Energy-Efficient MIMO Multiuser Systems: Nash Equilibrium Analysis

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Abstract. In this paper, an energy efficiency (EE) game in a MIMO multiple access channel (MAC) communication system is considered. The existence and the uniqueness of the Nash Equilibrium (NE) is affirmed. A bisection search algorithm is designed to find this unique NE. Despite being sub-optimal for deploying the ε -approximate NE of the game when the number of antennas in transmitter is unequal to receiver's, the policy found by the proposed algorithm is shown to be more efficient than the classical allocation techniques. Moreover, compared to the general algorithm based on fractional programming technique, our proposed algorithm is easier to implement. Simulation shows that even the policy found by proposed algorithm is not the NE of the game, the deviation w.r.t. to the exact NE is small and the resulted policy actually Paretodominates the unique NE of the game at least for 2-user situation.

Keywords: Energy efficiency · Multiple access channel · MIMO · Game theory · Nash Equilibrium · Approximate Nash Equilibrium.

1 Introduction

With the release of first 5G package, it turns out that the number of devices in the upcoming wireless network will increase tremendously, e.g., Internet of Things (IoT). Consequently, classical paradigm which merely aims at optimizing the quantitative performance, e.g., data-rate, bit-error-rate and latency faces extreme difficulty in many domains in both academic research and industrial application. Thus the issue of energy-efficient design of the wireless system tends to be crucial. Different definition of energy efficiency (EE) has been proposed in recent years in [12–15]. Amongst which the most popular one is defined as the total benefit obtained under the unit consumption of energy or power known as global energy efficiency (GEE) e.g., in [3–5,11]. Taken the bits-per-second type rate function as benefit function, one will obtain well-known the bits-per-joule energy efficiency.

One of the pioneer works of studying the maximization of EE in Multiple-Input Multiple-Output (MIMO) system is [5]. In [5], the optimal precoding scheme is studied and divided into different cases with different assumptions on the systems. Till now the optimal precoding matrix for general condition is merely conjectured and unproved. Hereafter, optimal precoding matrix design for single user MIMO system is performed for imperfect channel state information (CSI) scenario in [9]. Then it is later widely realized that the problem of EE maximization actually belongs to the category of fractional programming. Techniques such as Dinkelbach's algorithm (see [8]) is used to solve EE maximization in [9,10]. These algorithms are generally based on the idea that the optimal solution can be found by solving a sequence of convex optimization problems related to the original one. The main difficulty of EE maximization OP is usually due to the non-convexity of energy efficiency function. Under some assumption on the benefit function, the EE function is well-known as being quasi-concave or even pseudo-concave. However, it is generally difficult to trace the Nash Equilibrium (NE) of a game where the individual utility function of player is of EE type. In [3], it is shown that there always exists an unique NE for scalar power allocation game in a relay-assisted MIMO systems due to the standard property of the best response dynamics. Similar results in MIMO-MAC system will be given latter in the paper.

The contribution of this paper is twofold: 1) we first extend the work in [3] to a more general situation where each user is allowed to choose its covariance matrix to maximize its individual EE instead of tuning its scalar power merely. The existence and uniqueness of the NE is proved under some assumptions. 2) An algorithm is proposed to to find the unique NE of this MIMO-MAC game. When the number of antennas of transmitter is equal the one of receiver, proposed algorithm leads to exact NE. Otherwise it leads to the ε -approximate NE defined latter in the paper for replacing the exact best response dynamic by its linear approximation.

The remaining parts of the paper is organized as follows: the MIMO-MAC system and the EE game are first presented in Sec. 2. Then some basics of game theory are given and the existence and the uniqueness of NE of the EE game is proved in Sec. 3. In Sec. 4, a basic algorithm is proposed and an improved bisection search algorithm is given which yields an ε -approximate NE slightly Pareto-dominating the exact NE. The numeric results of proposed algorithms are compared with classical allocation policy and analyzed in Sec. 5. The paper concludes by several remarkable conclusions in Sec. 6.

Notations: $(\cdot)^{H}$ and $(\cdot)^{\dagger}$ denote matrix transpose and Moore-Penrose inverse respectively. \mathbf{I}_{N} stands for identity matrix of size N. det (\cdot) and Tr (\cdot) denote the determinant and the trace of a matrix respectively. Denote the natural number set inferior or equal than N as $[N] \triangleq \{1, \ldots, N\}$.

2 System model

Consider a multiple access channel (MAC) with one base station (BS) and K users (players) to be served. BS is equipped with N_r receive antennas and each user terminal is equipped with N_t transmit antennas. We assume a block fading channel where the realization of channel remains a constant during the coherence time of transmission and randomly generated according to some statistical

distributions from period to period. The received signal at BS is given by:

$$\boldsymbol{y} = \sum_{k=1}^{K} \mathbf{H}_k \boldsymbol{x}_k + \boldsymbol{z}, \qquad (1)$$

where $\mathbf{H}_{k} \triangleq [\mathbf{H}_{k,i,j}]_{i,j=1}^{N_{r},N_{t}} \in \mathbb{C}^{N_{r} \times N_{t}}$ is the channel transmit matrix of k-th user and $\mathbf{H}_{k,i,j}$ is the channel from *i*-th transmit antenna of k-th user to *j*-th receive antenna at BS which is assumed to be i.i.d. complex Gaussian distributed according to $\mathcal{CN}(0,1)$. $\mathbf{x}_{k} = (x_{k,1},\ldots,x_{k,N_{t}})^{T}$ is the transmit symbol of k-th user and \mathbf{z} is the noise observed by the receiver with complex Gaussian distribution distribution $\mathcal{CN}(\mathbf{0},\sigma^{2}\mathbf{I}_{N_{r}})$. For the sake of simplicity, we assume that single user decoding is implemented for each user. Then the capacity the k-user can be achieved is

$$R_{k} = \log \frac{\det \left(\sigma^{2} \mathbf{I}_{N_{r}} + \sum_{j=1}^{K} \mathbf{H}_{j} \mathbf{Q}_{j} \mathbf{H}_{j}^{H}\right)}{\det \left(\sigma^{2} \mathbf{I}_{N_{r}} + \sum_{j \neq k}^{K} \mathbf{H}_{j} \mathbf{Q}_{j} \mathbf{H}_{j}^{H}\right)},$$
(2)

where $\mathbf{Q}_k = \mathbb{E} \left[\mathbf{x}_k \mathbf{x}_k^H \right] \in \mathbb{C}^{N_t \times N_t}$ is the covariance matrix of symbol \mathbf{x}_k which determines how power should be allocated for each antenna and $P_c > 0$ is the power dissipated in transmitter circuit to operate the devices. It is reasonable to assume that each user has perfect knowledge about its own channel, e.g., through downlink pilot training. Therefore user k is able to perform the singular value decomposition (SVD) of its own channel \mathbf{H}_k and its covariance matrix \mathbf{Q}_k as well. The SVD of \mathbf{H}_k and \mathbf{Q}_k is given by $\mathbf{H}_k = \mathbf{U}_k \mathbf{\Lambda}_k \mathbf{V}_k^H$ and $\mathbf{Q}_k = \mathbf{W}_k \mathbf{P}_k \mathbf{W}_k^H$ respectively. To simplify the problem, we assume that user k always adapts its covariance matrix to \mathbf{H}_k , i.e., choosing $\mathbf{W}_k = \mathbf{V}_k$. \mathbf{P}_k is a diagonal matrix with $\mathbf{P}_k = \text{diag}(\mathbf{p}_k) = \text{diag}(p_{k1}, \dots, p_{kN_t})$ where we use diag (\cdot) to generate a diagonal matrix from a vector or vice versa. Thus user k's only legal action is represented by \mathbf{p}_k or \mathbf{P}_k and the action set of k-th user is

$$\mathcal{P}_{k} = \left\{ \boldsymbol{p}_{k} \left| \sum_{i=1}^{N_{t}} p_{ki} \leq \overline{P}_{k}, \ p_{ki} \geq 0 \right. \right\}$$
(3)

where \overline{P}_k is power budget of k-th user. Through out the paper, we will use the matrix \mathbf{P}_k or its diagonal \boldsymbol{p}_k interchangeably to represent user k's action depending on the context. Further more, we denote $\boldsymbol{p} = (\boldsymbol{p}_k, \boldsymbol{p}_{-k})$ with $\boldsymbol{p}_{-k} \triangleq (\boldsymbol{p}_1, \ldots, \boldsymbol{p}_{k-1}, \boldsymbol{p}_{k+1}, \ldots, \boldsymbol{p}_K) \in \mathcal{P}_{-k}$ and $\mathcal{P}_{-k} \triangleq \mathcal{P}_1 \times \cdots \times \mathcal{P}_{k-1} \times \mathcal{P}_{k+1} \times \cdots \times \mathcal{P}_K$. In this paper energy efficiency defined as the ratio between a benefit function and the power consumed by producing it can be proven to has the following expression for user k after some simplifications:

$$u_k\left(\mathbf{P}_k, \mathbf{P}_{-k}\right) = \frac{\log \frac{\det\left(\sigma^2 \mathbf{I}_{N_r} + \sum_{j=1}^{K} \mathbf{U}_j \mathbf{\Lambda}_j \mathbf{P}_j \mathbf{\Lambda}_j^H \mathbf{U}_j^H\right)}{\det\left(\sigma^2 \mathbf{I}_{N_r} + \sum_{j\neq k}^{K} \mathbf{U}_j \mathbf{\Lambda}_j \mathbf{P}_j \mathbf{\Lambda}_j^H \mathbf{U}_j^H\right)}}{\operatorname{Tr}\left(\mathbf{P}_k\right) + P_c}$$
(4)

To this end, the MIMO MAC EE game is thus given by the following strategic form in triplet:

$$\mathcal{G} = \left(\mathcal{K}, \left(\mathcal{P}_k \right)_{k \in \mathcal{K}}, \left(u_k \right)_{k \in \mathcal{K}} \right)$$
(5)

3 Game-theoretic Analysis

In this section, we will firstly give some basic concepts of any game-theoretic analysis. The central concept of game-theoretic analysis is Nash Equilibrium (NE) defined as:

Definition 1. For game $\mathcal{G} = (\mathcal{K}, (\mathcal{P}_k)_{k \in \mathcal{K}}, (u_k)_{k \in \mathcal{K}})$, an action profile $\mathbf{p} = (\mathbf{p}_k, \mathbf{p}_{-k})$ is called a Nash Equilibrium if for $\forall k \in \mathcal{K}$ and $\forall \mathbf{p}' = (\mathbf{p}'_k, \mathbf{p}_{-k})$:

$$u_k\left(\boldsymbol{p}_k, \boldsymbol{p}_{-k}\right) \ge u_k\left(\boldsymbol{p}'_k, \boldsymbol{p}_{-k}\right) \tag{6}$$

The meaning of NE is that any unilateral change of action at this point won't lead to an enhance of individual benefit. Furthermore, we introduce an important conception in game-theoretic analysis known as best response dynamics.

Definition 2. (Best Response): In a non-cooperative game \mathcal{G} , the correspondence $\operatorname{BR}_k(p_{-k}): \mathcal{P}_{-k} \to \mathcal{P}_k$ s.t.

$$BR_{k}\left(\boldsymbol{p}_{-k}\right) \triangleq \arg \max_{\boldsymbol{p}_{k} \in \mathcal{P}_{k}} u_{k}\left(\boldsymbol{p}_{k}, \boldsymbol{p}_{-k}\right)$$
(7)

is called the best response (BR) of player $k \in \mathcal{K}$ given the action profile of other player \mathbf{p}_{-k} . From the definition of best response, one has immediately the following characterization for NE:

Proposition 1. [Nash,1950] An action profile \mathbf{p}^* is an NE if and only if : $\forall k \in \mathcal{K}, \ \mathbf{p}_k^* \in BR_k(\mathbf{p}_{-k}^*).$

To identify the NE of game in (5), the properties of individual utility function should be identified as first step. We define two critical properties satisfied by the individual utility function.

Definition 3. (Quasi-concavity) Let $\mathcal{X} \in \mathbb{R}^n$ be a convex set, a function $f : \mathcal{X} \to \mathbb{R}$ is said to be quasi-concave if

$$f(\lambda \boldsymbol{x} + (1 - \lambda) \boldsymbol{y}) \ge \min \{f(\boldsymbol{x}), f(\boldsymbol{y})\}$$
(8)

for any $\boldsymbol{x}, \boldsymbol{y} \in \mathcal{X}$ with $\boldsymbol{x} \neq \boldsymbol{y}$ and $0 < \lambda < 1$.

Definition 4. (Pseudo-concavity) Let $\mathcal{X} \in \mathbb{R}^n$ be a convex set, a function $f : \mathcal{X} \to \mathbb{R}$ is said to be quasi-concave if it is differentiable and for any $\mathbf{x}, \mathbf{y} \in \mathcal{X}$, it holds:

$$f(\boldsymbol{y}) < f(\boldsymbol{x}) \implies \nabla f(\boldsymbol{y})^{T}(\boldsymbol{x} - \boldsymbol{y}) > 0$$
(9)

With the definition of quasi-concavity and the pseudo-concavity, Prop. 2 shows that the individual utility function does possess these important properties:

Proposition 2. R_k is a concave functions w.r.t. p_k and u_k is a pseudo-concave (quasi-concave) function w.r.t. p_k for $\forall k \in \mathcal{K}$; For any fixed $p_{-k} \in \mathcal{P}_{-k}$ and p_{kj} with $j \neq i$, only one of following statements is true for all $i \in [N_t]$:

i) $\exists p_{ki}^{\star} > 0$ s.t. u_k is an increasing function in $(0, p_{ki}^{\star})$ and a decreasing function in $(p_{ki}^{\star}, +\infty)$ w.r.t. p_{ki} .

ii) u_k is a decreasing function in $(0, +\infty)$ w.r.t. p_{ki} .

Proof. It is well-known that R_k is a concave function for p_k . Then the pseudoconcavity (quasi-concavity) of u_k comes from the fact that it is a ratio of a concave function and an affine function of p_k . For more details of the proof, see [2]. Now we prove the second part of this proposition. Rewrite the individual utility function as $u_k = \frac{R_k(\gamma_k)}{\sum_{i=1}^{N_t} p_{ki} + P_c}$ with $R_k(\gamma_k) = \log(1 + \gamma_k)$. Then we can prove that $\frac{\partial^2 u_k}{\partial p_{ki}^2} \leq 0$ due to the fact that R_k is an increasing concave function w.r.t. γ_k and γ_k is a also increasing concave function w.r.t. p_{ki} . However we can't conclude directly of the sign of $\lim_{p_{ki} \to +\infty} \frac{\partial u_k}{\partial p_{ki}}$. It can be positive or negative depending on the value of p_{kj} with $j \neq i$. Therefore, if $\lim_{p_{ki} \to +\infty} \frac{\partial u_k}{\partial p_{ki}} \geq 0$ then we are in case ii), otherwise we are in case i).

Before stating the best response dynamics of the game, we define the following boundary of set \mathcal{P}_k indicated by an index subset $\mathcal{E} \subset [N_t]$:

$$\mathcal{P}_{k}\left[\mathcal{E}\right] \triangleq \left\{\boldsymbol{p}_{k} \in \mathcal{P}_{k}, p_{ki} = 0 \text{ for } i \in \mathcal{E}\right\}$$
(10)

and the non-negative index set for a given action \mathbf{P}_k :

$$\mathcal{I}(\mathbf{P}_k) \triangleq \{ i \in [N_t] \ s.t. \ p_{ki} \ge 0 \}$$
(11)

Proposition 3. For any given \mathbf{P}_{-k} and provided that the power budget \overline{P}_k is sufficiently large, denote the unique solution of the following equation as \mathbf{P}_k^* :

$$diag\left(\mathbf{\Lambda}_{k}^{H}\left(\mathbf{\Lambda}_{k}\mathbf{P}_{k}\mathbf{\Lambda}_{k}^{H}+\mathbf{F}_{k}+\sigma^{2}\mathbf{I}_{r}\right)^{-1}\mathbf{\Lambda}_{k}\right)=u_{k}\left(\mathbf{P}_{k},\mathbf{P}_{-k}\right)\mathbf{I}_{N_{t}}$$
(12)

with $\mathbf{F}_k = \sum_{j \neq k} \mathbf{S}_j \mathbf{P}_j \mathbf{S}_j^H$ is the interference matrix of k-th user with $\mathbf{S}_j = \mathbf{U}_k^H \mathbf{U}_j \mathbf{\Lambda}_j$. Then the BR of \mathbf{P}_k w.r.t. \mathbf{P}_{-k} is standard and converges to the unique NE admitted by game (5); The BR is the unique solution of (12) restricted to the boundary of \mathcal{P}_k indicated by $\mathcal{I}(\mathbf{P}_k^*)$ with $\mathcal{I}(\mathbf{P}_k^*) \neq \emptyset$.

Proof. our proof consists of two parts: i) existence of NE; ii) uniqueness of NE. i) Existence of NE: it is easy to prove that the action set \mathcal{P}_k for each player is compact (closed and bounded), combining the quasi-concavity of u_k claimed in Prop. 2, the existence is due to Debreu-Fan-Glicksberg theorem [7]. Moreover, Prop. 2 claims that u_k is a pseudo-concave function w.r.t. \mathbf{P}_k . Due to the property of pseudo-concave function, the unique stationary point (points where derivative vanishes) is the global optimizer of the utility function if the stationary point is in the feasible action set. We first calculate the stationary point of u_k for $\forall k \in \mathcal{K}$ using matrix calculus which leads to (12). However, the stationary point might not belong to the feasible action set \mathcal{P}_k . Denote \mathbf{P}_k^* the unique solution of (12) in \mathbb{R}^{N_t} . It is easy to prove that for given \mathbf{P}_{-k} , p_{ki}^* is a decreasing function w.r.t. $\forall p_{kj}$ with $j \neq i$ by contradiction. Due to this monotonicity of the BR and knowing that the feasible action set \mathcal{P}_k is a polyhedron, BR must be on the boundary of \mathcal{P}_k except $\mathbf{0}_{N_t \times N_t}$ defined as (10) which corresponds to the index set $\mathcal{I}(\mathbf{p}_k^*) \neq \emptyset$, which completes the proof for existence.

ii) Now we would like to prove that the BR converges to a point which is the unique NE of the game. We will achieve that by showing that the best response is a standard function 3 , i.e.,

1) Positivity: $\forall \mathbf{P}_{-k} \succeq 0$, $BR_k(\mathbf{P}_{-k}) \succeq 0$;

2) Monotonicity: if $\mathbf{P}'_{-k} \succeq \mathbf{P}_{-k}$, then $\mathrm{BR}_k \left(\mathbf{P}'_{-k} \right) \succeq \mathrm{BR}_k \left(\mathbf{P}_{-k} \right)$;

3) Scalability: BR_k ($\alpha \mathbf{P}_{-k}$) $\prec \alpha BR_k (\mathbf{P}_{-k})$ for any $\alpha > 1$.

Positivity is obviously observed in its form given by Prop. 3. The proof for monotinicity and scalability is similar to [3]. The strict proof is omitted due to the limit of space.

4 Algorithm for finding NE

Prop. 3 actually provides an approach for us to find the NE of the game (5). One can easily deduce an iterative equation according to (12):

$$\operatorname{diag}\left(\boldsymbol{\Lambda}_{k}^{H}\left(\boldsymbol{\Lambda}_{k}\mathbf{P}_{k}^{(t)}\boldsymbol{\Lambda}_{k}^{H}+\mathbf{F}_{k}^{(t-1)}+\sigma^{2}\mathbf{I}_{r}\right)^{-1}\boldsymbol{\Lambda}_{k}\right)=u_{k}\left(\mathbf{P}_{k}^{(t-1)},\mathbf{P}_{-k}^{(t-1)}\right)\mathbf{I}_{N_{t}}$$
(13)

However, due to Prop. 3, this stationary point might not be in the feasible action set. One can design the following basic algorithm to find NE of the game (5) based on Prop. 3 summarized in alg. 1.

Algorithm 1 Basic Algorithm for finding NE of MIMO-MAC EE game

Initialization: $\mathbf{P}_{k}^{(0)} = \frac{1}{N_{t}} \mathbf{I}_{N_{t}}, \forall k.$ Choose T and ϵ For t = 1 to T, do For k = 1 to K, do Compute $\mathbf{P}_{k}^{(t)}$ using (13) If $\mathcal{I}\left(\mathbf{P}_{k}^{(t)}\right) \neq [N_{t}]$ Compute $\mathbf{P}_{k}^{(t)}$ using (13) restricted to $\mathcal{I}\left(\mathbf{P}_{k}^{(t)}\right)$ End If End For If $\sum_{k} \left\|\mathbf{P}_{k}^{(t)} - \mathbf{P}_{k}^{(t-1)}\right\| < \epsilon$ Break End If End For Output: $\mathbf{P}_{k}^{NE} = \mathbf{P}_{k}^{(t)}$ for $\forall k.$

Nevertheless, alg. 1 is not satisfatory way to find the NE of the game. More precisely, to find the BR for given \mathbf{P}_{-k} , one actually need to solve an optimization problem. However, if $h = U(\mathbf{P}_{-k}) = \max_{\mathbf{P}_k \in \mathcal{P}_k} u_k(\mathbf{P}_k, \mathbf{P}_{-k})$ is known as a

³ The generalized inequality for matrix defined here is referred to its diagonal and takes the non-negative orthant as the underlying cone.

priori information, (13) can be transformed into following equation which is relatively easy to be solved compared to (13):

diag
$$\left(\mathbf{\Lambda}_{k}^{H}\left(\mathbf{\Lambda}_{k}\mathbf{P}_{k}^{(t)}\mathbf{\Lambda}_{k}^{H}+\mathbf{F}_{k}^{(t-1)}+\sigma^{2}\mathbf{I}_{r}\right)^{-1}\mathbf{\Lambda}_{k}\right)=h\mathbf{I}_{N_{t}}$$
 (14)

Introducing an auxiliary parameter h, one obtains an iterative equation of \mathbf{P}_k . Without loss of generality, we assume that the solution of (13) belongs to the feasible action set for given \mathbf{P}_{-k} . Otherwise, similar analysis can applied for \mathbf{P}_k but restricted on a boundary given by Prop. 3. For the sake of simplicity, we omit the discussion here and restrict ourselves to the situation where the BR is strictly included in the interior of the feasible action set. Therefore for all $i \in [N_t]$, there exists p_{ki}^* such that individual utility function $u_k(\mathbf{P}_k, \mathbf{P}_{-k})$ is an increasing function in $(0, p_{ki}^*)$ and a decreasing function in $(p_{ki}^*, +\infty)$ with respect to p_{ki} , where p_{ki}^* is the *i*-th component of user k's BR for given \mathbf{P}_{-k} . Then u_k is also an increasing function in $(0, U(\mathbf{P}_{-k}))$ and a decreasing function in $(U(\mathbf{P}_{-k}), +\infty)$ w.r.t. parameter h. In other words, to find $\mathbf{P}_k = \text{BR}(\mathbf{P}_{-k})$, it is sufficient to find $U(\mathbf{P}_{-k})$ by a bisection search due to the special monotonicity of the utility function.

However, it is worth mentioning that it is still difficult to directly find the solution of iterative equation (14). because this solution is actually implicitly given. We would like to further simplify (14) to facilitate the calculation of BR or NE. To start with, we assume that $N_t = N_r$. Firstly, we remove the diagonal operator of LHS of (14). Therefore we have:

$$\mathbf{P}_{k}^{(t)} = \frac{1}{h} \mathbf{I}_{N_{t}} - \mathbf{\Lambda}_{k}^{-1} \left(\mathbf{F}_{k}^{(t-1)} + \sigma^{2} \mathbf{I}_{N_{r}} \right) \mathbf{\Lambda}_{k}^{-1}$$
(15)

If $N_t > N_r$ or $N_t < N_r$ then Λ_k is not directly invertible, then we should consider the pseudo-inverse matrix of Λ_k . Without loss of generality, we assume that $N_t > N_r$, denoting the right pseudo-inverse of Λ_k as Λ_k^{\dagger} then one has $\Lambda_k \Lambda_k^{\dagger} = \mathbf{I}_{N_r}$ and $(\Lambda_k^{\dagger})^H \Lambda_k^H = \mathbf{I}_{N_r}$. Similarly, one has:

$$\mathbf{\Lambda}_{k}^{H} \left(\mathbf{\Lambda}_{k} \mathbf{P}_{k}^{(t)} \mathbf{\Lambda}_{k}^{H} + \mathbf{F}_{k}^{(t-1)} + \sigma^{2} \mathbf{I}_{r} \right)^{-1} \mathbf{\Lambda}_{k} = h \mathbf{I}_{N_{t}}$$
$$\left(\mathbf{\Lambda}_{k} \mathbf{P}_{k}^{(t)} \mathbf{\Lambda}_{k}^{H} + \mathbf{F}_{k}^{(t-1)} + \sigma^{2} \mathbf{I}_{r} \right)^{-1} = h \left(\mathbf{\Lambda}_{k}^{\dagger} \right)^{H} \mathbf{\Lambda}_{k}^{\dagger}$$
(16)

However, it is generally impossible to have $\mathbf{\Lambda}_{k}^{\dagger}\mathbf{\Lambda}_{k} = \mathbf{I}_{N_{t}}$. Thus the equality does not always holds when we multiply $\mathbf{\Lambda}_{k}^{\dagger}$ on left and $(\mathbf{\Lambda}_{k}^{\dagger})^{H}$ on the right on both sides of the equation. Nevertheless, this operation will yield a linear approximation of the BR dynamics:

$$\widehat{\mathbf{P}}_{k}^{(t)} = \frac{\mathbf{\Lambda}_{k}^{\dagger} \left[\left(\mathbf{\Lambda}_{k}^{\dagger} \right)^{H} \mathbf{\Lambda}_{k}^{\dagger} \right]^{-1} \left(\mathbf{\Lambda}_{k}^{\dagger} \right)^{H}}{h} - \mathbf{\Lambda}_{k}^{\dagger} \left(\mathbf{F}_{k}^{(t-1)} + \sigma^{2} \mathbf{I}_{N_{r}} \right) \left(\mathbf{\Lambda}_{k}^{\dagger} \right)^{H}$$
(17)

Similarly, if $N_t < N_r$ we can obtain exactly the same iterative equation as (17). This type of dynamics belongs to the so-called ε -approximate best response which generally leads to the ε -approximate Nash Equilibrium defined as:

Definition 5. For game $\mathcal{G} = (\mathcal{K}, (\mathcal{P}_k)_{k \in \mathcal{K}}, (u_k)_{k \in \mathcal{K}})$, an action profile $\mathbf{p} = (\mathbf{p}_k, \mathbf{p}_{-k})$ is called an ε -approximate Nash Equilibrium if for $\forall k \in \mathcal{K}$ and $\mathbf{p}' = (\mathbf{p}'_k, \mathbf{p}_{-k})$ for $\varepsilon \geq 0$:

$$u_k\left(\boldsymbol{p}_k, \boldsymbol{p}_{-k}\right) - u_k\left(\boldsymbol{p}'_k, \boldsymbol{p}_{-k}\right) \ge -\varepsilon \tag{18}$$

Obviously ε -approximate Nash Equilibrium is actually an extension of the concept of Nash Equilibrium. Notice that when $\varepsilon = 0$ then we are exactly back to the definition of Nash Equilibrium. If one deploys (17) as the BR dynamics to compute NE according to alg. 2, one may only result in ε -approximate Nash Equilibrium of the game. To this end, we obtain a sub-optimal algorithm summarized in alg. 2 by using the iterative equation deduced in (17) instead of using (13).

Algorithm 2 Bisection Search Algorithm for find the NE of MIMO-MAC EE game

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Initialization: \mathbf{P}_{k}^{(0)} = \frac{1}{N_{t}} \mathbf{I}_{N_{t}}, \forall k. choose T, \epsilon_{1} and \epsilon_{2}
For t = 1 to T, do
   For k = 1 to K, do
     Initialization: h = 0 and \overline{h} = h_{max}
      Repeat Until \overline{h} - h \leq \epsilon_1
        h_L = \max\left(0, h_M - \frac{\epsilon_1}{2}\right) \text{ and } h_M = \frac{\underline{h} + \overline{h}}{2}h_R = \min\left(h_{max}, h_M + \frac{\epsilon_1}{2}\right)
        Compute \mathbf{P}_{k}(h_{i}) using (17), i \in \{L, M, R\}
       U_i = u_k \left( \mathbf{P}_k(h_i), \mathbf{P}_{-k}^{(t-1)} \right), \ i \in \{L, M, R\}
        If U_L < U_M < U_R
              \underline{h} = h_L
              Else If U_L > U_M > U_R
                    \overline{h} = h_R
              End If
         Else
              \underline{h} = h_L and \overline{h} = h_R
        End If
        Compute \mathbf{P}_{k}^{(t)} by (17) with h = h_{M}
  End For
If \sum_{k} \left\| \mathbf{P}_{k}^{(t)} - \mathbf{P}_{k}^{(t-1)} \right\| < \epsilon_{2}
         Break
  End If
End For
Output: \mathbf{P}_k^{\text{NE}} = \mathbf{P}_k^{(t)} for \forall k.
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Remark 1. Alg. 2 actually works for general utility function possessing the same property as u_k . Moreover, this algorithm should be slightly faster than general bisection search. The reason is once by coincidence that the case neither $U_L > U_M > U_R$ nor $U_L < U_M < U_R$ occurs, we are surely to be very close to the stationary point of function. Otherwise we are in the monotonic region of the function, then this algorithm works as regular bisection search algorithm. In the worst case, this algorithm should have same complexity as the general bisection algorithm. Finally, alg. 2 merely requires the value of utility function instead of derivative of the utility function. Notice that alg. 2 is actually an off-line learning algorithm. Therefore a online-learning-version of alg. 2 by combing it with some deep learning techniques could be an important extension of this paper.

5 Numeric Results

The goal of this part is to show the performance of the proposed algorithms. Notice if $N_t = N_r$, (17) degenerates to (15) which conserves the optimality of best response. For this situation, we choose $N_t = N_r = 2$ with K = 2 users. A sufficient large power budget is chosen so that the BR is included in the feasible action set $\overline{P}_k = 10mW$ for $\forall k \in \{1, 2\}$ and the circuit power is $P_c = 1mW$. The error tolerance for alg. 2 is $\epsilon_1 = \epsilon_2 = 0.001$.

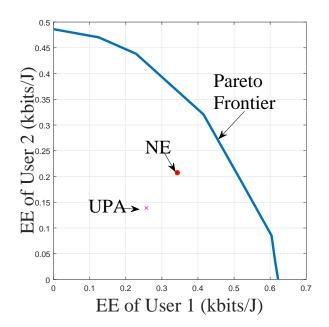


Fig. 1. Energy Efficiency under NE and uniform power allocation with $N_t = N_r = 2$ for 2-user situation. Policy found by our algorithms outperforms UPA policy.

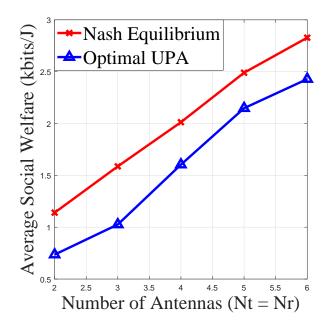


Fig. 2. Average social welfare under NE and UPA as function of number of antennas $(N_t = N_r)$ with $\overline{P}_k = 10mW$ for 2-user situation.

In Fig. 1, the achievable utility region, the average performance under NE found by alg. 2 and the averaged performance achieved by uniform power allocation (UPA) are depicted. All results are averaged over 1000 randomly generated channel samples. It is observed that the performance achieved by deploying UPA is Pareto-dominated by NE which can be found by alg. 2. Furthermore, the NE found by alg. 2 is closed to the Pareto frontier achieved by some centralized algorithms which suggest the efficiency using alg. 2 is higher than UPA.

Moreover, define the social welfare for a given action profile as $w(\mathbf{p}) = \sum_{k \in \mathcal{K}} u_k(\mathbf{p}_k, \mathbf{p}_{-k})$. Then the average social welfare as function of number of number of antennas (still we keep $N_t = N_r$) and the power budget in Fig. 2 and Fig. 3 respectively. For Fig. 2, the averaged social welfare of both UPA policy and our proposed algorithm is increased quasi-linearly as the number of antennas grows. However our proposed algorithm always outperforms the optimal UPA policy which is allowed to tune the power but always equally shared among each transmit antenna. In Fig. 3, we would like to show the influence of user's power budget. There are two different regions for social welfare. In the first region where the power budget is sufficiently large, the NE found by our proposed algorithm is increase of the power budget. In the second region where the power budget is relatively small, Using proposed algorithm, it is not sure to converge to the NE of the game because Prop. 3 is no more valid in this

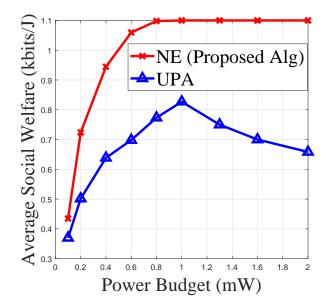


Fig. 3. Performance under NE and UPA as function of the power budget of user with $N_t = N_r = 2$ for 2-user situation. There are two different regions: one corresponds to Prop. 3. In the region uncovered by Prop. 3, proposed algorithm still dominates UPA.

region. Nevertheless, the performance achieved by our algorithm is still better than UPA which prove the superiority of our algorithm.

Then a more probable situation is considered where $N_t < N_r$ meaning that the number of antennas in user terminal is less than the one in base station. The discussion in Sec. 4 shows that the proposed suboptimal algorithm is actually suboptimal due to the usage of ε -approximate best response. For numeric demonstration, we choose $N_t = 2 < N_r = 4$. The performance of alg. 2 is illustrated in Fig. 4. The sub-optimality is clearly demonstrated in this figure. However, the resulted policy actually Pareto-dominates the exact NE found by alg. 1 and the dispersion is relatively small in terms of average performance. This remark entails that even the policy found by alg. 2 is not the NE of the game in its sub-optimal region however its performance does slightly outperforms the exact NE. Moreover the proposed algorithm is easy to implement for using explicit iterative equation even if it is approximated.

6 Conclusions

In this paper, a game where the individual utility function is the energy efficiency in a MIMO multiple access channel system is considered. The existence and the uniqueness of Nash Equilibrium is proved and an exact algorithm and a suboptimal algorithm is proposed to find the NE of this game. Simulation results

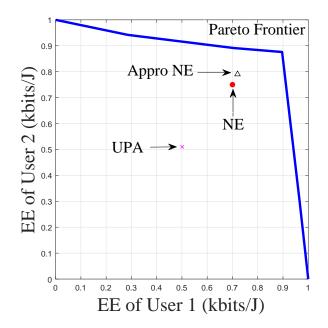


Fig. 4. Performance achieved by alg. 1 (NE) and alg. 2 (Approximate NE) and UPA with $N_t = 2$ and $N_r = 4$ for 2-user situation. Policy found by alg. 2 is very near to the exact NE and Pareto-dominates it. Moreover, two policies found by proposed algorithms both outperform UPA.

show that if the the number of transmit antennas and the number of receiving antennas is the same, performance under NE found by proposed algorithms is always better than uniform power allocation policy for both inside or outside the range covered by the main proposition of the paper. When the condition for antennas is not met, our proposed algorithm actually deploys an ε -approximate best response which might leads to an ε -approximate Nash Equilibrium. Quiet surprisingly the approximate NE found by our sub-optimal algorithm slightly Pareto-dominates the exact NE of the game. This observation shows that the performance of proposed algorithm is acceptable while it is relatively easy to implement. Other techniques such as pricing might be useful to improve the efficiency of the overall system. The situation where each user is allowed to freely choose its covariance matrix merely constrained to the maximum power is the natural extension of this paper. Moreover, the discussion over the effect of successive interference cancellation and multiple carrier seems to be complicated and serve as the challenge of the future works.

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