ($s_1 \succ s_4$). However, in the long term, both DM realize that the dispute will have a negative impact on future economic development. Therefore, in this dimension, the two DMs prefer to not impose tariffs on each other rather than not ($s_4 \succ s_1$). In this scenario, the orders of preference of the two DMs according to the two objectives are shown in Table 2.

Table 2. The orders of preference of US and China in short and long term in Scenario 1

Scenario 1	US	
	Short term ($1-\omega$)	$s_2 \succ s_1 \succ s_4 \succ s_3$
	Long term (ω)	$s_2 \succ s_4 \succ s_1 \succ s_3$
	China	
	Short term ($1-\theta$)	$s_3 \succ s_1 \succ s_4 \succ s_2$
	Long term (θ)	$s_3 \succ s_4 \succ s_1 \succ s_2$

The UM matrices for US and China are as follows

$$\mathbf{J}_{US} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \mathbf{J}_C = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

According to Definition 1, the preference matrices of US and China in the short and long term are

$$\mathbf{P}_{US}^S = \begin{pmatrix} 0 & 1 & -1 & -1 \\ -1 & 0 & -1 & -1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & -1 & 0 \end{pmatrix}, \mathbf{P}_{US}^L = \begin{pmatrix} 0 & 1 & -1 & 1 \\ -1 & 0 & -1 & -1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & -1 & 0 \end{pmatrix},$$

$$\mathbf{P}_C^S = \begin{pmatrix} 0 & -1 & 1 & -1 \\ 1 & 0 & 1 & 1 \\ -1 & -1 & 0 & -1 \\ 1 & -1 & 1 & 0 \end{pmatrix}, \mathbf{P}_C^L = \begin{pmatrix} 0 & -1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ -1 & -1 & 0 & -1 \\ -1 & -1 & 1 & 0 \end{pmatrix}.$$

Based on Eq. (1), UMP matrices for US and China can be calculated as follows

$$\mathbf{H}_{US}(\omega) = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \mathbf{H}_C(\theta) = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

According to Definition 4, we get one Nash equilibrium of the dispute in Scenario 1, which is shown in Fig. 1.

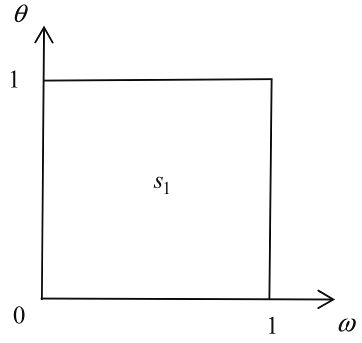

Fig. 1. Nash equilibrium in Scenario 1

Figure 1 shows that, in Scenario 1, no matter in which objective the two DMs operate under, the final Nash equilibrium is state s_1. That is, the two DMs will always be caught in a fierce battle of imposing tariffs on each other. In this circumstance, neither of them has any incentive to move away from state s_1.

(2) In Scenario 2, in the short term, the orders of preference of US and China are the same as in Scenario 1. In the long term, the difference between Scenario 2 and Scenario 1 is that the orders of preference of two DMs changes. They both regard state s_4 as their most preferred state, they don't want to impose tariffs on each other. Both sides uphold the concept of harmony. It means that in the long term the US changes its preference order from $s_2 \succ s_4$ in Scenario 1 to $s_4 \succ s_2$ in Scenario 2 and China could change its preference order from $s_3 \succ s_4$ in Scenario 1 to $s_4 \succ s_3$ in Scenario 2. The other preferences remain the same as in Scenario 1. In this Scenario, the orders of preference of the two DMs in the short and long term are shown in Table 3.

Table 3. The orders of preference of US and China in short and long term in Scenario 2

Scenario 2	US	
	Short term $(1-\omega)$	$s_2 \succ s_1 \succ s_4 \succ s_3$
	Long term (ω)	$s_4 \succ s_2 \succ s_1 \succ s_3$
	China	
	Short term $(1-\theta)$	$s_3 \succ s_1 \succ s_4 \succ s_2$
	Long term (θ)	$s_4 \succ s_3 \succ s_1 \succ s_2$

Based on the Eq. (1), UMP matrices of US and China in Scenario 2 can be calculated as follows

$$\mathbf{H}_{US}(\omega) = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 2\omega - 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 - 2\omega & 0 & 0 \end{pmatrix}, \mathbf{H}_C(\theta) = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\theta - 1 \\ 0 & 0 & 1 - 2\theta & 0 \end{pmatrix}.$$

According to Definition 4, s_1 is a Nash equilibrium when $0 \leq \omega \leq 1$ and $0 \leq \theta \leq 1$, and s_4 is a Nash equilibrium iff $0.5 \leq \omega \leq 1$ and $0.5 \leq \theta \leq 1$. Thus, the Nash equilibria of the dispute in Scenario 2 can be shown in Fig. 2.

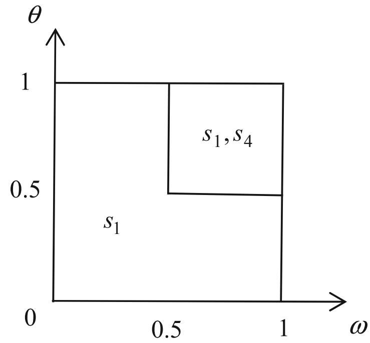

Fig. 2. Nash equilibrium in Scenario 2

Figure 2 indicates that, in Scenario 2, both DMs may change their orders of preference, that is, both DMs may make concessions in the negotiations, and state s_4 without tariffs is likely to be a Nash equilibrium. Why is it possible, not certain? Because, as shown in Fig. 2, state s_4 is an equilibrium solution only when both DMs focus more on the long term, and state s_1 is also an equilibrium solution at this time.

(3) In Scenario 3, in the short term, the orders of preference of the US and China are the same as in Scenario 1 and 2. In the long term, the two DMs change further. They both regard state s_1 as their least preferred state, they don't want to impose tariffs on each other. Why do DMs make such changes? The two DMs have repeatedly imposed tariffs, and the damage and losses to the domestic economy have exceeded their capacity. Then Scenario 3 will occur. In the long term, the US changes its preference from $s_1 \succ s_3$ in Scenario 2 to $s_3 \succ s_1$ in Scenario 3 and China changes its preference from $s_1 \succ s_2$ in Scenario 2 to $s_2 \succ s_1$ in Scenario 3. The other preferences remain the same as Scenario 2. In this Scenario, the orders of preference of the two DMs in the short and long term are shown in Table 4.

Table 4. The orders of preference of US and China in short and long term in Scenario 3

Scenario 3	US	
	Short term $(1-\omega)$	$s_2 \succ s_1 \succ s_4 \succ s_3$
	Long term (ω)	$s_4 \succ s_2 \succ s_3 \succ s_1$
	China	
	Short term $(1-\theta)$	$s_3 \succ s_1 \succ s_4 \succ s_2$
	Long term (θ)	$s_4 \succ s_3 \succ s_2 \succ s_1$

Based on the Eq. (1), UMP matrices of US and China in Scenario 3 can be calculated as follows

$$\mathbf{H}_{US}(\omega) = \begin{pmatrix} 0 & 0 & 2\omega-1 & 0 \\ 0 & 0 & 0 & 2\omega-1 \\ 1-2\omega & 0 & 0 & 0 \\ 0 & 1-2\omega & 0 & 0 \end{pmatrix},$$

$$\mathbf{H}_{C}(\theta) = \begin{pmatrix} 0 & 2\theta-1 & 0 & 0 \\ 1-2\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\theta-1 \\ 0 & 0 & 1-2\theta & 0 \end{pmatrix}.$$

According to Definition 4, s_1 is a Nash equilibrium when $0 \leq \omega \leq 0.5$ and $0 \leq \theta \leq 0.5$, s_2 is a Nash equilibrium when $0 \leq \omega \leq 0.5$ and $0.5 \leq \theta \leq 1$, s_3 is a Nash equilibrium when $0.5 \leq \omega \leq 1$ and $0 \leq \theta \leq 0.5$, and s_4 is a Nash equilibrium iff $0.5 \leq \omega \leq 1$ and $0.5 \leq \theta \leq 1$. Thus, Nash equilibria of the dispute in Scenario 3 are shown in Fig. 3.

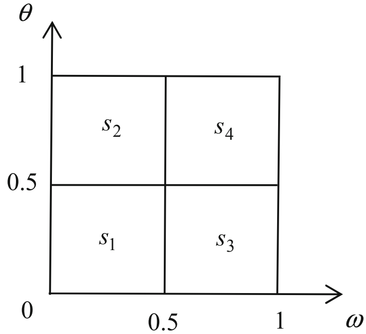

Fig. 3. Nash equilibrium in Scenario 3

The results of Scenario 3 have the following implications. First, the bilateral trade conflict can be stable in a peaceful state s_4 only if both DMs regard "Don't impose tariffs" as their preferred state and "Impose tariffs" on each other as the least desirable state. Second, if the weights ω and θ are large enough, that is, both sides pay enough attention to the long term interests, then peace state s_4 will be stable. Third, if one or both DMs are dissatisfied with the current peace state, s_4, concessions made by the opponent, or if the opponent does not implement the negotiation conditions in accordance with expectations, the current stable state may become unbalanced and the conflict enters into a state of struggle.

4 Multi-objective Graph Model with Interval Preference Weights

Crisp preference represents a certain (definite) preference between two states. For example, if the order of states for DM t is $s_1 \succ s_2 \succ s_3 \succ s_4$, then the preference matrix is

$$\mathbf{P}_t = \begin{pmatrix} 0 & -1 & -1 & -1 \\ 1 & 0 & -1 & -1 \\ 1 & 1 & 0 & -1 \\ 1 & 1 & 1 & 0 \end{pmatrix}.$$

Apparently, in this case, the degree or strength of preference of s_1 over s_3 is greater than the strength of preference of s_1 over s_2. However, in the structure of crisp preference, all degrees of preference are equal to one. In other words, crisp preference cannot express the degree or strength of preference between two states. Furthermore, in real management situations, there is commonly a great deal of fuzziness. DMs are often unclear or uncertain about their preference between two states for various reasons, such as cultural or educational factors, personal habits, lack of information, or the inherent vagueness of human judgment. For these two reasons, an interval number may be the most suitable data expression to describe preference.

An interval [25] is a special subset of the real number set $\Re$, denoted by $\hat{a} = [a, \bar{a}] = \{x \in \Re | a \leq x \leq \bar{a}\}$, where a and $\bar{a}$ are the left endpoint and the right endpoint of the interval $\hat{a}$, respectively. Sometimes a and $\bar{a}$ are called the lower and upper limits or bounds of the interval $\hat{a}$. If $\bar{a} \geq a \geq 0$, then $\hat{a}$ is called a positive interval number. In particular, if $a = \bar{a}$, then the interval number $\hat{a}$ reduces to a real number.

The basic arithmetic operations for intervals are defined as follows [25, 26].

Definition 5. Let $\hat{a} = [a, \bar{a}]$ and $\hat{b} = [b, \bar{b}]$ be two intervals, and let λ be a real number. Then

(1) $\hat{a} + \hat{b} = [a + b, \bar{a} + \bar{b}]$;

(2) $\lambda\hat{a} = \begin{cases} [\lambda a, \lambda\bar{a}], & if\ \lambda \geq 0 \\ [\lambda\bar{a}, \lambda a], & if\ \lambda < 0 \end{cases}$.

Definition 6. For a graph model G, the interval preference matrix for DM t is an $m \times m$ matrix, $\hat{\mathbf{P}}_t$ with (s_i, s_j) entries

$$\hat{\mathbf{P}}_t(s_i, s_j) = \begin{cases} \hat{a}, & \textit{if } s_j \succ_t s_i, \\ 0, & \textit{if } s_j \sim_t s_i, \\ -\hat{a}, & \textit{if } s_j \prec_t s_i, \end{cases}$$

where $\hat{a}$ is a positive interval number. The preferences of DM t for s_i over s_j are represented by intervals. A value $\hat{\mathbf{P}}_t(s_i, s_j) = \hat{a} = [a, \bar{a}]$ in the interval preference matrix indicates the degree or strength of the preference for s_i over s_j for DM t. $\hat{\mathbf{P}}_t(s_i, s_j) = 0$ indicates that DM t is indifferent between s_i and s_j, while $\hat{\mathbf{P}}_t(s_i, s_j) = -\hat{a} = [-\bar{a}, -a]$ implies that DM t prefers state s_j to state s_i.

The interval preference matrix for DM t on an objective o_k, $k = 1, 2, \cdots, K$ is denoted by a matrix $\hat{\mathbf{P}}_{tk} = ([\underline{\mathbf{P}}_{tk}, \bar{\mathbf{P}}_{tk}])_{m \times m}$. In order to incorporate all objectives, the comprehensive preference matrix $\hat{\mathbf{P}}_t(\boldsymbol{\omega}_t)$ with parameter $\boldsymbol{\omega}_t$ for DM t can be calculated by

$$\hat{\mathbf{P}}_t(\boldsymbol{\omega}_t) = \sum\nolimits_{k=1}^{K} \omega_{tk} \hat{\mathbf{P}}_{tk}. \tag{4}$$

Then, for a MOGM, the UM matrix including interval preference weights (UMIP) for DM t can be calculated by

$$\hat{\mathbf{H}}_t(\boldsymbol{\omega}_t) = \mathbf{J}_t \circ \hat{\mathbf{P}}_t(\boldsymbol{\omega}_t), \tag{5}$$

where "$\circ$" denotes the Hadamard product. Note that $\hat{\mathbf{H}}_t(\boldsymbol{\omega}_t)$ is an $m \times m$ matrix with parameter $\boldsymbol{\omega}_t$. The (i, j) entry in the matrix $\hat{\mathbf{H}}_t(\boldsymbol{\omega}_t)$ is $\hat{\mathbf{H}}_{t,\boldsymbol{\omega}_t}(i,j) = [\underline{\mathbf{H}}_{t,\boldsymbol{\omega}_t}(i,j), \bar{\mathbf{H}}_{t,\boldsymbol{\omega}_t}(i, j)]$.

The logical definition of Nash stability of the MOGM with interval preference weights is given as follows.

Definition 7. For MOGM with interval preference weights, state $s_i \in S$ is interval Nash stable for DM t iff $\boldsymbol{\omega}_t$ satisfies $\hat{\mathbf{H}}_{t,\boldsymbol{\omega}_t}(i, j) \leq 0$ for all $j \neq i, j = 1, 2, \cdots, m$.

Since $\hat{\mathbf{H}}_{t,\boldsymbol{\omega}_t}(i, j)$ is an interval number, one can obtain different results on $\hat{\mathbf{H}}_{t,\boldsymbol{\omega}_t}(i, j) = [\underline{\mathbf{H}}_{t,\boldsymbol{\omega}_t}(i, j), \bar{\mathbf{H}}_{t,\boldsymbol{\omega}_t}(i, j)] \leq [0, 0]$ by using different ranking methods on interval numbers. In this paper, assume that $\hat{\mathbf{H}}_{t,\boldsymbol{\omega}_t}(i,j) \leq 0$ iff $\underline{\mathbf{H}}_{t,\boldsymbol{\omega}_t}(i, j) \leq \bar{\mathbf{H}}_{t,\boldsymbol{\omega}_t}(i, j) \leq 0$. Thus, the following Definition 8 on interval Nash equilibrium is proposed for MOGM with interval preference weights.

Definition 8. Fix $s_i \in S$. If s_i is interval Nash stable for DM t, then interval preference weight $\boldsymbol{\omega}_t$ satisfies $\bar{\mathbf{H}}_{t,\boldsymbol{\omega}_t}(i,j) \leq 0$ for all $j \neq i, j = 1, 2, \cdots, m$. Moreover, for all $\boldsymbol{\omega}_t, t \in N$, the area defined by the intersection $\Delta = \bigcap_{\substack{t \in N \\ j \neq i}} \{\boldsymbol{\omega}_t : \bar{\mathbf{H}}_{t,\boldsymbol{\omega}_t}(i,j) \leq 0\}$ is the location of the Nash equilibrium.

5 US-China Trade Dispute with Interval Preference

We analyze Scenario 3 as an example. For this case, using Definition 3 and Definition 4, each preference relation can be characterized by a degree or strength of preference and a relative degree or intensity of preference as shown in Table 5.

Table 5. Interval preference of four states relation

Preference structure	Interval preference degree
$\succ\succ\succ$	[0.8, 1]
$\succ\succ$	[0.6, 0.8]
$\succ$	[0.2, 0.6]
$\sim$	[0, 0]
$\prec$	[−0.6, −0.2]
$\prec\prec$	[−0.8, −0.6]
$\prec\prec\prec$	[−1, −0.8]

Then, the interval preference matrices for the US and China in the short and long term are

$$\hat{\mathbf{P}}_{US}^{S} = \begin{pmatrix} [0,0] & [0.2,0.6] & [-0.8,-0.6] & [-0.6,-0.2] \\ [-0.6,-0.2] & [0,0] & [-1,-0.8] & [-0.8,-0.6] \\ [0.6,0.8] & [0.8,1] & [0,0] & [0.2,0.6] \\ [0.2,0.6] & [0.6,0.8] & [-0.6,-0.2] & [0,0] \end{pmatrix},$$

$$\hat{\mathbf{P}}_{US}^{L} = \begin{pmatrix} [0,0] & [0.6,0.8] & [0.2,0.6] & [0.8,1] \\ [-0.8,-0.6] & [0,0] & [-0.6,-0.2] & [0.2,0.6] \\ [-0.6,-0.2] & [0.2,0.6] & [0,0] & [0.6,0.8] \\ [-1,-0.8] & [-0.6,-0.2] & [-0.8,-0.6] & [0,0] \end{pmatrix},$$

$$\hat{\mathbf{P}}_{C}^{S} = \begin{pmatrix} [0,0] & [-0.8,-0.6] & [0.2,0.6] & [-0.6,-0.2] \\ [0.6,0.8] & [0,0] & [0.8,1] & [0.2,0.6] \\ [-0.6,-0.2] & [-1,-0.8] & [0,0] & [-0.8,-0.6] \\ [0.2,0.6] & [-0.6,-0.2] & [0.6,0.8] & [0,0] \end{pmatrix},$$

$$\hat{\mathbf{P}}_{C}^{L} = \begin{pmatrix} [0,0] & [0.2,0.6] & [0.6,0.8] & [0.8,1] \\ [-0.6,-0.2] & [0,0] & [0.2,0.6] & [0.6,0.8] \\ [-0.8,-0.6] & [-0.6,-0.2] & [0,0] & [0.2,0.6] \\ [-1,-0.8] & [-0.8,-0.6] & [-0.6,-0.2] & [0,0] \end{pmatrix}.$$

Based on Eq. (5), UMIP matrices for the US and China in Scenario 3 can be calculated as follows

$$\hat{\mathbf{H}}_{US}(\omega) = \begin{pmatrix} 0 & 0 & [\omega - 0.8, 1.2\omega - 0.6] & 0 \\ 0 & 0 & 0 & [\omega - 0.8, 1.2\omega - 0.6] \\ [-1.2\omega + 0.6, -\omega + 0.8] & 0 & 0 & 0 \\ 0 & [-1.2\omega + 0.6, -\omega + 0.8] & 0 & 0 \end{pmatrix},$$

$$\hat{\mathbf{H}}_{C}(\theta) = \begin{pmatrix} 0 & [\theta - 0.8, 1.2\theta - 0.6] & 0 & 0 \\ [-1.2\theta + 0.6, -\theta + 0.8] & 0 & 0 & 0 \\ 0 & 0 & 0 & [\theta - 0.8, 1.2\theta - 0.6] \\ 0 & 0 & [-1.2\theta + 0.6, -\theta + 0.8] & 0 \end{pmatrix}.$$

According to Definition 8, s_1 is an interval Nash equilibrium when $0 \leq \omega \leq 0.5$ and $0 \leq \theta \leq 0.5$, s_2 is an interval Nash equilibrium when $0 \leq \omega \leq 0.5$ and $0.8 \leq \theta \leq 1$, s_3 is an interval Nash equilibrium when $0.8 \leq \omega \leq 1$ and $0 \leq \theta \leq 0.5$, and s_4 is an interval Nash equilibrium iff $0.8 \leq \omega \leq 1$ and $0.8 \leq \theta \leq 1$. Thus, the interval Nash equilibria of the dispute in Scenario 3 can be shown in Fig. 4.

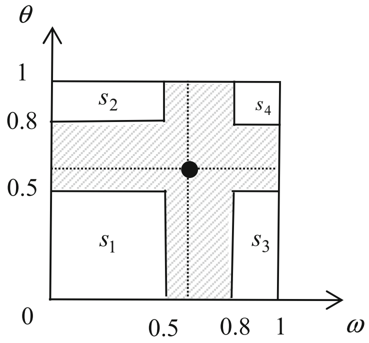

Fig. 4. Interval Nash equilibrium in Scenario 3

There is a shaded hole in Fig. 4. The hole means that it is uncertain if the Nash stability s_1 is going to become s_2 with a change from short term to long term for China. For example, at the point $(0.6, 0.6)$, $\hat{\mathbf{H}}_{US}(1,3) = [-0.2, 0.12]$ indicates the degree or strength of the US is likely to leave state s_1 to state s_3. If the US uses the average of interval number, then US won't stay in state s_1, US would change its strategy to state s_3 since $\mathbf{H}_{US}(1,3) = -0.04$. This result is consistent with our idea of introducing an uncertain preference. This kind of uncertainty is more suitable for real world disputes. It is indeed a challenging problem in this uncertain environment.

From the above, we get the following management enlightenment for this kind of bilateral trade disputes.

(1) This shows that whether US and China can stabilize in the peace state s_4 for a long time mainly depends on whether the two sides can reasonably take a long term view, not be tempted by the immediate short term, and no longer adopt

sophisticated sanctions and counter sanctions such as deterrence and temptation. It is obvious that achieving win-win cooperation between the US and China in both economy and trade is not only the rational choice for China but also for the US.

(2) Any unilateral concession won't reverse the stable state of the dispute. The most ideal state is that the two DMs, through peaceful consultations, focus on the long term, make concessions, and maintain stability.

(3) No matter how sophisticated sanctions and anti-sanction strategies are, if either of the DMs in the dispute lacks the goodwill to cooperate and promote mutual well-being and prosperity, then both DMs of the dispute are likely to fall into a swirl of fierce fighting, which will eventually lead to the undesirable result of both losing.

(4) Even if the "Thucydides trap" is unavoidable, we still need to analysis and comprehend the situation and understand that mutual concessions and cooperation are the paths to prevent economic and social decline.

6 Conclusion

A MOGM is defined in this paper to incorporate each DM's objectives. Mathematical matrix representations of preference are introduced by using values 1, 0, and -1. It can express three concepts of preference by DM: positive preference, indifference, and negative preference in one matrix. The comprehensive preference matrix with weight parameters on objectives and the UMP preference for MOGM is developed. Furthermore, the subjective uncertainty of DMs is considered. Interval numbers are used to express the degree of uncertainty of preference. Subsequently, Nash equilibrium and interval Nash equilibrium solution methods are developed for MOGMs and the dependence of equilibria on weights of objectives is also shown. A detailed modeling and calculation process in US-China trade dispute is also explained and demonstrated. The implications and in-depth result analysis for DMs have been given. The MOGM provides DMs with guidance for how to act strategically in bilateral trade disputes that occur in the real world.

Further studies can be carried out within MOGM. A preference structure to incorporate random uncertainty and probability deserves further research. Nash stability for MOGM is developed in this paper. For other classical stabilities for MOGM, such as GMR, SMR and SEQ, the definitions and the solution methods still need to be studied. Furthermore, MOGM could be expanded by taking into account coalitions among DMs.

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