

Lecture Notes in Computational Science and Engineering

140

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DUNE — The Distributed and Unified Numerics Environment



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Preface

The idea that a book about DUNE would be a good thing to have has been on my mind for a long time. Many people have expressed interest in DUNE, right from the start. Many have started to use it, but it was a steep learning curve for a lot of them, because DUNE is a complex piece of software, and not every part of it is well documented.

There were two things that kept me from considering to write the book myself. First, it was obvious that it would be a large book, and I was intimidated by the amount of work that this would mean. Secondly, DUNE is the joint work of a large group of people, and I did not know how to properly and sufficiently give them credit.

For a while, I thought that I had a plan that would elegantly solve both worries at once. When that plan failed, I had already produced too much of a book to simply throw it away again. Therefore I simply kept going, and the result is what you hold in hands. I hope it will be useful to you.

Many people helped me write this book. Some proof-read parts of it and suggested improvements, others explained technical details to me that I hadn't known before. Several people donated text fragments and images that existed previously, and some fixed bugs in DUNE at my request. For all this I would like to thank Peter Bastian, Markus Blatt, Ansgar Burchardt, Andreas Dedner, Martin Drohmann, Christian Engwer, Jö Fahlke, Bernd Flemisch, Sebastian Götschel, Carsten Gräser, Dominic Kempf, Robert Klöfkorn, Sven Marnach, Dmitry Mazilkin, Steffen Müthing, Martin Nolte, Benedikt Oswald, Simon Praetorius, Susanne Schmidt, Justus Schock, Michaël Sghaïer, Henrik Stolzmann, Edscott Wilson Garcia, and Jonathan Youett. Special thanks go to Christoph Grüninger for reading the entire manuscript twice. Finally, I would also like to thank Jobst Hoffmann for his help with the L^AT_EX listings package, and Leonie Kunz and Martin Peters from Springer Verlag for their support.

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Oliver Sander

Contents

Preface	v
1. Introduction	1
1.1. The Case for Standardization	1
1.2. Goal of the Book	3
1.3. Structure of the Book	4
1.4. Source Code in this Book	6
I. Preliminaries	9
2. Mathematical Concepts	11
2.1. The Finite Element Method	11
2.2. The Finite Volume Method	28
2.3. Local Grid Adaptivity	33
3. Getting Started with Dune	43
3.1. Installation of DUNE	43
3.2. A First DUNE Application	45
3.3. Example: Solving the Poisson Equation Using Finite Elements	48
3.4. Example: Solving the Transport Equation with a Finite Volume Method	61
4. The Design and Structure of Dune	71
4.1. Software Functionality for Finite Element and Finite Volume Methods	71
4.2. The Structure of DUNE	73
4.3. The DUNE Core Modules	76
4.4. Designing Efficient Interfaces	78
4.5. Coding Style	84
4.6. Interface Stability and Backward Compatibility	87
II. The Core Modules	89
5. Grids and the Dune Grid Interface	91
5.1. Hierarchical Grids and Grid Views	93
5.2. Iterating over Vertices and Elements	97
5.3. Entities and Geometries	101

5.4. Intersections Between Elements	116
5.5. Reference Elements	127
5.6. Attaching Data to Grids	137
5.7. Creating Grids	151
5.8. Writing Grids and Data to VTK Files	165
5.9. Local Grid Adaptivity	171
5.10. Some Existing Grid Managers	183
6. Dune Grids on Distributed Machines	195
6.1. The DUNE Data Decomposition Model	195
6.2. Setting up a Distributed Grid	202
6.3. Dynamic Load-Balancing	206
6.4. Communication	213
6.5. MPI Setup with the MPIHelper Class	221
6.6. Writing Distributed Grids to VTK Files	222
6.7. Example: The Poisson Equation on a Distributed Grid	223
7. Linear Algebra with dune-istl	237
7.1. Constructing Matrix and Vector Types by Nesting	238
7.2. Data Structures for Vectors	241
7.3. Data Structures for Matrices	254
7.4. Solvers and Preconditioners	277
7.5. The Algebraic Multigrid Preconditioner	291
8. Local Finite Elements and the dune-localfunctions Module	301
8.1. Finite Elements and Affine Families	302
8.2. The Interface for Finite Elements Defined on the Reference Element	303
8.3. Implementations of the Local Finite Element Interface	312
8.4. Run-Time Selection of Local Finite Elements	323
9. Quadrature Rules	325
9.1. Numerical Integration	325
9.2. The DUNE Quadrature Rule Interface	328
III. Solving Partial Differential Equations	333
10. Function Space Bases and Discrete Functions	335
10.1. Function Space Bases	336
10.2. Programming Interface for Function Space Bases	349
10.3. Constructing Trees of Function Space Bases	359
10.4. Treating Subtrees as Separate Bases	365
10.5. Functions	366
10.6. Combining Global Bases and Coefficient Vectors	374

10.7. Writing Functions to a VTK File	380
10.8. Example: Solving the Stokes Equation with dune-functions	383
11. Discretizing Partial Differential Equations with dune-pdelab	397
11.1. Example: Linear Reaction–Diffusion Equation	398
11.2. Implementing Element Assemblers	405
11.3. Dirichlet Boundary Conditions	451
11.4. Linear Algebra Backends	461
11.5. Local Grid Adaptivity	476
 IV. Appendix	 497
A. The Dune Build System	499
A.1. Building and Installing DUNE Modules	499
A.2. DUNE Modules	508
 B. Complete Source Codes of the Example Programs	 519
B.1. The Geometry of the Example Domain	519
B.2. Finite Element Method for the Poisson Equation	520
B.3. Finite Volume Method for the Linear Transport Equation	525
B.4. Local Grid Adaptivity Without Data Transfer	528
B.5. Local Grid Adaptivity with Data Transfer	530
B.6. The Poisson Equation on a Distributed Grid	532
B.7. The Poisson Equation on a Distributed Grid Using ISTL Interfaces . .	540
B.8. The Sequential AMG Preconditioner	547
B.9. Dual Mortar Finite Elements	548
B.10. The Stokes Equation Using Taylor–Hood Elements	550
B.11. The Linear Reaction–Diffusion Equation Using dune-pdelab	556
B.12. p -Laplace Problem Using dune-pdelab and Finite Elements	558
B.13. The Linear Reaction-Diffusion Equation Using a DG Method	563
B.14. The Poisson Equation with Dirichlet Boundary Conditions	573
B.15. Demonstrating the Linear Algebra Backends	575
B.16. Error Estimation and Adaptive Grid Refinement Using dune-pdelab .	580
 Bibliography	 587
 Index	 599