Pricing and Quality Investments in a Mixed Brown-Green Product Market

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Abstract. Sustainable Supply Chain Management (SSCM) has assumed a position of prominence for academics and industry over the last two decades. The sustainability literature shows that typically manufacturers aim to optimize their pricing and greening level decisions in a mixed (green and brown) consumer market. In this work, we capture a manufacturer's classic dilemma on the pricing of green and brown products, and greening investments, while subject to budget constraint. We compute and analyze the variations of optimal decisions over time. Our findings underscore the importance of investing in greening technologies and learning for the survival of green products. Furthermore, we show that a manufacturer's optimal pricing strategy is to enter the market with a lower price for the green product and to increase it over time, eventually, surpassing the price for the brown product. Our analysis reveals that the greening level attraction can nullify the effect of a high price on the green product, resulting in higher green demand than brown. Higher green product demand is a win-win situation for both the manufacturer and the environment.
Keywords: Green products, Pricing, Greening level, Learning, Sustainability, Optimal control.

1 Introduction

The global emphasis on sustainable environmental practises in the last two decades has encouraged many manufacturers and retailers to envisage future consumer behaviour to be progressively green. Consequently, these manufacturers have invested in green practices and have been seeking to use green manufacturing, products, services or processes as means of competitive advantage. Manufacturing is one of the salient activities influencing the cost and environmental impact on supply chains [15,14], yet there are many drivers and barriers to green manufacturing practices. For example, [8] used a Delphi Survey method to explore the various stimulus and obstacles. While it was found that elements such as organizational commitment for Good Manufacturer Practice (GMP),

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eco-knowledge, society influences (green demand), financial incentives (government support), and innovation are drivers of green initiatives, few barriers also hinder them. Examples of such barriers are weak organizational structure to support GMP, lack of GMP knowledge, weak market positions for GMP-based products/processes, a society with low green attitudes, inadequate technology, the existence of sunk costs in GMP which can cause organizations to incur losses, inadequate supplier commitments towards GMP.

Greening of a product comes at a cost and, while greening the processes is more achievable, the subsequent deeper dive into green practises is costlier [18]. As a result, price of green products is usually much higher than the brown counterpart. Many consumers may be environmentally conscious. However, due to the price premium for green products, many consumers refrain from buying green despite their intentions to do so [1]. Thus, for most of the consumer goods, green products remain as a mere product line extension, while the brown counterparts far exceed the demand for green products. Deloitte conducted a direct study of more than 6,000 shopper experiences in 11 major retailers of varying formats to investigate the characteristics of the green shopper and to examine their shopping responses to sustainability issues. The survey revealed that while 95% of the respondents say they will buy green, only 22% have bought green, and 75% know what a green product is. The 2013 survey "*Consumer Trends in sustainability*", by Solarcity [17], shows that consumers are becoming more environmentally conscious and aware, and they wish to buy sustainable products and brands, but they are not always willing to pay more for the green products. Thus, there is a conflict between willingness to pay and willingness to act.

Motivated by the necessity of objectively understanding the conditions that potentiate the survival of green products, we propose an optimal control model that accounts for a manufacturer producing both green and brown substitutable products. The goal is to determine the optimal pricing and greening strategies over time under the following assumptions: *(i)* the manufacturer benefits from economies of scale; *(ii)* the manufacturer learning is increasing with green demand and this reduces greening costs; *(iii)* the manufacturer cannot exceed an investment budget. Through the determination of the model optimal outcomes, we provide theoretical results and numerical experiments allowing to identify the critical parameters for the success of the green product.

The rest of the paper is arranged in the following manner. In section 2, we provide a literature review of the related papers. Section 3 develops the model and section 4 provides the analytical results. In section 5, we present a comprehensive numerical study of different influential parameters of our model. Finally, in section 6, we conclude with the implication of our findings and directions for future research.

2 Literature Review

A considerable part of the literature in Green Supply Chain management focus on the pricing and green quality of these products. In a comprehensive literature review, [11] shows that the recent articles on sustainability focus on different dimensions like economical, social, environmental, ethical and temporal. Many decision-support models are derived, leading to a myriad of solutions for sustainable societies. In the same line, our work also articulates a decision problem of pricing and quality, integrating the economic and environmental aspects. In particular, the literature can be broadly classified into two categories: *(i)* research on green products only (for example, [9,4,3,23]); *(ii)* research on both green and brown products (for example, [19,20,21]). In the first category (green products), these works investigate different directions such us the development and introduction of green products [4], the competition between green products [22,4,12,5] or chain to chain competition [13], green pricing and greening investment decisions under supply chain collaboration [9,10], government intervention for green product developments or promotions [7,6,5,24,5], eco-labelling [3] and diffusion of green products [16].

Our paper contributes in the second category - brown and green products. Therefore, we detail the literature in this direction. Various research problems have been explored in this area. In [21], it is investigated how a manufacturer's choice of green and brown products is determined in a market with environmentally conscious customers. The authors also shed light on fraudulent green attributes. Precisely, a manufacturer might have a strong inclination to behave fraudulently leading to the enforcement of strict supervision for such behaviour. Using a consumer utility based model, [19] shows that while extending a brown product line with green products, manufacturers can strategically avoid product cannibalization with a two-level pricing structure based on consumer segmentation. The static game theoretic model also incorporates learning and capacity constraints. The green quality appeal can also downsize and eliminate the brown demand under certain conditions. [20] considers the manufacturer strategies of *greening out* or *greening up*. Based on market segmentation of consumers into three different groups - the traditional segment, the fence sitter segment and the green segment - the authors find that better economic and environmental performance can persist together. Yenipazarli and Vakharia [23] consider a competition between a green and a brown manufacturer where there are two groups of consumers - brown consumers who are not willing to pay for green products and green consumers who are willing to pay for the green products. The authors highlight that the size of each consumer group can result in different findings. For example, the manufacturers are in a loss-loss situation when the size of the green customer group is larger. On the other hand, an increasing premium for the green product may lead to a win-win situation for both manufacturers.

In this paper, we propose a novel model where the demand for green and brown products is a dynamic function of both product prices and greening level. There is a vast literature focusing in this area of pricing and greening level investments, and some studies treat this classic dilemma of a manufacturer (*e.g.* [19,20,23]). However, to the best of our knowledge, our paper is one of the first to simultaneously incorporate a market with green and brown products, pricing and greening investments, budgets, learning of green practices and mutual interactions among these elements. We account for time when analyzing the optimal manufacturer decisions related to green and brown products. We emphasize the appropriateness of a dynamic model since greening level and learning evolve with time. Thus a dynamic model accurately enables an understanding and planing of long term optimal decisions (strategies). Moreover, since we derive feedback strategies, the manufacturer does not use static or pre-committed pricing and greening policies. Instead, given that its decisions are a function of greening level and learning, they enable the manufacturer to adjust them to improve its profit.

3 The Model

In this section, we present the manufacturer model within a monopolist context selling both green and brown products. Contrary to most of the previous literature, we consider a dynamic setup, *i.e.*, the decisions to be taken by the manufacturer and hence, the consumers' demands and manufacturer's budget restrictions are all dynamic entities depending on time and on the evolving state of the economy. In this context, the manufacturer sells a green and a brown products which are substitutable (*e.g.*, electric vehicles and conventional cars, organic food and GMO food, thermal or renewable power source). The manufacturer has as decision variables $p_g(t)$ and $p_b(t)$ which represent the price of the green and brown products, respectively, at time t, as well as, $I(t)$ corresponding to the investment efforts in green quality at time t. The state variables are $S(t)$, describing the greening level (or green quality), and $\lambda(t)$, modeling the learning of green practices. In the remainder of the paper, the subscript indices $_g$ and $_b$ will be used to designate association with green and brown products, respectively. Next, we devise and explain the manufacturer's optimization model.

Demand. The demand for the products depends on their prices. In particular, the demand functions for the green and brown products are:

$$\begin{aligned} D_g(t) = &\quad \alpha_g - \beta_g p_g(t) + \gamma_g p_b(t) + \eta S(t) \\ D_b(t) = &\quad \alpha_b - \beta_b p_b(t) + \gamma_b p_g(t) - \delta S(t). \end{aligned} \tag{1}$$

The parameters α_g and α_b are the market potentials, β_g and β_b are the individual price sensitivities, γ_g and γ_b are the cross price sensitivities, and, η and δ are the green and brown consumer's attraction, respectively, towards green products. We assume that the following relations hold: $\beta_k > \gamma_k$, where $k \in \{g, b\}$, *i.e.*, the green product demand $D_g(t)$ is more sensitive to its price $p_g(t)$ than to the brown product price $p_b(t)$; similarly, for $D_b(t)$. Thus, this is reasonable assumption in practice. As the demand functions (1) reflect, it is considered that green customers are not restricted to buying green products neither the reverse. Customers can indeed switch to brown products from green if they are not satisfied by the green level obtained at a premium price rises due to manufacturer's constraints. Similarly, brown customers can be attracted towards the green product due to green quality $S(t)$. However, following some of the existing literature (*e.g.*[19]), we assume that the influence of greening level on green customers is superior to the negative influence of greening level on brown customers, *i.e.*, $\eta > \delta$. The total instantaneous demand for the manufacturer is $D(t) = D_g(t) + D_b(t)$.

Learning. We assume that the manufacturer can reduce the production cost of green products by economies of scale. In other words, as demand increases, learning from more production takes place. Consequently, learning is a state variable of this model. We define learning by the following equation:

$$\dot{\lambda}(t) = \phi D_g(t) - \omega\lambda(t) \text{ and } \lambda(0) = 0, \tag{2}$$

where ϕ and ω are non-negative numbers providing the rate of learning increase due to green demand and decrease due to decay in learning. Let us further discuss their interpretation. There are several factors influencing these parameters - (i) a firm failing to adopt a new technology, resulting in learning decay, (ii) improper development of business processes may result in learning loss, (iii) information asymmetry or lack of information in a supply chain can cause a higher decay (ω) or a lower ϕ, (iv) stringent budget constraints may result in learning decay and lastly, (v) efficiency of employees or technology being used can also be determinant for the values of ϕ and ω. While the above list is insightful it might not be exhaustive.

Learning will occur if $\lambda(t) > 0$. The complexity of our parametric space does not allow a straightforward derivation of a condition for positive learning. However, we have numerically verified that for a set of parameter values providing feasible equilibrium solutions, learning is always positive and increasing, as long as the initial value of learning is non-negative.

Green Quality. The greening level is given by the following equation:

$$\dot{S}(t) = k_1 I(t) + k_2\lambda(t) - \epsilon S(t) \text{ and } S(0) = 0, \tag{3}$$

where k_1, k_2 and ϵ are non-negative numbers reflecting the rate of greening level increase due to investments and learning, and decay in greening level. The above evolution of greening quality shows that marginal greening is positively influenced by the direct greening investments and by learning. One may argue that a higher price for the green product than for the brown one can lead consumers to perceive the green product as of superior quality. Nevertheless, [2] reveals that this is not significant and thus, we do not consider it in equation (3).

Green Product Budget. In the industry, manufacturers are always constrained by budgets that limit investments and thus, the greening level. Therefore, we assume that there is a budget constraint given by

$$0 \leq I(t) \leq M_g(t). \tag{4}$$

The above equation enforces a budget controlling the investment effort.

This captures the classic trade off between the economic and environmental dimensions of sustainability. Since managers of manufacturers are aware that learning reduces greening costs. Therefore, we assume that the budget $M_g(t)$ is not constant but reduces with learning increases with greening level. Therefore we assume the following structure for the budget:

$$M_g(t) = \xi_s S(t) - \xi\lambda(t). \tag{5}$$

The non-negative parameter ξ_s is the initial constant budget for green quality and green production. The rationale behind choosing such a function is that it allows us to capture two important aspects of the greening budget: *(i)* if learning is high management may decrease the budget, *(ii)* if greening quality is high, the management my increase the budget to avoid to limit it.

Cost Structure. The cost of greening is quadratic in the investment efforts and is given by:

$$\frac{\mu}{2} I^2(t) - \sigma\lambda(t), \tag{6}$$

where the non-negative parameters μ and σ are the proportionality constant for investment costs and cost reduction per unit learning. Given that the accumulated learning at time t, $\lambda(t)$, is increasing with the green demand, cost structure signifies that with an increasing green demand, we have a reducing greening cost due to learning. The higher the learning $\lambda(t)$, the lower the cost (6). Here σ can be interpreted as how efficiency of a manufacturer in leveraging the learning. If a manufacturer is efficient, it can employ proper resources to reduce more the cost (*i.e.*, σ is high).

Model. Therefore, the manufacturer's infinite horizon decision problem is given by:

$$\begin{aligned} V(S(t), \lambda(t)) = \underset{p_g, p_b, I}{\text{Max}} \int_0^\infty e^{-rt} [D_g p_g + D_b p_b - \left(\frac{\mu}{2} I^2 - \sigma\lambda\right)] dt, \\ \text{Subject to } (2), (3), (4) \\ p_g \geq 0, p_b \geq 0. \end{aligned} \tag{7}$$

For sake of simplicity, we have dropped the dependence on the time in the objective function of (7), and we will do it in the remaining of the paper, whenever this dependence is clear from the context. We stress that the consideration of an infinite time horizon is suitable for the questions addressed in this work as it enables a long-term analysis and thus, planning.

The above problem can be solved using the Hamilton-Jacobi-Bellman (HJB) equations, in order to obtain the optimal feedback strategies:

$$\begin{aligned} rV(S, \lambda) = \underset{p_g, p_b, I}{\text{Max}} \Bigg(D_b p_b + D_g p_g + \frac{\partial V}{\partial \lambda} (\phi D_g - \omega\lambda) + \\ \frac{\partial V}{\partial S} (\lambda k_2 + k_1 I - \epsilon S) - \frac{\mu I^2}{2} + \sigma\lambda \Bigg). \end{aligned} \tag{8}$$

The HJB equation (8) does not incorporate the budget constraint. However, we consider the feasible solutions obtained by solving (8) which are characterised by Lemma (2) established in section 4.

We conjecture that the value function is of the following form and later prove it in Proposition (1):

$$V(S, \lambda) = A_1 S^2 + A_2 \lambda^2 + A_3 \lambda S + A_4 S + A_5 \lambda + A_6. \tag{9}$$

4 Analytical Results

In this section, we present our analytical findings. First, we find the manufacturer's optimal pricing of the green and brown products and the optimal investment for the green product. We show how the prices and investments vary with time, the greening level and learning. Fundamental sensitivity analysis on the effect of our model parameters is performed, namely, learning sensitivity, price sensitivity and green level sensitivity on the pricing and investment strategies. We have conjectured the structure of the value function in equation (9) and, in Proposition 1, we will show how to solve for this value function. Before presenting Proposition 1, we provide the following technical lemma which is essential for establishing the non-negativity of the optimal decision variables values.

Lemma 1 *The following condition holds:* $4\beta_g\beta_b - (\gamma_g + \gamma_b)^2 > 0$.

Proof. According to our model assumptions, we have the relations $\beta_g > \gamma_g, \gamma_b$ and $\beta_b > \gamma_b, \gamma_g$. Note that $4\beta_g\beta_b - (\gamma_g + \gamma_b)^2 = (2\beta_g\beta_b - 2\gamma_b\gamma_g) + (\beta_g\beta_b - \gamma_g^2) + (\beta_g\beta_b - \gamma_b^2)$, where each term in the brackets is positive.

Proposition 1 *In the problem* (7)*, the optimal prices of the green and brown products and the greening investment decisions of the manufacturer are given by:*

$$p_g(t) = \frac{1}{4\beta_b\beta_g - (\gamma_b + \gamma_g)^2}\Big(2\beta_b\left(-\phi\beta_g\left(2A_2\lambda + A_3S + A_5\right) + \alpha_g + \eta S\right) + (\gamma_b + \gamma_g)\left(\phi\gamma_g\left(2A_2\lambda + A_3S + A_5\right) + \alpha_b + \delta(-S)\right)\Big), \quad (10)$$

$$p_b(t) = \frac{\beta_g\left(\phi\left(2A_2\lambda + A_3S + A_5\right)(\gamma_g - \gamma_b) + 2\alpha_b - 2\delta S\right) + (\gamma_b + \gamma_g)(\alpha_g + \eta S)}{4\beta_b\beta_g - (\gamma_b + \gamma_g)^2}, \quad (11)$$

$$I(t) = \frac{k_1\left(A_3\lambda + 2A_1S + A_4\right)}{\mu}.$$

(12)

The value function of the manufacturer is given by

$$V(S, \lambda) = A_1S^2 + A_2\lambda^2 + A_3\lambda S + A_4S + A_5\lambda + A_6 \quad (13)$$

where A_i*s,* $i \in \{1, 2, 3, 4, 5, 6\}$ *are constant coefficients of the state variable associated terms of the value function* $V(S, \lambda)$ *given in equation (9).*

Proof. The right hand side of the HJB equations (8) is given by
$HJB_{RHS} = D_bp_b + D_gp_g + \frac{\partial V}{\partial \lambda}(\phi D_g - \lambda\omega) + \lambda\sigma + \frac{\partial V}{\partial S}(\lambda k_2 + k_1Z - S\epsilon) - \frac{\mu Z^2}{2}$.
Next, as standard, we take the first order conditions:

$$\frac{\partial(HJB_{RHS})}{\partial p_g} = -\phi\beta_g\left(2A_2\lambda + A_3S + A_5\right) + p_b\gamma_g + \gamma_bp_b + \alpha_g - 2\beta_gp_g + \eta S = 0 \quad (14)$$

$$\frac{\partial(HJB_{RHS})}{\partial p_b} = \phi\gamma_g\left(2A_2\lambda + A_3S + A_5\right) + \alpha_b + \gamma_bp_g - 2\beta_bp_b + \gamma_gp_g + \delta(-S) = 0 \quad (15)$$

$$\frac{\partial(HJB_{RHS})}{\partial I} = k_1\left(A_3\lambda + 2A_1S + A_4\right) - \mu I = 0. \quad (16)$$

The system of equations given by (14) and (15), leads to the solutions in equations (10) and (11) for p_g and p_b. Solving equation (16) for I, gives the expression in equation (13).

Thereafter, these expressions of p_g, p_b and I are placed in the right-hand-side of the HJB equation. Finally, we compare the coefficients of the state variables or its associations from the two sides of the equation:

$$r \times (A_1 S^2 + A_2 \lambda^2 + A_3 \lambda S + A_4 S + A_5 \lambda + A_6) = D_b p_b + D_g p_g + \frac{\partial V}{\partial \lambda}(\phi D_g - \lambda \omega) + \lambda \sigma + \frac{\partial V}{\partial S}(\lambda k_2 + k_1 Z - S\epsilon) - \frac{\mu Z^2}{2} \quad (17)$$

to obtain a set of six non-linear equations which are solved to get the coefficients A_i. We used Mathematica software (version 12) to solve the equations as these are not solvable manually[1].

Remark: For our model, the complexity of the parametric space is such that we are able to solve the value function only numerically. We have assumed suitable numerical values (given in the GitHub code) of the parameters which are well articulated to our model assumptions and the reality. Therefore, we confidently assert that our numerical results are quite robust. Moreover, the coefficients of the state variables (A_is) in our value function have multiple solutions and therefore, the value function is not unique. Hence, we have chosen the most suitable solution in which all A_is are positive. All other solutions result in a negative investment or pricing decision which is infeasible.

The above are the feedback pricing and green investment decisions of the manufacturer. One can note that the optimal prices and investment decisions are linear in the state variables, greening level S and learning λ. Therefore, an important question is: how do the optimal decisions vary with the state variables and other model parameters? Another, is on the comparison between the green and brown demands under the optimal policies. Nevertheless, before answering to these questions in the next sections, it remains to ensure that the optimal investment decision satisfies the budget constraint (4).

The parameters ξ_s and ξ in the budget constraint are known ex-ante by the manufacturer. In addition, the optimal decisions are also known to the manufacturer. Therefore, for a green product to exist in the market, the values of ξ_s and ξ must be suitable to design an admissible investment strategy. The following lemma shows the conditions feasible for producing green products.

Lemma 2 *The manufacturer will produce green products only if the following relationship between greening level and learning holds*

$$S(t) \geq \frac{\left(\xi_s + \frac{A_3}{\mu}\right)\lambda(t) + \frac{A_4 k_1}{\mu}}{\left(\xi - \frac{2A_1 k_1}{\mu}\right)}. \quad (18)$$

[1] The code used in this paper for solving the HJB equations and for the sensitivity analysis of section 5 is publicly available on GitHub: https://github.com/arkamukherjee80/RagingSun.

Proof. We know that the green investment constraint is given by $0 \leq I(t) \leq M_g(t) = \xi_s S(t) - \xi\lambda(t)$. From Proposition (1) we note that $I(t) = \frac{k_1(A_3\lambda + 2A_1 S + A_4)}{\mu}$. Therefore, for the the budget constraint to be satisfied, we must have $\frac{k_1(A_3\lambda+2A_1S+A_4)}{\mu} \leq M_g(t)$. Rearranging the terms, we get the inequality (18).

In inequality (18), ξ_s represents the manufacturer's (or management's) enthusiasm to invest in green quality and ξ represents the management's anticipation of how demand learning will decrease the budget for green investment. The above relation, therefore, signifies that given any values of ξ_s and ξ an admissible investment policy $I(t)$ might not exist at any given point of time.

4.1 Qualitative Analysis of Optimal Policies

In this section, we investigate how the decisions of the manufacturers vary with the state variables, greening level and learning, and the different model parameters. While we provide a comprehensive analysis of the optimal decisions, it is often more tractable to consider symmetry of the system parameters (*e.g.*, $\beta_g = \beta_b = \beta$ and $\gamma_g = \gamma_b = \gamma$) without compromising the quality of the insights obtained. We carry out such simplifications in some cases.

Lemma 3 *The price of the green product is:*

(i) increasing with the greening level $S(t)$ if and only if $\beta_g < \frac{1}{A_3}\left(\gamma_g + \frac{\eta - \delta A_3}{\phi}\right)$, and decreasing otherwise,
(ii) always decreasing in the learning $\lambda(t)$,
(iii) always increasing with η and decreasing with δ

where A_3 is the coefficient of the term λS of the value function given in equation (9).

Proof. The lemma follows from the first order condition of the pricing decision p_g as given in Proposition (1).

$$\frac{\partial p_g}{\partial S} = \frac{2\beta_b(\eta - A_3\phi\beta_g) + (\gamma_b + \gamma_g)(A_3\phi\gamma_g - \delta)}{4\beta_b\beta_g - (\gamma_b + \gamma_g)^2} > 0, \quad \frac{\partial p_g}{\partial \lambda} = \frac{2A_2\phi\gamma_g(\gamma_b + \gamma_g) - 4A_2\phi\beta_b\beta_g}{4\beta_b\beta_g - (\gamma_b + \gamma_g)^2} < 0$$

$$\frac{\partial p_g}{\partial \eta} = \frac{2S\beta_b}{4\beta_b\beta_g - (\gamma_b + \gamma_g)^2} > 0, \quad \frac{\partial p_g}{\partial \delta} = -\frac{S(\gamma_b + \gamma_g)}{4\beta_b\beta_g - (\gamma_b + \gamma_g)^2} < 0.$$

Algebraic manipulations of the above inequalities together with the assumption $\beta_i > \gamma_i$ for $i \in \{g, b\}$ yield the inequalities above.

From the Lemma (3), we conclude that the price of the green product is not necessarily always increasing in the greening level. This is apparently a counter-intuitive finding. The condition for a higher price premium for a higher greening level is a complex interrelationship among the price sensitivities, efficiency of demand learning ϕ, A_3 (the coefficient of λS in the profit (9)), η and δ. Learning helps in reducing the greening costs. The direct effect of this is a price reduction with a higher learning. Lastly, while the "green appeal", η, increases the green product price, the attraction of brown consumers, δ can reduce the same.

Lemma 4 *The price of the brown product is:*

(i) decreasing with the greening level $S(t)$ *if and only if* $\beta_g > \dfrac{\eta(\gamma_g+\gamma_b)}{2\delta - \phi A_3(\gamma_g-\gamma_b)}$
(ii) increasing in the learning $\lambda(t)$ *if and only if* $\gamma_g > \gamma_b$
(iii) increasing with η *and decreasing with* δ.

Proof. The proof of this lemma is similar to the proof of Lemma (3).

The price of a brown product decreases if the price sensitivity of the green productivity is greater than a certain threshold which depends on γ_g and γ_b. We can interpret γ_g as the increase in demand for the green product per unit increase in the brown product price and γ_b as the increase in demand for the brown product per unit increase in price of the green product. Surprisingly the equilibrium price of a brown product increases with the green preference level (η) of green consumers and decreases with a brown consumers' attraction (δ) towards green products.

The manufacturer's pricing strategies for brown and green products seems similar with respect to η and δ. In our model, though we consider a single manufacturer, there is a competitive environment between the green and brown products. Therefore, with an increase in greening level and subsequent green product price enhancement, the price of brown product rises because the manufacturer wants to reduce product differentiation by bridging the gap between the brown and green prices. Secondly, when brown consumers' attraction (δ) towards the green product is high, the approach for the manufacturer is more of retaining consumer loyalty by lowering brown product's price.

4.2 Comparative Analysis of Equilibrium Price and Demand

Typically, the green products are priced higher than the brown products [19,7]. However, under our model assumption of learning and budget constraints, we address the following questions:
(i) Is it possible to have a higher green demand?
(ii) Is there any situation in which a manufacturer may charge lower prices for the green product?
To answer the above questions we assume symmetry of the model parameters (*i.e.*, $\beta_g = \beta_b = \beta; \gamma_g = \gamma_b = \gamma$). Examining and simplifying the condition $D_g > D_b$, we obtain:

$$\begin{aligned} &\phi(\beta+\gamma)\left(2A_2\lambda + A_3S + A_5\right) - \alpha_b + \alpha_g + S(\delta+\eta) > 0 \\ \implies &\alpha_g > \alpha_b - \phi(\beta+\gamma)\left(2A_2\lambda + A_3S + A_5\right) - S(\delta+\eta). \end{aligned} \tag{19}$$

All the terms in the RHS of equation (19) are positive. Therefore, if the market potential for the green product is high enough, the demand for green product can be higher than that of the brown product at any point of time. However, in most industries brown consumption far surpasses the green consumption. We may conclude here that if the green market potential is higher, the green demand is higher.

Similarly for $p_b > p_g$, we have:

$$\phi(\beta+\gamma)\left(2A_2\lambda + A_3S + A_5\right) + \alpha_b - \alpha_g - S(\delta+\eta) > 0$$
$$\alpha_g < \alpha_b + \phi(\beta+\gamma)\left(2A_2\lambda + A_3S + A_5\right) + \alpha_b - \alpha_g + S(\delta+\eta). \quad (20)$$

From equations (19) and (20), we note that the marketing potential α_g and α_b have an important role to play in determining green product price and demand. The finding implies that the manufacturer should strategically focus on increasing the marketing potential which in turn will enhance the green demand (*e.g.* by advertising, eco labelling, promotions, etc.).

Lemma 5 *The manufacturer's greening investments are increasing in the greening level and learning.*

Proof. Recall that $I(t) = \dfrac{k_1\left(A_3\lambda + 2A_1S + A_4\right)}{\mu}$. Therefore, $\frac{\partial I}{\partial S} = \frac{k_1 2A_1}{\mu} > 0$ and $\frac{\partial I}{\partial \lambda} = \frac{k_1 A_3}{\mu} > 0$. The positivity follows from $A_i > 0$. Numerically, it was found that a feasible solution for the HJB equation requires $A_i > 0$.

While high green investment will enhance green quality, one may comprehend that green investments should decrease with learning. However, learning λ indirectly affects the investments in two opposite ways: *(i)* by decreasing greening costs through the expression $\frac{\mu}{2}I^2 - \sigma\lambda$ in equation (7) and *(ii)* by enhancing greening level and, consequently, raising greening through the term $k_2\lambda(t)$ in the state evolution equation (3). At optimality, the "incremental force" of learning influences the investments more significantly.

Using the method of equating the coefficients of two equivalent polynomials, we found that each coefficient has four solutions. We want to reiterate that the only feasible solution is the one where all the coefficients are positive. Clearly, the state variables, learning and greening level, as well as their multiplicative association have a positive effect on the value function.

5 Sensitivity Analysis

Our model has many parameters each of which influences the behaviour of the decisions and the state variables. We identify ϕ, σ, β_g, β_b,γ_g, γ_b, k_1 and k_2, as the most important parameters of the model which enables us to answer our salient research questions.

In the numerical analysis, the parameter values were fixed or their variation was restricted as follows: $\alpha_g = \alpha_b = 10, k_1 = k_2 = 1, \mu = 10, r = 0.05, \epsilon = 0.1, \omega = 0.1,\ \phi \in \{0.001, 0.005\}, \delta \in \{0.03, 0.04\}, \sigma \in (0, 3), \beta = \beta_g = \beta_b \in \{1.8, 2.2, 2.6\}, \lambda = \lambda_g = \lambda_b \in \{1.8, 2.2, 2.6\}$.

Efficiency of Demand Learning. The demand learning efficiency ϕ can be interpreted as the manufacturer's ability to capitalize economies of scale. A manufacturer may be a highly efficient learner due to, *e.g.*, technology innovation or the skills of its employees. On the other hand, a manufacturer can be a slow learner when it lacks such technology or skills. Succinctly, a higher ϕ represents a quick learner.

It is straightforward that learning and greening level will increase with ϕ. As per our model, learning has a positive impact on greening level. Therefore, the greening level also increases with the learning efficiency as shown in Figure (1). Less direct, it is the impact of ϕ in the optimal decisions. Recall that the optimal greening investment is $\frac{k_1(A_3\lambda+2A_1S+A_4)}{\mu}$. Therefore, the investment increases with λ and S given that A_3 and A_1 are positive. Since learning λ and greening level S increase with ϕ, then, by a transitive relationship, the investment also increases with ϕ; see Figure 3 for an illustration. A counter intuitive result is that the investment and the price of green product is increasing with the efficiency ϕ. One may apprehend that an efficient learning may decrease price while keeping the greening level high. This, in turn, increases the greening level, the learning and the corresponding green product price. A high greening level ensures more green demand. In this particular case, the attractiveness of a high greening level outplays the negative effect of the higher price on demand. On the other hand, the price of the brown counterpart decreases, although the effect of ϕ is much less marked for this decision variable. As the learning level increases, the firm makes more profit by enhancing the price difference rather than by bridging the gap. These intricate relations are illustrated in Figures 2 and 4.

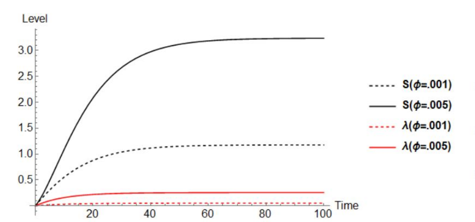

Fig. 1: Variation of Green Level and Learning with ϕ

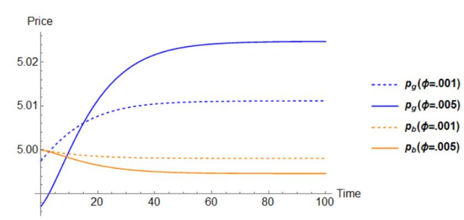

Fig. 2: Price Variation with ϕ

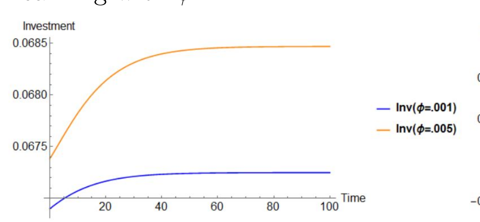

Fig. 3: Variation of Investment with ϕ

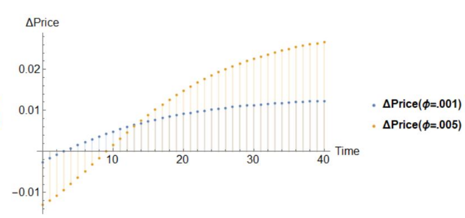

Fig. 4: Price Difference with ϕ

Green Level Appeal for Brown Consumers. The parameter δ in our model is the brown consumer's attraction towards green products. In Figure 5, we observe that the greening level and learning both drop as this attraction increases[2]. If the brown consumers are considerably attracted by the green quality, the manufacturer looses motivation to increase the greening level. The greening investment therefore also decreases as δ becomes higher (Figure 7). A higher value of δ also means more brown consumers are opting against brown products and increasing the demand for green products. This results in a price drop for the green product. The price of the brown product also decreases as a reaction to drop in green product price (Figure 6). A higher efficiency of demand learning results in a higher price difference between green and brown products with $p_g(t) > p_b(t)$ for most of the life cycle of the product (Figure 8).

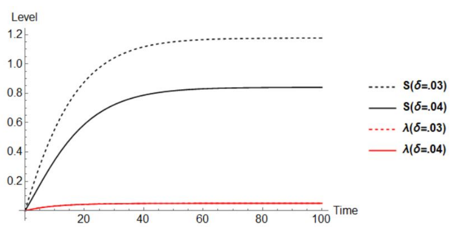

Fig. 5: Variation of Green Level and Learning with δ

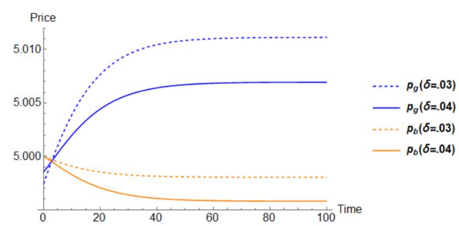

Fig. 6: Price Variation with δ

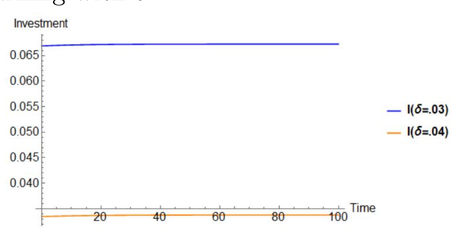

Fig. 7: Variation of Investment with δ

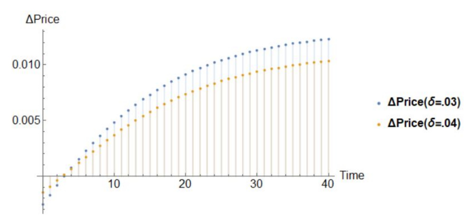

Fig. 8: Price Difference with δ

The Cost Efficiency of Learning. The parameter σ, the cost efficiency of learning, represents how responsive a company's cost savings are to the learning level λ. In Figure 9, we observe that the greening level slowly increases with σ. This is not surprising, because at a given point of time, a manufacturer would achieve higher greening level if costs are lesser (the other parameters are fixed). A high cost efficiency can lead to a lower cost of green product than the brown product and the investments in greening level increase with the cost efficiency

[2] Due to the scale difference of greening level and learning it is not obvious from the figure that the learning curve for $\delta = .04$ is below the learning curve for $\delta = .03$. However, this is true.

(Figure 11). Obviously, a higher σ implies a lower cost of greening. When the cost efficiency increases, the greening investments increase. We posit that a higher cost efficiency (higher σ) has an overall positive impact on the manufacturer's profit. Therefore, the manufacturer can afford and is motivated to make higher greening investments when σ is high. The importance of the parameter is strategic. A manufacturer may strategically save more costs given a certain learning level. Judicious resource allocation, optimization of operational techniques and so on are examples of such strategies. [3]

While in modern business in most industries we usually see that the prices of green products are higher, we show theoretically it is possible to reduce green product price below the brown price when cost efficiency of learning is high enough (Figure 10). The significance of this result is that manufacturers can adopt strategies to increase cost efficiency of learning. This will help them in reducing the price of green products and, in the long term, the green and brown products can have similar prices resulting in increasing demand for green products and, consequently, environmental benefits.

Price Sensitivity and Decisions. The price sensitivity, here assumed $\beta = \beta_g = \beta_b$, has a vital role in pricing decisions. From Figures 13, 14, 15 and 16, we observe that higher price sensitivities of green and brown products *(i)* reduce greening level and learning, *(ii)* reduce price of both green and brown products, *(iii)* reduce the price difference between green and brown products and *(iv)* reduce the optimal greening investments. From the Figures 2, 6, 10 and 14, we can see a trend for all optimal decisions. For the manufacturers offering both green and brown products from the start of the planning horizon, it is the equilibrium policy to start with a lower price for the green product and then increase the green product price, while simultaneously decreasing the brown product price. In general, the green investments should also increase with time. Finally, in what concerns the profits, from equation (13), we see that all the state variables positively influence the profit function since the coefficients of the states are positive. We also see from the above Figures 1, 5, 9 and 13 that the state variables are increasing with time. Therefore, the profit function will also increase with time.

6 Conclusion

The urge to benefit the environment by enhancing green demand while operating under a strict budget often places a manufacturer in a strategic dilemma. A number of decisions encompassing quantity of brown and green products to be produced, the optimal product prices and the optimal greening investments, have been widely explored in the extant literature, but mostly under a static setting or

[3] In regards to the observations, Mathematica software was not able to solve the value function without exact parametric values like $\phi = .001$ and $\delta = .03$ However, for σ, we were able to obtain parametric solutions of the value function coefficients in terms of σ. Consequently, we are able to depict the variation of $S(t), \lambda(t), p_g(t), p_b(t), I(t)$ for the the entire range ($[0, 10]$ in this case) of values of σ.

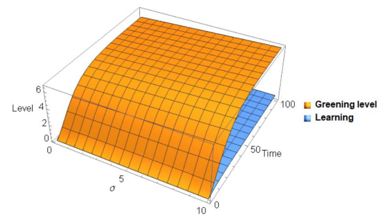

Fig. 9: Green Level and Learning with σ

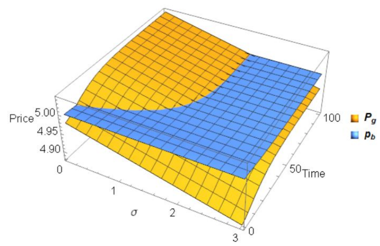

Fig. 10: Price variation with σ

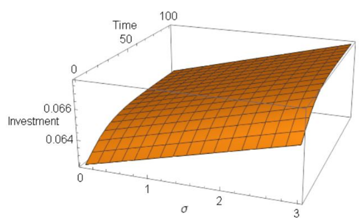

Fig. 11: Investments vs σ

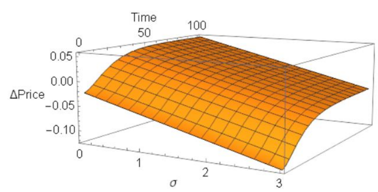

Fig. 12: Price difference with σ

not accounting simultaneously for all thosw elements. In our study, we explored the optimal pricing and investment decisions of a firm, selling both green and brown products. We contributed incrementally by considering dynamic decisions. We have also considered demand learning, investment budget and cost efficiency parameters to be some of the determinants of the optimal decisions. These give us several insights, both surprising and confirming empirical evidence.

Shortly, the most fundamental findings of our study are the following:

- Under the assumption that green and brown products have the same market potential ($\alpha_g = \alpha_b$), the optimal pricing policy of a firm is to enter the market with a lower price of for the green product and then increase the same over time. At steady state, the green prices are higher than the brown product prices. Market potentials, α_g and α_b, of unequal values can change our findings.

– A fast learning firm can profit more by charging a higher price premium for a greener "product" rather than bridging the gap between the brown and green product price.
– If the cost efficiency of learning σ is very high, theoretically, the firm can afford to have a lower green product price than the brown product's price. From a managerial perspective, this highlights the importance of firms investing in technology or strategies to reduce greening costs in order to ensure the successful stay of such products in the market.

Future research can extend our work by introducing multiple players of a supply chain, considering competition in a dynamic setting and by incorporating cooperative strategies such as contracts establishing the sharing of greening costs. Moreover, an analysis of consumer surplus or a possible government's intervention in our model will be of utmost interest for the cases where market potential α_g is not sufficiently high.

References

1. Benveniste, A.: Average Americans Can't Afford to Buy Green (2019), https://www.bloomberg.com/news/articles/2019-03-07/it-s-not-cheap-being-a-green-consumer
2. Berger, J.: Signaling can increase consumers' willingness to pay for green products. theoretical model and experimental evidence. Journal of consumer behaviour **18**(3), 233–246 (2019)

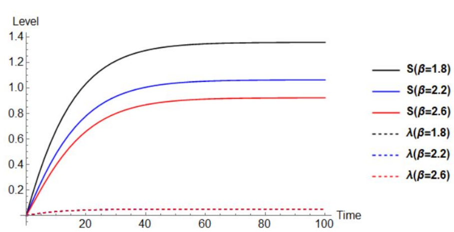

Fig. 13: Variation of Green Level and Learning with β

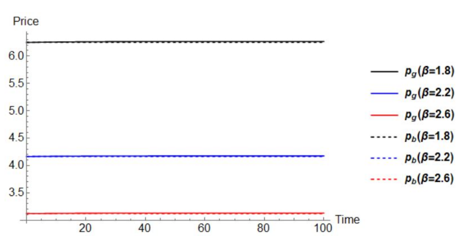

Fig. 14: Price Variation with β

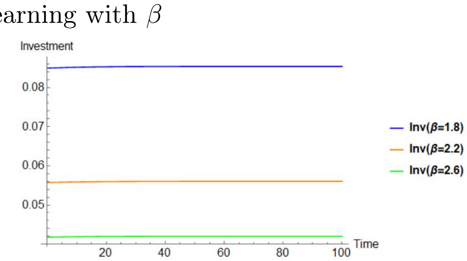

Fig. 15: Green Investments with β

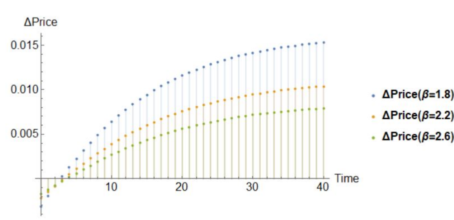

Fig. 16: Price Difference with β

3. Brécard, D.: Consumer misperception of eco-labels, green market structure and welfare. Journal of Regulatory Economics **51**(3), 340–364 (2017)
4. Chen, C.: Design for the environment: A quality-based model for green product development. Management Science **47**(2), 250–263 (2001)
5. Chen, Y.J., Sheu, J.B.: Environmental-regulation pricing strategies for green supply chain management. Transportation Research Part E: Logistics and Transportation Review **45**(5), 667–677 (2009)
6. Cohen, M.A., Cui, S., Gao, F.: The effect of government support on green product design and environmental impact. Available at SSRN 3291017 (2019)
7. Ding, J., Wang, W.: Information sharing in a green supply chain with promotional effort. Kybernetes (2020)
8. Ghazilla, R.A.R., Sakundarini, N., Abdul-Rashid, S.H., Ayub, N.S., Olugu, E.U., Musa, S.N.: Drivers and barriers analysis for green manufacturing practices in malaysian smes: a preliminary findings. Procedia Cirp **26**(1), 658–663 (2015)
9. Ghosh, D., Shah, J.: Supply chain analysis under green sensitive consumer demand and cost sharing contract. International Journal of Production Economics **164**, 319–329 (2015)
10. Ghosh, D., Shah, J., Swami, S.: Product greening and pricing strategies of firms under green sensitive consumer demand and environmental regulations. Annals of Operations Research pp. 1–30 (2018)
11. Gonzalez, E.D., Sarkis, J., Huisingh, D., Huatuco, L.H., Maculan, N., Montoya-Torres, J.R., de Almeida, C.M.: Making real progress toward more sustainable societies using decision support models and tools: introduction to the special volume. Journal of Cleaner Production **105**, 1–13 (2015)
12. Li, B., Zhu, M., Jiang, Y., Li, Z.: Pricing policies of a competitive dual-channel green supply chain. Journal of Cleaner Production **112**, 2029–2042 (2016)
13. Li, X., Li, Y.: Chain-to-chain competition on product sustainability. Journal of Cleaner Production **112**, 2058–2065 (2016)
14. Montoya-Torres, J.R., Gutierrez-Franco, E., Blanco, E.E.: Conceptual framework for measuring carbon footprint in supply chains. Production Planning & Control **26**(4), 265–279 (2015)
15. Neto, J.Q.F., Bloemhof-Ruwaard, J.M., van Nunen, J.A., van Heck, E.: Designing and evaluating sustainable logistics networks. International journal of production economics **111**(2), 195–208 (2008)
16. Peng, H.: Optimal subsidy policy for accelerating the diffusion of green products. Journal of Industrial Engineering and Management (JIEM) **6**(2), 626–641 (2013)
17. solarcity: Solar City Inside Energy. (2013). Consumer trends in sustainability: Insights to grow your market share and defend your brand. (2013), https://www.solarcity.com/sites/default/files/reports/reports-consumer-trends-in-sustainability.pdf
18. Walley, N., Whitehead, B.: It's not easy being green. Harvard Business Review **72**(3), 46–52 (1994)
19. Yenipazarli, A., Vakharia, A.: Pricing, market coverage and capacity: Can green and brown products co-exist? European Journal of Operational Research **242**(1), 304–315 (2015)
20. Yenipazarli, A., Vakharia, A.J.: Green, greener or brown: choosing the right color of the product. Annals of Operations Research **250**(2), 537–567 (2017)
21. Zhang, Q., Zhao, Q., Zhao, X.: Manufacturer's product choice in the presence of environment-conscious consumers: brown product or green product. International Journal of Production Research **57**(23), 7423–7438 (2019)

22. Zhang, Q., Zhao, Q., Zhao, X., Tang, L.: On the introduction of green product to a market with environmentally conscious consumers. Computers & Industrial Engineering **139**, 106190 (2020)
23. Zhou, Y.: The role of green customers under competition: A mixed blessing? Journal of Cleaner Production **170**, 857–866 (2018)
24. Zu, Y., Chen, L., Fan, Y.: Research on low-carbon strategies in supply chain with environmental regulations based on differential game. Journal of cleaner production **177**, 527–546 (2018)