Multilevel Parallel Computations for Solving Multistage Multicriteria Optimization Problems

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Abstract. In the present paper, a novel approach for solving the computationally costly multicriteria optimization problems is considered. Within the framework of the developed approach, the obtaining of the efficient decisions is ensured by means of several different methods for the scalarization of the efficiency criteria. The proposed approach provides an opportunity to alter the scalarization methods and the parameters of these ones in the course of computations that results in the necessity of multiple solving the time-consuming global optimization problems. Overcoming the computational complexity is provided by reusing the computed search information and efficient parallel computing on high-performance computing systems. The performed numerical experiments confirmed the developed approach to allow reducing the amount and time of computations for solving the time-consuming multicriteria optimization problems.

Keywords: Multicriteria optimization · Criteria scalarization · Global optimization · Search information · Parallel computations · Numerical experiment

1 Introduction

The paper discusses a novel approach for solving the time-consuming multicriteria optimization (MCO) problems. Such problems arise in many applications that is confirmed by a wide spectrum of research on this subject – see, for example, monographs [1–5] and reviews of scientific and practical results [6–8].

The solving of the MCO problems is usually reduced to finding the efficient (non-dominated) decisions[1]. In the limiting case, it may appear to be necessary to obtain the whole set of the efficient decisions (the Pareto set) that may

[1] The solutions, which cannot be improved with respect to any criteria without worsening of the efficiency values with respect to other criteria are understood as the efficient (non-dominated) decisions.

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require a large amount of computations. Another approach used widely consists in finding a relatively small set of efficient decisions only. As a rule, to find particular efficient decisions, the vector criterion is transformed to a single (scalar) criterion, which can be optimized by some algorithms of nonlinear programming. Among such approaches, one can outline various types of criteria convolutions, the lexicographic optimization methods, the reference point algorithms, etc. [1–3].

In the present paper it is supposed that the efficiency criteria can be multiextremal and computing the values of criteria and constraints can be time-consuming. But the computational complexity is even higher as it is supposed that the applied scalarization method can be varied in the course of computations that leads to a necessity of multiple solving of the global optimization problems [9–11]. In the framework of developed approach efficient global optimization methods are proposed for solving such computationally expensive MCO problems and different multilevel parallel computation schemes are investigated for executing these methods on high-performance supercomputer systems.

Further structure of the paper is organized as follows. In Sect. 2, the statement of the multicriteria optimization problem is given and a general scheme for the criteria scalarization is proposed. In Sect. 3, efficient global optimization methods utilizing the computed search information are considered. In Sect. 4, the multilevel schemes of parallel computations for the multistage solving of the computational-costly MCO problems are given. Section 5 presents the results of numerical experiments confirming the developed approach to be promising. In Conclusion, the obtained results are summarized and main directions of further investigations are outlined.

2 Multicriteria Optimization Problem Statement

Multicriteria optimization (MCO) problem can be formulated as follows [1–5]

$$f(y) \to \min, y \in D, \tag{1}$$

where $y = (y_1, y_2, \ldots, y_N)$ is a *vector of varied parameters*,

$f(y) = (f_1(y), f_2(y), \ldots, f_s(y))$ is a vector efficiency criterion, and $D \subset R^N$ is a search domain

$$D = \{y \in R^N : a_i \le y_i \le b_i, 1 \le i \le N\}, \tag{2}$$

for the given vectors a and b. Without loss of generality, in further consideration the criteria $f_i(y)$, $1 \le i \le s$ are suggested to be non-negative and the decrease of these ones corresponds to the increase of the decision efficiency.

In the most general case, the criteria $f_i(y)$, $1 \le i \le s$ can be multiextremal, and the procedure of computing the values of these ones can appear to be time consuming. Also, the criteria $f_i(y)$, $1 \le i \le s$ are supposed to satisfy the Lipschitz condition

$$|f_i(y_1) - f_i(y_2)| \le L_i \|y_1 - y_2\|, 1 \le i \le s, \tag{3}$$

where L_i, $1 \le j \le s$ are the Lipschitz constants for the criteria $f_j(y)$, $1 \le j \le s$ and $\| * \|$ denotes the Euclidean norm in R^N.

The efficiency criteria of the MCO problems are usually contradictory, and the decisions $y^* \in D$ with the best values with respect to all criteria simultaneously may be absent. In such situations, for the MCO problems it is appropriate to find the efficient (non-dominated) decisions, for which the improvement of the values with respect to any criterion results in worsening the efficiency values with respect to other criteria. Obtaining of the whole set of efficient decisions (the Pareto set) may require to perform a large amount of computations. As a result, another approach is applied for solving the MCO problems often – finding only a relatively small set of efficient decisions defined according to the requirements of the decision maker.

An approach to obtaining particular efficient decisions used widely consists in the transformation of a vector criterion into some general scalar criterion of efficiency[2] [1–5]

$$\min \varphi(y) = F(\alpha, y), y \in D, \tag{4}$$

where F is the objective function generated as a result of scalarization of the criteria f_i, $1 \le i \le s$, α is a vector of parameters of the criteria convolution applied, and D is the search domain from (2). Because of (3), the function $F(\alpha, y)$ also satisfies the Lipschitz condition with some constant L i.e.

$$|F(\alpha, y') - F(\alpha, y'')| \le L\|y_1 - y_2\|. \tag{5}$$

To construct a general scalar efficiency function $F(\alpha, y)$ from (4), one can use, in particular, the following methods of the criteria scalarization.

1. One of the scalarization methods used often consists in the use of the minmax convolution of criteria [1–3]:

$$\begin{gathered} F_1(\lambda, y) = \max(\lambda_i f_i(y), 1 \le i \le s), \\ \lambda = (\lambda_1, \lambda_2, \ldots, \lambda_s) \in \Lambda \subset R^s : \sum_{i=1}^{s} \lambda_i = 1, \lambda_i \ge 0, 1 \le i \le s. \end{gathered} \tag{6}$$

2. Another approach used widely is applied if there are some *a priori* estimates of the criteria values for the required decision (for example, based on some ideal decision or any existing prototype). In such cases, the solving of a MCO problem may consist in finding an efficient decision corresponding to given criteria values most completely. The scalar criterion $F_2(\lambda, y)$ can be presented as root-mean-square deviation of a point $y \in D$ from the ideal decision y^* [3]:

$$F_2(\theta, y) = \frac{1}{s} \sum_{i=1}^{s} \theta_i (f_i(y) - f_i(y^*))^2, y \in D, \tag{7}$$

where the parameters $0 \le \theta_i \le 1$, $1 \le i < s$ are the indicators of importance of the approximation precision with respect to each varied parameter y_i, $1 \le i \le N$ separately.

[2] It is worth noting that such an approach provides an opportunity to use a wide set of already existing global optimization methods for solving the MCO problems.

3. In the case when the criteria can be arranged in the importance, the method of successive concessions (MSC) [2–4] is applied often, according to which the solving of a MCO problem is reduced to a multistage solving of the global optimization problems with nonlinear constraints:

$$\begin{gathered} f_1^* = \min_{y \in D} f_1(y), \\ f_i^* = \min_{y \in D} f_i(y), f_j(y) \le f_j^* + \delta_j, 1 \le j < i, 1 < i \le s, \\ P_{lex}^{\delta}(f, \delta, D) = Arg \min_{y \in D} f_s(y), f_j(y) \le f_j^* + \delta_j, 1 \le j < s, \end{gathered} \tag{8}$$

where the notation Arg means the set of all points $y \in D$, at which the minimum value of the optimized criterion is achieved and δ_i, $1 \le i \le s$ are the feasible concessions from the minimum values of the efficiency criteria. At that, the multi-step computations in the scheme (8) may be reduced to the minimization of the general scalar criterion $F_3(\lambda, y)$ [11]:

$$F_3(\delta, y) = f_s(y), f_i(y) \le f_i^{min} + \delta_i (f_i^{max} - f_i^{min}), 1 \le i < s, y \in D, \tag{9}$$

where f_i^{min}, f_i^{max}, $1 \le i < s$ are the minimum and maximum values of the criteria[3] in the domain D from (2) and $0 \le \delta_i \le 1$, $1 \le i < s$ are the concessions normalized to the interval $[0, 1]$.

It is worth noting that due to the possibility of the changing of the requirements to the optimality in the course of computations, the form of the scalar criterion $F(\alpha, y)$ from (4) may vary. Thus, it may turn to be necessary to alter the scalarization method used (6)–(9) and/or to change the convolution parameters λ, θ, and δ [9–11]. Such variations form a set of scalar global optimization problems (4)

$$\mathbb{F}_T = \{F_k(\alpha_i, y) : 1 \le i \le T, k = 1, 2, 3\}. \tag{10}$$

This set of problems may be formed progressively in the course of computations; the problems from the set may be solved strictly sequentially or simultaneously in the time-share mode. Besides, the problems from the set $\mathbb{F}_T$ may be solved in parallel using high-performance computer systems. An opportunity of forming the set $\mathbb{F}_T$ allows to formulate a *new approach to the multistage solving of the multicriteria optimization problems* (MMCO).

3 Methods of Multistage Solving of Multicriteria Optimization Problems

In the general case, the problems of the set $\mathbb{F}_T$ from (10) are the global optimization problems, the solving of which implies constructing some grids covering the search domain D – see, for example, [12–17]. The necessity to construct the coverage of the search domain D leads to the "curse of dimensionality" – the

[3] Since the magnitudes f_i^{min}, f_i^{max}, $1 \le i < s$, may be unknown *a priori*, the values of these ones may be replaced by some numerical estimates, which may be obtained using the available search information.

computational complexity of solving the global optimization problems increases exponentially with increasing dimensionality. This computational complexity can be decreased by reducing the dimensionality of the optimization problems being solved with the use of Peano *space-filling curves or evolvents* $y(x)$ mapping the interval $[0,1]$ onto a N-dimensional hypercube D unambiguously and continuously – see, for example, [12,15]. As a result of such a reduction, initial multidimensional problem of multicriteria optimization (4) is reduced to a one-dimensional problem:

$$f(y(x)) = (f_1(y(x)), f_2(y(x)), \ldots, f_s(y(x))) \rightarrow min, x \in [0,1]. \quad (11)$$

It is worth noting that the one-dimensional functions obtained as a result of the reduction satisfy the uniform Hölder condition (see [12,15]) i. e.

$$|f_i(y(x')) - f_i(y(x''))| \leq H_i|x' - x''|^{1/N}, x', x'' \in [0,1], 1 \leq i < s \quad (12)$$

where the constants H_i are defined by the relation $H_i = 2L_i\sqrt{N+3}$, $1 \leq i \leq m$, L_i are the Lipschitz constants from (4) and N is the dimensionality of the optimization problem (1).

As a result of the dimensionality reduction, the search information obtained in the course of computations can be represented in the form

$$A_k = \{(x_i, z_i, f_i = f(y(x_i)) : 1 \leq i \leq k\}, \quad (13)$$

where x_i, $1 \leq i \leq k$ are the points of performed global search iterations, z_i, f_i, $1 \leq i \leq k$ are the values of scalar criterion $F(\alpha, y(x))$ from (4) and of the criteria $f_i(y)$ from (11), $1 \leq i \leq s$ computed in the points x_i, $1 \leq i \leq k$. Note that the data in the set A_k are arranged in the order[4] of increasing of the point coordinates x_i, $1 \leq i \leq k$ i.e.

$$x_1 < x_2 < \cdots < x_k \quad (14)$$

for more efficient execution of the global search algorithm.

The availability of the set A_k from (13) allows recalculating the results of all computations of the criteria values performed earlier to the values of the current optimization problem $F(\alpha, y(x))$ from (4) being solved without repeating the time-consuming computations of the criteria values i.e.

$$(x_i, f_i) \rightarrow z_i = F(\alpha, y(x_i)), 1 \leq i \leq k. \quad (15)$$

In this way, the search information A_k from (13) recalculated according to (15) can be reused to continue the solving of the MCO problem. Such an opportunity can provide an essential decrease of the amount of computations performed to solve every next problem of the set $\mathbb{F}_T$ from (10) down to performing some limited set of the global search iterations.

In the proposed approach, the Multidimensional Algorithm of Global Search (MAGS) developed within the framework of the information-statistical theory

[4] The arrangement of the data is reflected by the use of the lower index in (14).

of global search [12,15,18,19] is applied to solve the multiextremal optimization problems of the $\mathbb{F}_T$ from (10). The general computational scheme of MAGS can be presented as follows.

At the initial iteration of MAGS, a minimized function value $\varphi(y(x^0))$ from (4) is computed in some arbitrary point x^0 from the interval $(0, 1)$ (the computing of the function value will further be called a *trial*). Then, let us assume k, $k > 1$ global search iterations to be completed. The choice of the trial point of the next $(k+1)^{th}$ iteration is determined by the following rules.

Rule 1. For each interval (x_{i-1}, x_i), $1 < i \leq k$ compute a magnitude $R(i)$ called further a *characteristic* of the interval.

Rule 2. Determine the interval (x_{t-1}, x_t), which the maximum characteristic corresponds to[5]

$$R(t) = \max\{R(i) : 1 < i \leq k\}. \tag{16}$$

Rule 3. Perform the new trial in the interval with the maximum characteristic

$$x^{k+1} \in (x_{t-1}, x_t). \tag{17}$$

The stopping condition, according to which the trials are terminated, is defined by the condition

$$(x_t - x_{t-1})^{1/N} \leq \varepsilon, \tag{18}$$

where t is from (15), N is the dimensionality of the problem being solved from (1), and $\varepsilon > 0$ is the predefined accuracy of the problem solution. If the stopping condition is not fulfilled, the number of iterations k is incremented by unity, and new global search iteration is performed.

The convergence conditions for the algorithms developed within the framework of the information-statistical theory of global search were considered in [12]. Thus, at appropriate numerical estimates of the Hölder constants H_i, $1 \leq i \leq m$ from (12), MAGS converges to all available global minima points of the minimized function $\varphi(y(x))$.

It is worth noting also that the application of the MAGS algorithm after solving the current problem $F(\alpha, y)$ from (4) to solving the next problems from the set $\mathbb{F}_T$ from (10) allows reusing the search information A_k from (13) obtained in the course of all preceding computations.

4 Multilevel Parallel Computations for Solving Multistage Multicriteria Optimization Problems

The applied general approach to parallel solving the global optimization problems is the following – parallel computations is provided by simultaneous computing the minimized function values $F(\alpha, y)$ from (4) in several different points of the search domain D – see, for example, [12,20]. Such an approach provides

[5] The characteristics $R(i)$, $1 < i \leq k$ may be interpreted as some measures of importance of the intervals with respect to the probability to find the global minimum point in respective intervals.

the parallelization of the most time-consuming part of the global search process and is a general one – it can be applied to almost all global search methods for a variety of global optimization problems [18,19,21,22].

This approach can be applied at different computation levels – either at the level of solving of one of the problems of the set $\mathbb{F}_T$ from (10) or at the level of parallel solving of several problems of this set. These methods of parallel computations will be considered below in relation to multiprocessor computer systems with shared memory.

4.1 Parallel Computations in Solving Single Multicriteria Optimization Problem

Since the characteristics $R(i)$, $1 < i \leq k$ of the search intervals (x_{i-1}, x_i), $1 < i \leq k$ play the role of the measures of importance of the intervals with respect to the probability to find the global minimum points, the MAGS algorithm can be extended for the parallel execution at the following generalization of the rules (16)–(17) [12,18,20,23]:

Rule 2′. Arrange the characteristics of the intervals in the decreasing order

$$R(t_1) \geq R(t_2) \geq \cdots \geq R(t_{k-2}) \geq R(t_{k-1}) \tag{19}$$

and select p intervals with the indices t_j, $1 \leq j \leq p$ having the maximum values of the characteristics (p is the number of processors (cores) employed in the parallel computations).

Rule 3′. Perform new trials (computing of the minimized function values $F(\alpha, y(x))$ in the points x^{k+j}, $1 \leq j \leq p$ placed in the intervals with the maximum characteristics from (19) in parallel.

The stopping condition for the algorithm (18) should be checked for all intervals, in which the scheduled trials are performed

$$(x_{t_j} - x_{t_j-1})^{1/N} \leq \varepsilon, 1 \leq t_j \leq p. \tag{20}$$

As before, if the stopping condition is not satisfied, the number of iterations k is incremented by p, and new global search iteration is performed.

This extended version of the MAGS algorithm will further called *Parallel Multidimensional Algorithm of Global Search for solving Single* MCO problems (PMAGS-S).

4.2 Parallel Computations in Solving Several Multicriteria Optimization Problems

Another possible method of parallel computations consists in simultaneous solving several problems $F(\alpha, y)$ of the set $\mathbb{F}_T$ from (10). In this approach, the number of problems being solved simultaneously is determined by the number of processors (computational cores) available. The solving of each particular problem $F(\alpha, y)$ is performed using the MAGS algorithm. Then, taking into account

that all the problems of the set $\mathbb{F}_T$ are generated from the same MCO problem (the values of the scalar criterion $F(\alpha, y)$ are computed on the basis of the criteria values $f_i(y)$, $1 \le i \le s$ from (1)), it is possible to provide the interchange of the computed search information. For this purpose, the computational scheme of the MAGS algorithm should be appended by the following rule:

Rule 4. After completing a trial (computing the values of the function $F(\alpha, y)$ and criteria $f_i(y)$, $1 \le i \le s$) by a processor, the point of current trial $y^{k+1} \in D$ and the values of criteria $f(y^{k+1})$) are transferred to all processors. Then the availability of the data transferred from other processors is checked, and new received data is included into the search information A_k from (13).

Such a mutual use of the search information A_k from (13) obtained when solving particular problems of the set $\mathbb{F}_T$ from (10) allows to reduce significantly the number of global search iterations performed for each problem $F(\alpha, y)$ – see Sect. 5 for the results of the numerical experiments.

This version of the MAGS algorithm will be further called *Parallel Multidimensional Algorithm of Global Search for solving Multiple* MCO problems (PMAGS-M).

4.3 Parallel Computations in Joint Solving of Several Multicriteria Optimization Problems

The computational scheme of the PMAGS-S algorithm can be applied for the parallel solving of several problems of the set $\mathbb{F}_T$ as well. In this case, the choice of the intervals with the maximum characteristics $R(i)$, $1 < i \le k$ from (16) must be performed taking into account all simultaneously solved problems $F(\alpha, y)$:

$$R_{l_1}(t_1) \ge R_{l_2}(t_2) \ge \cdots \ge R_{l_{K-2}}(t_{K-2}) \ge R_{l_{K-1}}(t_{K-1}), 1 \le l_i \le p, 1 \le i \le K-1, \tag{21}$$

where l_i, $1 \le i \le K-1$ is the index of the problem, which the characteristic R_{l_i} belongs to and K is the total number of trials for all problems being solved simultaneously.

In this approach, the problems, for which the trials are performed, are determined dynamically in accordance with (21) – at each current global search iteration for a problem $F(\alpha, y)$, the trials may be absent or all p trials may be performed.

This version of the MAGS algorithms will further be called *Parallel Multidimensional Algorithm of Global Search for Joint solving of Multiple* MCO problems (PMAGS-JM).

5 Results of Numerical Experiments

The numerical experiments were carried out using the "Lobachevsky" supercomputer at University of Nizhny Novgorod (operating system – CentOS 6.4, managing system – SLURM). Each supercomputer node had 2 Intel Sandy Bridge E5-2660 processors 2.2 GHz, 64 GB RAM. Each processor had 8 cores (i.e. total

16 CPU cores per node were available). To obtain the executable program code, Intel C++ 17.0 compiler was used. The numerical experiments were performed using the Globalizer system [24].

The comparison of the efficiency of the sequential version of the developed approach with other approaches to solving the MCO problems was performed in [10,11]. This paper presents the results of numerical experiments for the evaluation of the efficiency of the parallel generalization of the developed approach. Each experiment consisted of the solving of 100 two-dimensional bi-criterial test MCO problems, in which the criteria were defined as the multiextremal functions [12]

$$\begin{gathered} f(y_1, y_2) = -(AB + CD)^{1/2} \\ AB = \left(\sum_{i=1}^{7} \sum_{j=1}^{7} [A_{ij} a_{ij}(y_1, y_2) + B_{ij} b_{ij}(y_1, y_2)] \right)^2 \\ CD = \left(\sum_{i=1}^{7} \sum_{j=1}^{7} [C_{ij} a_{ij}(y_1, y_2) - D_{ij} b_{ij}(y_1, y_2)] \right)^2 \end{gathered} \tag{22}$$

where

$$a_{ij}(y_1, y_2) = \sin(\pi i y_1) \sin(\pi j y_2),\ b_{ij}(y_1, y_2) = \cos(\pi i y_1) \cos(\pi j y_2)$$

were defined in the ranges and the parameters $-1 \le A_{ij}, B_{ij}, C_{ij}, D_{ij} \le 1$ were independent random numbers distributed uniformly. The functions of this kind are multiextremal essentially and are used often in the evaluation of the efficiency of the global optimization algorithms [10–12,18,19,21].

When performing the numerical experiments, the construction of a numerical approximation of the Pareto domain was understood as a solution of a MCO problem. To construct an approximation of a Pareto domain for each MCO problem with the criteria from (22), 64 scalar global optimization subproblems $F(\alpha, y)$ from (4) were solved with different values of the criteria convolution coefficients (i.e. total 6400 global optimization problems were solved in each experiment). The obtained results of experiments were averaged over the number of solved MCO problems. It should be noted that since the developed approach is oriented onto the MCO problems, in which computing the criteria values requires a large amount of computations, in all tables presented below the computational costs of solving the MCO problems is measured in the numbers of global search iterations performed.

In carrying out the numerical experiments, the following values of parameters of the applied algorithms were used: the required accuracy of the problem solutions $\varepsilon = 0.01$ from (18) and (20), the reliability parameter[6] $r = 2.3$. The experiments were carried out using a single supercomputer node (two processors, 16 computational cores with shared memory). In Table 1, the indicators of the computational costs (the numbers of the performed global search iterations) for all considered schemes of parallel computations (see Sect. 4) are presented (Fig. 1).

[6] The reliability parameter is used in the construction of the numerical estimate of the constants L_j , $1 \le j \le s$ from (3), L from (5) and H from (12).

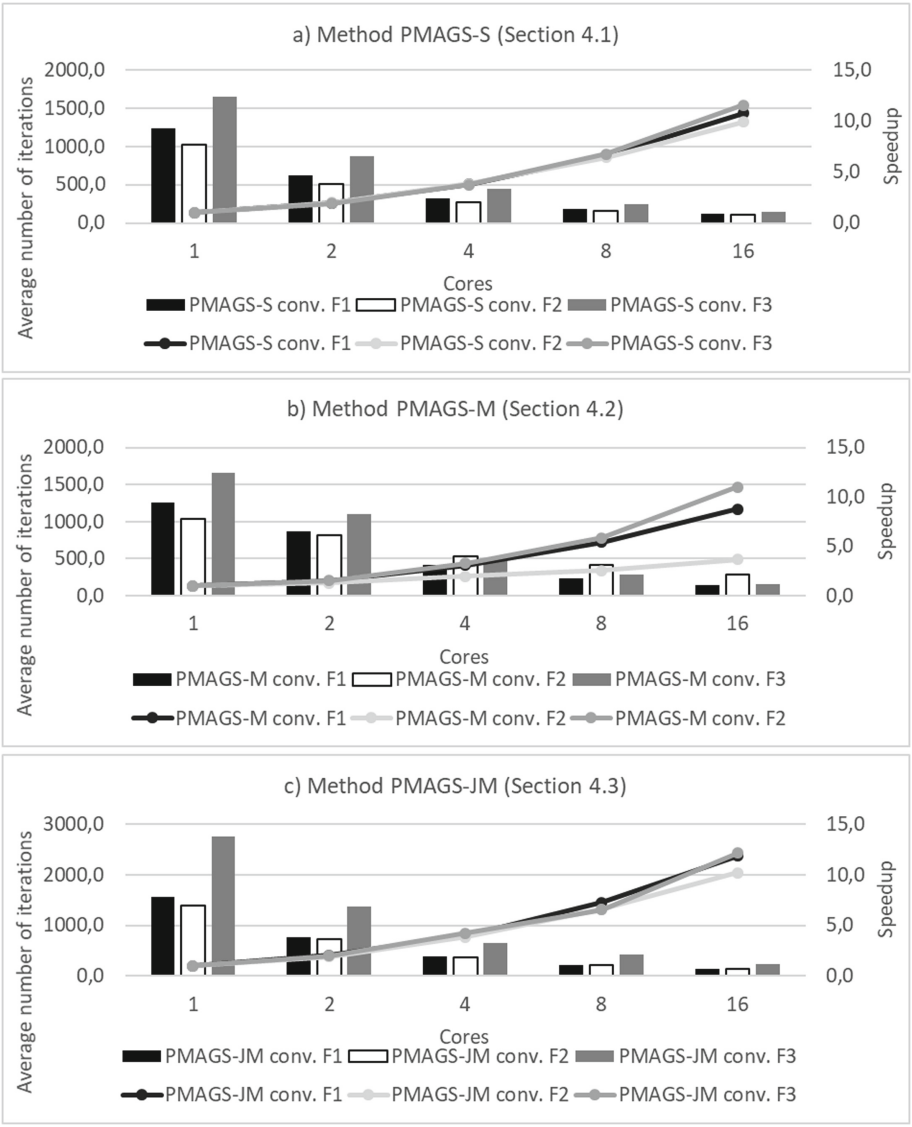

Fig. 1. Comparison of the averaged numbers of iterations executed for solving the MCO problems using various efficiency criteria convolutions

The results of experiments demonstrate the lowest computational costs (the number of performed global search iterations) to be achieved when using 16 cores for the PMAGS-S algorithm and the convolution F_2 from (7). Also, one can see from Table 1 that almost all computational schemes have a high efficiency from the viewpoint of parallelization. When using 16 cores, all algorithms except PMAGS-M with the convolutions F_1 from (6) and F_2 from (7) have demonstrated the speedup greater than 9.9.

Table 1. Comparison of performance of various schemes of parallel computations (the second column indicates the criteria convolution schemes F_1 from (6), F_2 from (7), F_3 from (9))

Method\Cores	Conv.	Average number of iterations					Speedup			
		1	2	4	8	16	2	4	8	16
Method	F_1	1238.3	625.7	324.5	186.7	115.3	**2.0**	3.8	6.6	10.7
PMAGS-S	F_2	**1018.6**	**512**	**264.2**	**158.5**	**102.9**	**2.0**	3.9	6.4	9.9
(Section 4.1)	F_3	1657.6	868.3	443.6	245.4	143.4	1.9	3.7	6.8	11.6
Method	F_1	1257.2	868.4	408.6	231.9	143.4	1.4	3.1	5.4	8.8
PMAGS-M	F_2	1035.9	811.2	532.1	407.6	284.3	1.3	1.9	2.5	3.6
(Section 4.2)	F_3	1653.3	1102.3	509.8	282.4	150.2	1.5	3.2	5.9	11.0
Method	F_1	1552.1	762.4	390.5	214	130.7	**2.0**	4.0	**7.3**	11.9
PMAGS-JM	F_2	1387.9	721.9	362.8	209.8	135.9	1.9	3.8	6.6	10.2
(Section 4.3)	F_3	2760.3	1371.7	652	421.4	227.6	**2.0**	**4.2**	6.6	**12.1**

Table 2. Comparison of the efficiency of the developed methods in solving the applied problem (the second row indicates the criteria convolution schemes F_1 from (6), F_2 from (7), F_3 from (9))

	Parallel scheme 4.1			Parallel scheme 4.2			Parallel scheme 4.3		
Convolution	F_1	F_2	F_3	F_1	F_2	F_3	F_1	F_2	F_3
Iterations	125	145	118	168	138	149	81	**82**	79

In order to demonstrate the efficiency of the proposed approach, a problem of vibration isolation for a system with several degrees of freedom consisting of an isolated base and an elastic body has been solved. In the considered problem statement, the protected object was represented as multi-mass mechanical system consisting of several material points connected by vibration damping elements. As the criteria, the maximum deformation and maximum displacement of the object relative to the base were minimized (for details, see [25]). The dimensionality of the space of the optimized parameters was selected to be 3.

The problem was solved by all considered methods using the Globalizer system. When solving the problem, the parameter $r = 3$ and the number of cores 16 were used. The number of convolutions was selected to be 16, and the accuracy of the method was set to $\varepsilon = 0.05$. The comparison of the efficiency of the methods by solving the applied problem is presented in Table 2.

The results of the numerical experiments demonstrate that all methods have found the sufficient approximation of the Pareto domain. The lowest number of iterations performed the PMAGS-JM method. In Fig. 2, the computed approximation of the Pareto domain obtained by the PMAGS-JM method with the convolution F_2 from (7) is presented.

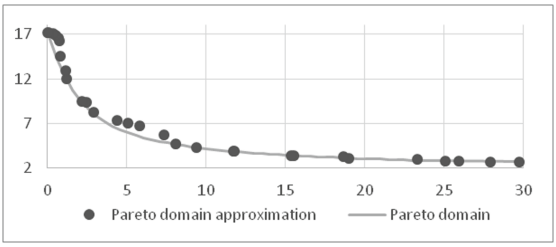

Fig. 2. Approximation of the Pareto domain for the problem of vibration isolation using PMAGS-JM method with the convolution F_2 from (7)

6 Conclusion

In the present paper, a novel approach for solving the time-consuming multicriteria optimization problems has been considered. Within the framework of the developed approach, obtaining the efficient decisions is provided by using several different efficiency criteria scalarization methods. The proposed approach allows altering the scalarization methods used and the parameters of these ones in the course of computations. In turn, such a variation of the problem statement leads to the need for solving multiple time-consuming global optimization problems. Overcoming the computational complexity is provided by means of the reuse of the whole search information obtained in the course of computations and efficient parallel computations on high-performance computational systems. The proposed methods of parallel computations can be used both for solving single MCO problems and for joint solving several ones.

The performed numerical experiments (total 6400 global optimization problems have been solved) and the example of solving the applied problem of vibration isolation confirm the developed approach to allow reducing the amount and time of computations for solving time-consuming MCO problems. In order to obtain more reliable evaluation of efficiency of the parallel computations, it is intended to continue carrying out the numerical experiments on solving the MCO problems with more efficiency criteria and for larger dimensionality.

Future research will also include investigations how to select the best parallel method automatically by using different computational platforms. Finally problems how to generalize the proposed approach for applying on multi-node cluster with GPU processors will be considered.

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