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Turbo Message Passing Algorithms for Structured Signal Recovery

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Preface

Message passing is an efficient iterative algorithm for solving inference, optimization, and satisfaction problems. It has found successful applications in numerous areas, including channel coding (e.g., low-density parity check (LDPC) codes and turbo codes), computer vision (e.g., stereo images), and free energy approximation. Recently, novel message passing algorithms, such as approximate message passing (AMP) and turbo compressed sensing (Turbo-CS), have been developed for sparse signal reconstruction. While the AMP algorithm exhibits guaranteed performance for sensing matrices with independent and identically distributed (i.i.d.) elements, Turbo-CS achieves a much better performance when the sensing matrix is constructed based on fast orthogonal transforms (such as the discrete Fourier transform and the discrete cosine transform). Inspired by the success of Turbo-CS, a series of message passing algorithms have been developed for solving various structured signal recovery problems with compressed measurements. We call these algorithms turbo message passing algorithms. In this book, we will undertake a comprehensive study on turbo message passing algorithms for structured signal recovery, where the structured signals include: (1) a sparse vector/matrix (which corresponds to the compressed sensing (CS) problem), (2) a low-rank matrix (which corresponds to the affine rank minimization (ARM) problem), and (3) a mixture of a sparse matrix and a low-rank matrix (which corresponds to the robust principal component analysis (RPCA) problem). In particular, the book is divided into the following parts: First, for the CS problem, we introduce a turbo message passing algorithm termed denoising-based Turbo-CS (D-Turbo-CS). We show that with D-Turbo-CS, signals without the knowledge of the prior distributions, such as images, can be recovered from compressed measurements by using message passing with a very low measurement rate. Second, we introduce a turbo message passing (TMP) algorithm for solving the ARM problem. We further discuss the impact of the use of various linear operators on the recovery performance, such as right-orthogonally invariant linear (ROIL) operators and random selectors. Third, we introduce a TMP algorithm for solving the compressive RPCA problem which aims to recover a low-rank matrix and a sparse matrix from their compressed mixture. We then apply the TMP algorithm to the video background subtraction problem and show that

TMP achieves much better numerical and visual recovery performance than its counterparts. With this book, we wish to spur new researches on applying message passing to various inference problems.

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