Self-stabilizing Uniform Reliable Broadcast (preliminary version)

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Abstract

We study a well-known communication abstraction called *Uniform Reliable Broadcast* (URB). URB is central in the design and implementation of fault-tolerant distributed systems, as many non-trivial fault-tolerant distributed applications require communication with provable guarantees on message deliveries. Our study focuses on fault-tolerant implementations for time-free message-passing systems that are prone to node-failures. Moreover, we aim at the design of an even more robust communication abstraction. We do so through the lenses of *self-stabilization*—a very strong notion of fault-tolerance. In addition to node and communication failures, self-stabilizing algorithms can recover after the occurrence of *arbitrary transient faults*; these faults represent any violation of the assumptions according to which the system was designed to operate (as long as the algorithm code stays intact).

This work proposes the first self-stabilizing URB solution for time-free message-passing systems that are prone to node-failures. The proposed algorithm has an $\mathcal{O}(\text{bufferUnitSize})$ stabilization time (in terms of asynchronous cycles) from arbitrary transient faults, where bufferUnitSize is a predefined constant that can be set according to the available memory. Moreover, the communication costs of our algorithm are similar to the ones of the non-self-stabilizing state-ofthe-art. The main differences are that our proposal considers repeated gossiping of $\mathcal{O}(1)$ bits messages and deals with bounded space (which is a prerequisite for self-stabilization). Specifically, each node needs to store up to bufferUnitSize $\cdot n$ records and each record is of size $\mathcal{O}(\nu + n \log n)$ bits, where n is the number of nodes in the system and ν is the number of bits needed to encode a single URB instance.

1 Introduction

We propose a self-stabilizing implementation of a communication abstraction called *Uniform Reliable Broadcast* (URB) for time-free message-passing systems whose nodes may fail-stop.

Context and Motivation. Fault-tolerant distributed systems are known to be hard to design and verify. Such complex challenges can be facilitated by high-level communication primitives. These high-level primitives can be based on low-level ones, such as the one that allows nodes to send a message to only one other node at a time. When an algorithm wishes to broadcast message m to all nodes, it can send m individually to every other node. Note that if the sender fails during this

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broadcast, it can be the case that only some of the nodes have received *m*. Even in the presence of network-level support for broadcasting or multicasting, failures can cause similar inconsistencies. To the end of simplifying the design of fault-tolerant distributed algorithms, such inconsistencies need to be avoided.

The literature has a large number of examples that show how fault-tolerant broadcasts can significantly simplify the development of fault-tolerant distributed systems via State Machine Replication [17, 21], Atomic Commitment [19], Virtual Synchrony [6] and Set-Constrained Delivery Broadcast [16], to name a few. The weakest variance, named *Reliable Broadcast* (RB), lets all non-failing nodes agree on the set of delivered messages. Stronger RB variants specify additional requirements on the delivery order. Such requirements can simplify the design of fault-tolerant distributed consensus, which allows reaching, despite failures, a common decision based on distributed inputs. Consensus algorithms and RB are closely related problems [15, 20], which have been studied for more than three decades.

Task description. Uniform Reliable Broadcast (URB) is a variance of the reliable broadcast problem, which requires that if a node delivers a message, then all non-failing nodes also deliver this message [15]. The task specifications consider an operation for URB broadcasting of message m and an event of URB delivery of message m. The requirements include URB-validity, *i.e.*, there is no spontaneous creation or alteration of URB messages, URB-integrity, *i.e.*, there is no duplication of URB messages, as well as URB-termination, *i.e.*, if the broadcasting node is non-faulty, or if at least one receiver URB-delivers a message, then all non-failing nodes URB-deliver that message. Note that the URB-termination property considers both faulty and non-faulty receivers. This is the reason why this type of reliable broadcast is named *uniform*. This work considers a URB implementation that is *quiescent* in the sense that every URB operation incurs a finite number of messages. Moreover, our implementation uses a bounded amount of local memory.

Fault Model. We consider a time-free (a.k.a asynchronous) message-passing system that has no guarantees on the communication delay. Moreover, there is no notion of global (or universal) clocks and we do not assume that the algorithm can explicitly access the local clock (or timeout mechanisms). Our fault model includes (i) detectable fail-stop failures of nodes, and (ii) communication failures, such as packet omission, duplication, and reordering. In addition to the failures captured in our model, we also aim to recover from *arbitrary transient faults, i.e.*, any temporary violation of assumptions according to which the system and network were designed to operate, *e.g.*, the corruption of control variables, such as the program counter, packet payload, and operation indices, which are responsible for the correct operation of the studied system, or operational assumptions, such as that the network cannot be partitioned for long periods. Since the occurrence of these failures can be arbitrarily combined, we assume that these transient faults can alter the system state in unpredictable ways. In particular, when modeling the system, we assume that these violations bring the system to an arbitrary state from which a *self-stabilizing algorithm* should recover the system.

Related Work. The studied problem can be traced back to Hadzilacos and Toueg [15] who consider asynchronous message-passing, where nodes may fail. They solved several variants to the studied problem with respect to the delivery order, *e.g.*, FIFO (first in, first out), CO (causal order), and TO (total order). They also showed that TO-URB and consensus have the same computability power in the context above. Here we focus only on the basic version of URB. To the end of satisfying the quiescent property, we consider a more advanced model, see the remark in [20, Section 4.2.1]. For a detailed presentation of existing non-self-stabilizing URB solutions and their applications, we

refer the reader to [1, 20]. (Due to the page limit, Section 3 of the Appendix brings some of these details.) We follow the design criteria of self-stabilization, which was proposed by Dijkstra [10] and detailed in [4, 11]. Delaët *et al.* [9] present a self-stabilizing algorithm for the propagation of information with feedback (PIF) that can be the basis for implementing a self-stabilizing URB. However, Delaët *et al.* do not consider node failures [9, Section 6]. To the best of our knowledge, there is no self-stabilizing algorithm that solves the studied problem for the studied fault-model.

Contributions. We present an important module for dependable distributed systems: a selfstabilizing algorithm for Uniform Reliable Broadcast (URB) for time-free message-passing systems that are prone to node failures. To the best of our knowledge, we are the first to provide a broad fault model that includes detectable fail-stop failures, communication failures, such as packet omission, duplication, and reordering as well as arbitrary transient faults. The latter can model any violation of the assumptions according to which the system was designed to operate (as long as the algorithm code stays intact).

The stabilization time of the proposed solution is in $\mathcal{O}(\text{bufferUnitSize})$ (in terms of asynchronous cycles), where bufferUnitSize is a predefined constant that can be set according to the available local memory. Our solution uses only a bounded amount of space, which is a prerequisite for self-stabilization. Specifically, each node needs to store up to bufferUnitSize $\cdot n$ records and each record is of size $\mathcal{O}(\nu + n \log n)$ bits, where n is the number of nodes in the system and ν is the number of bits needed to encode a single URB instance. Moreover, the communication costs of our algorithm are similar to the ones of the non-self-stabilizing state-of-the-art. The main difference is that our proposal considers repeated gossiping of $\mathcal{O}(1)$ bits messages.

Organization. We state our system settings in Section 2. Section 3 includes a brief overview of some of the earlier ideas that have led to the proposed solution. Our self-stabilizing algorithm is proposed in Section 4; it considers unbounded counters. The correctness proof appears in Section 5. We explain how to bound the counters of the proposed self-stabilizing algorithm in Section 7. We conclude in Section 8.

2 System settings

We consider a time-free message-passing system that has no guarantees on the communication delay. Moreover, there is no notion of global (or universal) clocks and the algorithm cannot explicitly access the local clock (or timeout mechanisms). The system consists of a set, \mathcal{P} , of *n* crash-prone nodes (or processors) with unique identifiers. Any pair of nodes $p_i, p_j \in \mathcal{P}$ have access to a bidirectional communication channel, *channel*_{j,i}, that, at any time, has at most **channelCapacity** $\in \mathbb{N}$ packets on transit from p_j to p_i (this assumption is due to a well-known impossibility [11, Chapter 3.2]).

Our analysis considers the *interleaving model* [11], in which the node's program is a sequence of *(atomic) steps.* Each step starts with an internal computation and finishes with a single communication operation, *i.e.*, a message send or receive. The state, s_i , of node $p_i \in \mathcal{P}$ includes all of p_i 's variables and *channel*_{j,i}. The term system state (or configuration) refers to the tuple $c = (s_1, s_2, \dots, s_n)$. We define an execution (or run) $R = c[0], a[0], c[1], a[1], \dots$ as an alternating sequence of system states c[x] and steps a[x], such that each c[x + 1], except for the starting one, c[0], is obtained from c[x] by a[x]'s execution.

2.1 Task specifications

The set of *legal executions* (*LE*) refers to all the executions in which the requirements of the task T hold. In this work, T_{URB} denotes the task of Uniform Reliable Broadcast (URB) and LE_{URB} denotes the set of executions in which the system fulfills T_{URB} 's requirements, which Definition 2.1 specifies. Definition 2.1 considers the operation, urbBroadcast(m), and the event urbDeliver(m). When processor $p_i \in \mathcal{P}$ URB-broadcasts message m, it does so by calling urbBroadcast(m). The specifications assume that every broadcasted message is unique, say, by associating a message identity, *i.e.*, the pair (*sender identifier*, *sequence number*), where the sequence number is an (integer) index that is locally generated by the sender.

Definition 2.1 (Uniform Reliable Broadcast [20]) Let R be a system execution. We say that the system demonstrates in R a construction of the URB communication abstraction if the validity, integrity, and termination requirements are satisfied.

- Validity. Suppose that p_i URB-delivers message m in step $a_i \in R$ with p_j as a sender. There is a step $a_j \in R$ that appears in R before a_i in which p_j URB-broadcasts m.
- Integrity. R includes at most one step in which processor p_i URB-delivers message m.
- Termination. Suppose that a non-faulty p_i takes a step in R that URB-broadcasts or URB-delivers message m. Each non-faulty $p_i \in \mathcal{P}$ URB-delivers m during R.

The URB implementation considered in this paper also satisfies the quiescent property (in a self-stabilizing manner). Our implementation uses MSG and MSGack messages for conveying information added to the system via urbBroadcast operations. We say that execution R satisfies the quiescent property if every URB-broadcast message that was USB-delivered incurs a finite number of MSG and MSGack messages. We note that the quiescent property does not consider all the messages that the proposed solution uses. Specifically, we use GOSSIP messages of constant size that the algorithm sends repeatedly. We note that self-stabilizing systems can never stop sending messages, because if they did, it would not be possible for the system to recover from transient faults [11, Chapter 2.3].

2.2 The Fault Model and Self-stabilization

We model a failure occurrence as a step that the environment takes rather than the algorithm.

Benign failures. When the occurrence of a failure cannot cause the system execution to lose legality, *i.e.*, to leave *LE*, we refer to that failure as a benign one (Figure 1).

Node failure. We consider *fail-stop failures*, in which nodes stop taking steps. We assume that there is a way to detect these failures, say, using unreliable failure detectors [8].

Communication failures and fairness. We consider solutions that are oriented towards time-free message-passing systems and thus they are oblivious to the time in which the packets arrive and depart. We assume that the communication channels are prone to packet failures, such as omission, duplication, reordering. However, we assume that if p_i sends a message infinitely often to p_j , node p_j receives that message infinitely often. We refer to the latter as the *fair communication* assumption. For example, the proposed algorithm sends infinitely often GOSSIP messages from any processor to

	Frequency	
Duration	Rare	Not rare
	Any violation of the assumptions according to	Packet failures: omissions,
Transient	which the system operates (but the code stays	duplications, reordering
	intact). This can result in any state corruption.	(assuming fair communications).
Permanent	Detectable fail-stop failures.	

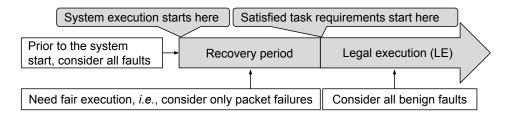


Figure 1: The table above details our fault model and the chart illustrates when each fault set is relevant. The chart's gray shapes represent the system execution, and the white boxes specify the failures considered to be possible at different execution parts and recovery guarantees of the proposed self-stabilizing algorithm. The set of benign faults includes both packet failures and fail-stop failures.

any other. Despite the possible loss of messages, the communication fairness assumption implies that every processor receives infinitely often GOSSIP messages from any non-failing processor.

Arbitrary transient faults. We consider any violation of the assumptions according to which the system was designed to operate. We refer to these violations and deviations as *arbitrary transient faults* and assume that they can corrupt the system state arbitrarily (while keeping the program code intact). The occurrence of an arbitrary transient fault is rare. Thus, our model assumes that the last arbitrary transient fault occurs before the system execution starts [11]. Moreover, it leaves the system to start in an arbitrary state.

Dijkstra's self-stabilization criterion An algorithm is *self-stabilizing* with respect to the task of *LE*, when every (unbounded) execution *R* of the algorithm reaches within a finite period a suffix $R_{legal} \in LE$ that is legal. That is, Dijkstra [10] requires that $\forall R : \exists R' : R = R' \circ R_{legal} \land R_{legal} \in LE \land |R'| \in \mathbb{N}$, where the operator \circ denotes that $R = R' \circ R''$ concatenates R' with R''.

2.3 Complexity Measures

The main complexity measure of self-stabilizing algorithms, called *stabilization time*, is the time it takes the system to recover after the occurrence of the last transient fault.

Message round-trips and iterations of self-stabilizing algorithms. The correctness proof depends on the nodes' ability to exchange messages during the periods of recovery from transient faults. The proposed solution considers communications that follow the pattern of request-reply, *i.e.*, MSG and MSGack messages, as well as GOSSIP messages for which the algorithm does not send replies. The definitions of our complexity measures use the notion of a message round-trip for the cases of request-reply messages and the term algorithm iteration.

We give a detailed definition of *round-trips* as follows. Let $p_i \in \mathcal{P}$ and $p_j \in \mathcal{P} \setminus \{p_i\}$. Suppose that immediately after system state c, node p_i sends a message m to p_j , for which p_i awaits a reply.

At system state c', that follows c, node p_j receives message m and sends a reply message r_m to p_i . Then, at system state c'', that follows c', node p_i receives p_j 's response, r_m . In this case, we say that p_i has completed with p_j a round-trip of message m.

It is well-known that self-stabilizing algorithms cannot terminate their execution and stop sending messages [11, Chapter 2.3]. Moreover, their code includes a do forever loop. Thus, we define a *complete iteration* of a self-stabilizing algorithm. Let N_i be the set of nodes with whom p_i completes a message round trip infinitely often in execution R. Moreover, assume that node p_i sends a gossip message infinitely often to $p_j \in \mathcal{P} \setminus \{p_i\}$ (regardless of the message payload). Suppose that immediately after the state c_{begin} , node p_i takes a step that includes the execution of the first line of the do forever loop, and immediately after system state c_{end} , it holds that: (i) p_i has completed the iteration it has started immediately after c_{begin} (regardless of whether it enters branches), (ii) every request-reply message m that p_i has sent to any node $p_j \in \mathcal{P}$ during the iteration (that has started immediately after c_{begin}) has completed its round trip, and (iii) it includes the arrival of at least one gossip message from p_i to any non-failing $p_j \in \mathcal{P} \setminus \{p_i\}$. In this case, we say that p_i 's complete iteration (with round-trips) starts at c_{begin} and ends at c_{end} .

Cost measures: asynchronous cycles and the happened-before relation. We say that a system execution is *fair* when every step that is applicable infinitely often is executed infinitely often and fair communication is kept. Since asynchronous systems do not consider the notion of time, we use the term (asynchronous) cycles as an alternative way to measure the period between two system states in a fair execution. The first (asynchronous) cycle (with round-trips) of a fair execution $R = R' \circ R''$ is the shortest prefix R' of R, such that each non-failing node executes at least one complete iteration in R'. The second cycle in execution R is the first cycle in execution R'', and so on.

Remark 2.1 For the sake of simple presentation of the correctness proof, when considering fair executions, we assume that any message that arrives in R without being transmitted in R does so within $\mathcal{O}(1)$ asynchronous cycles in R.

Remark 2.2 (Absence of transient faults implies no need for fairness assumptions) In the absence of transient faults, no fairness assumptions are required in any practical settings. Also, the existing non-self-stabilizing solutions (Section 3) do not make any fairness assumption, but they do not consider recovery from arbitrary transient fault regardless of whether the execution eventually becomes fair or not.

Lamport [18] defined the happened-before relation as the least strict partial order on events for which: (i) If steps $a, b \in R$ are taken by processor $p_i \in \mathcal{P}$, $a \to b$ if a appears in R before b. (ii) If step a includes sending a message m that step b receives, then $a \to b$. Using the happened-before definition, one can create a directed acyclic (possibly infinite) graph $G_R : (V_R, E_R)$, where the set of nodes, V_R , represents the set of system states in R. Moreover, the set of edges, E_R , is given by the happened-before relation. In this paper, we assume that the weight of an edge that is due to cases (i) and (ii) are zero and one, respectively. When there is no guarantee that execution R is fair, we consider the weight of the heaviest directed path between two system state $c, c' \in R$ as the cost measure between c and c'.

2.4 External building-blocks: self-stabilizing unreliable failure detectors

The concepts of failure patterns and failure detectors have been introduced in [8]. The failure detector Θ was introduced in [3], and the failure detector *HB* (heartbeat) has been introduced in [2]. A pedagogical presentation of these failure detectors is given in [20].

Any execution R := (c[0], a[0], c[1], a[1], ...) can have any number of failures during its run. R's failure pattern is a function $F : \mathbb{Z}^+ \to 2^{\mathcal{P}}$, where \mathbb{Z}^+ refers to an index of a system state in R, which in some sense represents (progress over) time, and $2^{\mathcal{P}}$ is the power-set of \mathcal{P} , which represent the set of failing nodes in a given system state. $F(\tau)$ denotes the set of failing nodes in system state $c_{\tau} \in R$. Since we consider fail-stop failures, $F(\tau) \subseteq F(\tau + 1)$ holds for any $\tau \in \mathbb{Z}^+$. Denote by $Faulty(F) \subseteq \mathcal{P}$ the set of nodes that eventually fail-stop in the (unbounded) execution R, which has the failure pattern F. Moreover, $Correct(F) = \mathcal{P} \setminus Faulty(F)$.

We assume the availability of self-stabilizing Θ failure detectors [2], which offer local access to trusted, which is a set that satisfies the Θ -accuracy and Θ -liveness properties. Let $trusted_i^{\tau}$ denote p_i 's value of trusted at time τ . Θ -accuracy is specified as $\forall p_i \in \mathcal{P} : \forall \tau \in \mathbb{Z}^+ : (trusted_i^{\tau} \cap Correct(F)) \neq \emptyset$, *i.e.*, at any time, $trusted_i$ includes at least one non-faulty node, which may change over time. Θ -liveness is specified as $\exists \tau \in \mathbb{N} : \forall \tau' \geq \tau : \forall p_i \in Correct(F) : trusted_i^{\tau'} \subseteq Correct(F)$, *i.e.*, eventually trusted_i includes only non-faulty nodes. A self-stabilizing Θ -failure detector appears in [7].

We also assume the availability of a class HB (heartbeat) self-stabilizing failure detector [2], which has the HB-completeness and HB-liveness properties. Let $HB_i^{\tau}[j]$ be p_i 's value of the jth entry in the array HB at time τ . HB-completeness is specified as $\forall p_i \in Correct(F), \forall p_j \in$ $Faulty(F) : \exists K : \forall \tau \in \mathbb{N} : HB_i^{\tau}[j] < K$, *i.e.*, any faulty node is eventually suspected by every non-failing node. HB-liveness is specified as $(1) \forall p_i, p_j \in \mathcal{P} : \forall \tau \in \mathbb{N} : HB_i^{\tau}[j] \leq HB_i^{\tau+1}[j]$, and (2) $\forall p_i, p_j \in Correct(F) : \forall K : \exists \tau \in \mathbb{Z}^+ : HB_i^{\tau}[j] > K$. In other words, there is a time after which only the faulty nodes are suspected. The implementation of the HB failure detector that appears in [1] and [20, Chapter 3.5] uses unbounded counters. A self-stabilizing variation of this mechanism can simply let $p_i \in \mathcal{P}$ to send HEARTBEAT $(HB_i[i], HB_i[j])$ messages to all $p_j \in \mathcal{P}$ periodically while incrementing the value of $HB_i[i]$. Once p_j receives a heartbeat message from p_i , it updates the *i*-th and the *j*-th entries in HB_j , *i.e.*, it takes the maximum of the locally stored and received entries. Moreover, once any entry reaches the value of the maximum integer, MAXINT, a global reset procedure is used (see Section 7).

Remark 2.3 For the sake of simple presentation of the correctness proof, during fair executions, we assume that $c_{\tau} \in R$ is reached within $\mathcal{O}(1)$ asynchronous cycles, such that $\forall_{p_i \in Correct(F)} : trusted_i^{\tau} \subseteq Correct(F)$ and for a given K, $\forall p_i, p_j \in Correct(F) : HB_i^{\tau}[j] > K$, where $\tau \in \mathbb{Z}^+$ is determined by the Θ - and HB-liveness properties.

3 Non-self-stabilizing URB with and without Failure Detectors

For the completeness' sake, we briefly review existing URB solutions. The following algorithms are from [2, 3]. We follow here their description as give in [20] by starting from the simplest model before considering more advanced ones.

In the absence of communication and node failures, one can implement the urbBroadcast(m) operation by running {foreach $p_j \in \mathcal{P}$ send $\mathsf{MSG}(m)$ to p_j } and calling urbDeliver(m) upon p_j 's reception of m. Algorithm 1 considers a model in which nodes can fail-stop without the possibility to detect it, but with reliable communications. Node p_i broadcasts message m by sending $\mathsf{MSG}(m)$

Algorithm 1: URB in the presence of reliable communications; code for $p_i \in \mathcal{P}$

- 1 operation urbBroadcast(m) do send MSG(m) to p_i ;
- 2 upon MSG(m) arrival from p_k begin
- **3 if** first reception of m **then**
- 4 | {foreach $p_j \in \mathcal{P} \setminus \{p_i, p_k\}$ do send MSG(m) to p_j ; urbDeliver(m);

to itself (line 1). Upon the message arrival (line 3), the receiver ignores the message if it got it before. This is possible due to the requirement of unique message identities (Definition 2.1). If it is the first reception, the receiver propagates MSG(m) to all other nodes (except itself and the sender) before calling urbDeliver(m) (lines 4 to 4).

Algorithm 2 considers a system in which at most t < n/2 nodes may crash without the possibility for detection as well as unreliable communications. Node p_i broadcasts message m by sending MSG(m) to itself (line 5) while assuming it has a reliable channel to itself). Upon the reception of MSG(m) for the first time (line 7), p_i creates the set $recBy[m] = \{i, k\}$ to contain the identities of nodes that receive MSG(m), before activating the Diffuse(m) task. In case this is not MSG(m)'s first arrival (line 8), p_i merely adds the sender identity, k, to recBy[m]. The task Diffuse(m) is responsible for transmitting (and retransmitting) MSG(m) to at least a majority of the nodes before URB-delivering m (lines 11 to 12).

Algorithm 2: URB in the presence of t < n/2 undetectable node failures; p_i 's code

- **5** operation urbBroadcast(m) do send MSG(m) to p_i ;
- 6 upon MSG(m) arrival from p_k begin
- 7 | if not the first reception of m then $recBy[m] \leftarrow recBy[m] \cup \{k\};$
- s | else allocate recBy[m]; $recBy[m] \leftarrow \{i, k\}$; activate Diffuse(m) task;

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9 do forever begin
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- 10 foreach active Diffuse(m) task do
- 11 foreach $p_j \in \mathcal{P} : j \notin recBy[m]$ do send MSG(j, seq) to p_j ;
- 12 | if $|recBy[m]| \ge t+1) \land (p_i \text{ has not yet } URB\text{-}delivered m)$ then urbDeliver(m);

Note that the task Diffuse(m) never stops transmitting messages. Using Θ failure detectors (Section 2.1), Algorithm 3 avoids such an infinite number of retransmissions by enriching Algorithm 2 as follows. (i) The URB-delivery condition, $trusted \subseteq recBy[m]$, of Algorithm 3's line 22 substitutes the condition, $|recBy[m]| \ge t + 1$), of Algorithm 2's line 12. (ii) Upon the reception of a $\mathsf{MSG}(m)$ message, p_i acknowledges the reception via a $\mathsf{MSGack}(m)$. Moreover, when p_i receives $\mathsf{MSGack}(m)$ from p_k , it marks the fact that p_k received m by adding k to recBy[m]. (iii) Node p_i can eventually avoid sending $\mathsf{MSG}(m)$ messages to a faulty processor p_j in the following manner. Processor p_i repeatedly transmits $\mathsf{MSG}(m)$ to p_j as long as p_j is trusted and $j \notin recBy[m]$ (line 21). Note that, eventually, either p_j will receive $\mathsf{MSG}(m)$ and acknowledge it to p_i , or in case p_j is faulty, $j \notin trusted_i$ due to the Θ -completeness property. Moreover, due to the strong Θ -accuracy, $j \notin trusted_i$ cannot hold before p_j fails (if it is faulty).

To the end of allowing the implementation of a quiescent URB solution and the unreliable failure detectors that it relies on, the underlying system needs to satisfy synchrony assumptions that can

Algorithm 3: Quiescent URB using Θ -failure detectors; code for $p_i \in \mathcal{P}$ 13 operation urbBroadcast(m) do send MSG(m) to p_i ; 14 upon MSG(m) arrival from p_k begin if not the first reception of m then $recBy[m] \leftarrow recBy[m] \cup \{k\};$ $\mathbf{15}$ else allocate recBy[m]; $recBy[m] \leftarrow \{i, k\}$; activate Diffuse(m) task; $\mathbf{16}$ send MSGack(m) to p_k ; 17 **18 upon** MSGack(m) **arrival from** p_k **do** { $recBy[m] \leftarrow recBy[m] \cup \{k\}$ } do forever begin 19 for each active Diffuse(m) task do 20 for each $j \in trusted \setminus recBy[m]$ do send MSG(m) to p_i ; 21 if $trusted \subseteq recBy[m] \land (p_i \text{ has not yet } URB\text{-}delivered m)$ then urbDeliver(m); 22

be captured by the combined use of the Θ - and *HB*-failure detectors [20, Chapter 3.5]. Algorithm 4 differs from Algorithm 3 only in the Diffuse(m) task (line 33). Specifically, p_i transmits MSG(m) to p_j only when $j \in recBy[m]$ (because from p_i 's perceptive, p_j has not yet received MSG(m)) and HB[j] has increased since the previous iteration (because from p_i 's perspective, p_j is not failing). Algorithm 4 is the basis for our proposal (Section 4).

Algorithm 4: Quiescent URB using Θ - and *HB*-failure detectors; code for $p_i \in \mathcal{P}$

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23 operation urbBroadcast(m) do send MSG(m) to p_i;
24 upon MSG(m) arrival from p_k begin
       if not the first reception of m then recBy[m] \leftarrow recBy[m] \cup \{k\};
\mathbf{25}
       else allocate recBy[m]; recBy[m] \leftarrow \{i, k\}; activate Diffuse(m, [-1, ..., -1]) task;
26
       send MSGack(m) to p_k;
27
   upon MSGack(m) arrival from p_k do {recBy[m] \leftarrow recBy[m] \cup \{k\}}
28
   do forever begin
29
        foreach active Diffuse(m, prevHB) task do
30
            let curHB := HB;
31
            foreach j \in trusted \setminus recBy[m] \land prevHB[m][j] < curHB[m][j] do
32
33
                send MSG(m) to p_i
            prevHB[m] \leftarrow curHB[m];
34
           if trusted \subseteq recBy[m] \land (p_i \text{ has not yet } URB\text{-}delivered m) then urbDeliver(m);
35
```

4 Unbounded Self-stabilizing Uniform Reliable Broadcast

Algorithm 5 allows $p_i \in \mathcal{P}$ to urbBroadcast message m in a way the guarantees that all non-failing nodes raise the event urbDeliver(m) according to the specifications (Section 2.1). The review in Section 3 can help the reader to understand the proposed solution. We note that the boxed code lines of Algorithm 5 are relevant only for an extension, which we discuss in Section 6.

Local variables and their purpose (lines 36 to 41). The task specifications assume that each processor $p_i \in \mathcal{P}$ can URB-broadcast unique messages. To that end, Algorithm 5 maintains

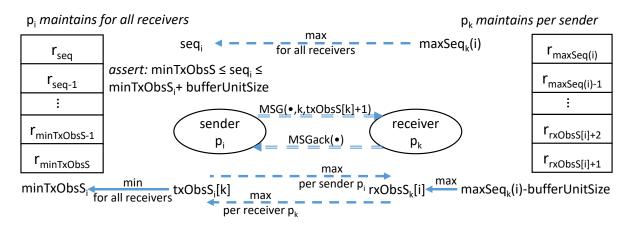


Figure 2: The self-stabilizing flow-control scheme between sender p_i and receiver p_k . The arrays on the figure sides represent the portion of peers' *buffer* variables that includes records r_s , where s is a sequence number of a message sent from p_i to p_k . The single-line arrows (dashed or not) and the text next to them represent a logical update, e.g., $x \xleftarrow{\max} y$ stands for $x \leftarrow \max\{x, y\}$. The text that appears below the arrow clarify whether a single variable aggregates these update or different entries in the array store the updated values. The dashed arrows refer to updates that require communication between p_i and p_k . The double-line arrows and the text above them depict MSG and MSGack messages.

the message index number, seq, that it increments upon urbBroadcast invocations.

The processors store all the currently processed messages as records in the variable *buffer*. Each record includes the following fields: (i) msq, which holds the URB message, (ii) id, which is the identifier of the node that invoked the URB-broadcast, (iii) seq, which is the message index number, (iv) *delivered*, which is a Boolean that holds False only when the message is pending delivery, (v) recBy, which is a set that includes the identifiers of nodes that have acknowledged msq, and (vi) prevHB, which is a value of the HB failure detector (Section 2.1) that Algorithm 5 uses for deciding when to transmit (and re-transmit) msg. Our proof shows that every node store at most $n \cdot \text{bufferUnitSize}$ records, where bufferUnitSize can be set according to the available local memory. When accessing records in *buffer*, we use a query-oriented notation, *e.g.*, $(\bullet, id = j, seq = s, \bullet) \in$ *buffer* considers all buffered records that their *id* and *seq* fields hold the values i and s, respectively. A self-stabilizing flow-control scheme for bounding *buffer*. Algorithm 5 bounds *buffer* using a flow-control technique. We say a record, with sequence number s, is obsolete if it had received acknowledgments from all trusted nodes and then it was URB-delivered. Moreover, since p_i needs to remove obsolete records from its buffer, we also define that any record with a sequence number lower than s to be also obsolete. This way, p_k can keep track of all the obsolete records it has deleted using a single counter $rxObsS_k[i]$, per sender p_i , which stores the highest sequence number of records that p_i considers to be obsolete. The array $txObsS_i$ facilitates the ability of sender (p_i) to control its sending flow since it can receive $rxObsS_k[i]$ from p_k and store it at $txObsS_i[k]$. (Note that we denote variable X's value at node p_i by X_i .) The flow-control mechanism can simply defer the processing a new URB-message when p_k 's message sequence number minus the minimum value stored in txObsS[] (that arrived from a node that p_k trusts) is smaller than the maximum number of records, **bufferUnitSize**, that a receiver can buffer for each node.

We use Figure 2 to describe in detail the flow-control scheme. The receiver p_k repeatedly sends to

the sender p_i the maximum p_i 's sequence number, $\max Seq_k(i)$, that it stores in its buffer, see the top dashed left arrow. This allows p_i to make sure that seq_i is greater than any sequence number in the system that is associated with p_i , as we show in Theorem 5.2's Argument (3). The buffer of p_k cannot store more than bufferUnitSize with messages from p_i . Therefore, p_k stores only messages that their sequence numbers are between $\max Seq_k(i)$ and $\max Seq_k(i) - \text{bufferUnitSize}$ and reports to p_i the highest sequence number, $rxObsS_k[i]$, of its obsolete records that are associated with p_i , see the lowest dashed arrow. The latter stores this value in $rxObsS_i[k]$ and makes sure it has the latest value from p_k by sharing $rxObsS_i[k]$ with it. The sender p_i also uses $rxObsS_i[k]$ for bounding $buffer_i$. Specifically, $\min T \times ObsS_i()$ aggregates the minimum value in $rxObsS_i[k]$ for any trusted receiver p_k (line 46). Using $\min T \times ObsS_i()$, the sender p_i can assert that $\min T \times ObsS_i() \le seq_i \le \min T \times ObsS_i()$ and seq_i (line 55). Since, due to a transient fault, p_i 's might indicate the reception of acknowledgment for a message that p_k 's state shows that it has never received, p_i repeatedly resends the message that has the sequence number s, such that $s = rxObsS_i[k] + 1$ (line63), see the double line arrows between p_i and p_k in Figure 2.

A detailed description of Algorithm 5. Upon the invocation of the urbBroadcast(m) operation, Algorithm 5 allows node p_i to process m without blocking as long as the flow-control mechanism can guarantee the available space at all trusted receivers (line 47). Such processing is done by creating a unique operation index, *seq*, and calling update().

The procedure update(m, j, s, k) receives a message, m, a unique message identifier, which is the pair (j, s) that includes the sender identifier (j) and the sequence number (s), and the identifier of the forwarding processor, p_k . The procedure considers first the case in which $buffer_i$ does not include a record with the identifier (j, s). In this case, p_i adds to $buffer_i$ the record $(m, j, s, False, \{j, k\}, [-1, ..., -1])$ (line 51), which stands for the message itself and its unique identifier, as well as stating that it was not yet been delivered but that the identifiers of the sending (j) and forwarding (k) processors appear in recBy. Moreover, the record holds a vector that is smaller than any value of the HB failure detector. For the case in which $buffer_i$ already includes a record with the identifier (j, s), p_i makes sure that recBy includes the identifiers of the sending and forwarding nodes (line 52).

Algorithm 5 includes a do forever loop (lines 53 to 65) that: (i) removes stale information (lines 54 to 58), (ii) processes URB messages (lines 59 to 64) and (iii) gossips information that is needed for flow-control and recovery from arbitrary transient faults (line 65).

(i) The removal of stale information includes the emptying the buffer whenever there are records for which the msg field is \perp or when there are two records with the same message identifier (line 54). Lines 55 to 57 implement recovery strategies that facilitate the bounds on the buffer size. Algorithm 5 tests for the case in which, due to an arbitrary transient fault, the sender does not store all of its messages such that their sequence number is between mS+1 and seq (line 55), where $mS := \min T \times ObsS()$ is the smallest obsolete sequence number that p_i had received from a trusted receiver. The recovery from such transient violations is done by allowing the sender to send bufferUnitSize URB messages without considering the space available on the receiver-side. Similarly, on the receiver-side, Algorithm 5 makes sure that the gap between the largest obsolete record, rxObsS[k] (of p_k 's messages) and the largest buffered sequence number, $\max Seq(k)$, is not larger than bufferUnitSize (line 56). Algorithm 5 updates the receiver-side counter that stores the highest obsolete message number per sender (line 57). To the end of bounding the memory use, p_i keeps in $buffer_i$ messages that it has sent but for which it has not yet received an indication from all trusted

Algorithm 5: Self-stabilizing quiescent uniform reliable broadcast; code for $p_i \in \mathcal{P}$ 36 global constants: bufferUnitSize; /* max records per node in *buffer* */ 37 local variables: (Initialization is optional in the context of self-stabilization.) **38** seq := 0;/* message index num. */ /* set of (msg, id, seq, delivered, recBy, prevHB) records */ **39** buffer := \emptyset ; 40 rxObsS[1..n] := [0, ..., 0];/* highest reciver's obsolete seq per node */ txObsS[1..n] := [0, ..., 0];/* highest sender's obsolete seq per node */ 41 $next[1..n] := [1, \ldots, 1]$; 42 /* next-to-deliver message indices; one entry per sender */ **43 interface required** *trusted* and *HB*; /* see Sec. 2.1 */ **macro** obsolete(r) := ($rxObsS[r.id] + 1 = r.seq \land trusted \subseteq r.recBy \land r.delivered$); 44 macro maxSeq(k) := max({s : (\bullet , id = k, seq = s, \bullet) \in buffer} \cup {next[k] - 1} \mid ; $\mathbf{45}$ **46** macro minTxObsS() := min{ $txObsS[k] : k \in trusted$ }; 47 operation urbBroadcast(m) do {wait(seq < minTxObsS() + bufferUnitSize); seq \leftarrow seq + 1; $update(m, i, seq, i); \};$ /* returns the transmission descriptor */ **48** procedure update(m, j, s, k) begin if $s \leq rxObsS[j]$ then return ; 49 if $(\bullet, id = j, seq = s, \bullet) \notin buffer \land m \neq \bot$ then 50 $buffer \leftarrow buffer \cup \{(m, j, s, \mathsf{False}, \{j, k\}, [-1, \dots, -1])\};$ 51else foreach $(\bullet, id = j, seq = s, \bullet, recBy = r, \bullet) \in buffer$ do $r \leftarrow r \cup \{j, k\};$ 5253 do forever begin if $(\exists r, r' \in buffer: r.msg = \bot \lor (r \neq r' \land ((r.id, r.seq) = (r'.id, r'.seq))))$ then $buffer \leftarrow \emptyset$; $\mathbf{54}$ if $\neg ((mS \leq seq \leq mS + bufferUnitSize) \land (\{mS + 1, \dots, seq\} \subseteq \{s : (\bullet, id = i, seq = s, \bullet) \in buffer\})$ 55 where $mS := \min \mathsf{TxObsS}()$ then $txObsS[] \leftarrow [seq, \ldots, seq];$ **foreach** $p_k \in \mathcal{P}$ **do** $(rxObsS[k]|, next[k]) \models (max\{rxObsS[k], maxSeq(k) - bufferUnitSize\},$ $\mathbf{56}$ $\max\{next[k], rxObsS[k]+1\}$); while $\exists r \in buffer : obsolete(r) \text{ do } rxObsS[r.id] \leftarrow rxObsS[r.id] + 1;$ $\mathbf{57}$ $buffer \leftarrow \{(\bullet, id = i, seq = s, \bullet) \in buffer : minTxObsS() < s\} \cup$ $\mathbf{58}$ $\{(\bullet, id = k, seq = s, \bullet) \in buffer : p_k \in \mathcal{P} \land ((rxObsS[k] < s \land \mathsf{maxSeq}(k) - \mathsf{bufferUnitSize} \le s))\};$ **foreach** $(msg = m, id = j, seq = s, delivered = d, recBy = r, prevHB = e) \in buffer$ **do** 59 if $(trusted \subseteq r) \land (\neg d) \land s = next[k]$ then 60 urbDeliver(m); $d \leftarrow \text{True}; next[k] \leftarrow next[k]+1$ 61 let u := HB: 62 for each $p_k \in \mathcal{P} : (k \notin r \lor (i = j \land s = txObsS[k] + 1)) \land (e[k] < u[k])$ do 63 $e[k] \leftarrow u[k]$; send MSG(m, j, s) to p_k ; /* piggyback lines 64 and 65 */ 64 for each $p_k \in \mathcal{P}$ do send GOSSIP(maxSeq(k), rxObsS[k], txObsS[k]) to p_k ; 65 66 upon MSG(m, j, s) arrival from p_k do {update(m, j, s, k); send MSGack(j, s) to p_k ; **67** upon MSGack(j, s) arrival from p_k do {update (\perp, j, s, k) ;} **68** upon GOSSIP(seqJ, txObsSJ, rxObsSJ) **arrival from** p_i **do** {(seq, txObsS[j], $rxObsS[j]) \leftarrow (\max\{seqJ, seq\}, \max\{txObsSJ, txObsS[j]\}, \max\{rxObsSJ, rxObsS[j]\}); \}$

receivers that they consider this message to be obsolete. It also keeps all non-obsolete messages (regardless of their sender).

(ii) Node p_i processes records by testing the field recBy of any not delivered message (line 59). The message is delivered when recBy encodes an acknowledgment from every trusted node (line 61). Processor p_i then marks the record as a delivered one and samples the HB failure detector (line 62). This sample is used to decide when a transmission (or retransmission) of a URB message is needed (line 63) in case an acknowledgment is missing or because the message sequence number is greater by one than the largest obsolete message number known to the sender. These messages are received and processed in line 66, which includes acknowledging the message arrival. These acknowledgments are processed in line 67.

(iii) At the end of the do-forever loop, p_i gossips to every p_k control information about the maximum seq value that p_i stores in a p_k record as well as p_k 's obsolete records (lines 65 and 68). The former value allows p_k to maintain the correctness invariant, *i.e.*, seq_k is not smaller than any other seq value in the system that is associated with p_k . The latter value allows p_k to control the flow of URB broadcasts according to the available space in $buffer_i$.

5 Correctness

This section brings the correctness proof of Algorithm 5. Theorem 5.2 demonstrates recovery after the occurrence of the last arbitrary transient fault. Theorem 5.3 demonstrates that Algorithm 5 satisfies the task specifications (Section 2.1).

5.1 Needed definitions

Definition 5.1 presents the necessary conditions for demonstrating that Algorithm 5 brings the system to a legal execution (Theorem 5.2).

Definition 5.1 (Algorithm 5's consistent sequence and buffer values) Let c be a system state and $p_i \in \mathcal{P}$ a non-faulty processor. Suppose that (i) $(\nexists r, r' \in buffer : r.msg = \bot \lor (r \neq r' \land ((r.id, r.seq) = (r'.id, r'.seq))))$, $((mS \leq seq_i \leq mS + bufferUnitSize) \land (mS+1, \ldots, seq_i) \subseteq$ $\{s : (\bullet, id = i, seq = s, \bullet) \in buffer_i\}$, $\forall p_k \in \mathcal{P} : (maxSeq_i(k) - rxObsS_i[k]) \leq bufferUnitSize$, $\nexists r \in buffer_i : obsolete(r), \forall (\bullet, id = i, seq = s, \bullet) \in buffer_i : mS < s, \forall (\bullet, id = k, seq = s, \bullet) \in$ $buffer_i : p_k \in \mathcal{P} \land rxObsS_i[k] < s \land maxSeq_i(k) \leq (s + bufferUnitSize)$, where $mS := minTxObsS_i()$. Moreover, (ii) seq_i is greater than or equal to any p_i 's sequence values in the variables and fields related to seq (including p_i 's records in $buffer_k$, where $p_k \in \mathcal{P}$ is non-failing, and incoming messages to p_k) and $\forall p_j \in \mathcal{P} : sMj \leq rxObsS_j[i]$, where sMj is either $txObsS_i[j]$ or the value of the fields txObsSJ and rxObsSJ in a GOSSIP($\bullet, txObsSJ, \bullet$) message in transit from p_j to p_i , and respectively, GOSSIP($\bullet, rxObsSJ$) message in transit from p_i to p_j . Also, (iii) $\forall k \in trusted_i : |\{(\bullet, id = i, \bullet) \in buffer_k\}| \leq bufferUnitSize \land seq_i \leq minTxObsS_i() + bufferUnitSize$. In this case, we say that p_i 's values in the variables and fields related to seq's sequence values and buffer are consistent in c.

We note that not every execution that starts from a system state that satisfies Definition 5.1 is a legal execution. For example, consider a system with $\mathcal{P} = \{p_i, p_j\}$ and an execution R, such that in its starting state it holds that $buffer_i = \{(m, i, 1, \bullet)\}$ and $buffer_j = \{(m', i, 1, \bullet)\}$, such that $m \neq m'$. The delivery of m and m' violates Definition 2.1's validity requirement because no legal execution

has R as a suffix. Theorem 5.3 circumvents this difficulty using Definition 5.2. Thus, definitions 5.1 and 5.2 provide the necessary and sufficient conditions for demonstrating self-stabilization.

Definition 5.2 (Complete execution with respect to urbBroadcast invocations) Let R be an execution of Algorithm 5. Let $c, c'' \in R$ denote the starting system states of R, and respectively, R'', for some suffix R'' of R. We say that message m is completely delivered in c if (i) the communication channels do not include $MSG(msg = m, \bullet)$ messages (or MSGack messages with a message identifier (id, seq) that refers to m), and (ii) for any non-failing $p_j \in \mathcal{P}$ and $r = (msg = m, \bullet) \in buffer_j$, it holds that r. delivered = True and for any non-failing $p_k \in \mathcal{P}$, we have $k \in r$.recBy. Suppose that $R = R' \circ R''$ has a suffix R'', such that for any urbBroadcast message m that is not completely delivered in c'', it holds that m either does not appear in c (say, due to an urbBroadcast(m) invocation during the prefix R') or it is completely delivered in c. In this case, we say that R'' is complete with respect to R's invocations of urbBroadcast messages. When the prefix R' is empty, we say that R is complete with respect to urbBroadcast invocations.

Theorems 5.2 and 5.3 consider Definition 5.3. For the sake of simple presentation, Definition 5.3 relies on the specifications' assumption (Section 2.1) that every broadcasted message is unique (even without an explicit assignment of the message identifier (id, seq) to m).

Definition 5.3 (The diffuse() **predicate)** Let $p_i \in \mathcal{P}$ and $c \in R$ be a system state. The predicate diffuse_i(m) holds in c if, and only if, $\exists (msg = m, \bullet, delivered = \mathsf{False}, \bullet) \in buffer_i$.

5.2 Basic facts

Both theorems 5.2 and 5.3 use Lemma 5.1.

Lemma 5.1 Let R be an execution of Algorithm 5 and $p_i, p_j \in \mathcal{P}$ be two non-failing processors. Suppose that in every system state $c \in R$, diffuse_i(m) holds, such that $(msg \neq \bot, \bullet, recBy = r, \bullet) \in$ buffer_i holds, but $j \in r$ does not. (i) Processor p_i sends, infinitely often, the message MSG(m, j, s)to p_j and p_j acknowledges, infinitely often, via the message MSGack(j, s) to p_i . (ii) The reception of any of these acknowledgments guarantees that $j \in r$ holds within cost measure of 2 (Section 2.3). (iii) Suppose that R is fair, then invariants (i) and (ii) occur within $\mathcal{O}(1)$ asynchronous cycles.

Proof. Invariants (i) and (ii). Since $j \in Correct$ and $j \notin r$ in c, processor p_i sends the message MSG(m, j, s) to p_j infinitely often in R (due to the do-forever loop, lines 59 and 64 as well as HB-liveness and this lemma's assumptions). Moreover, p_j receives p_i 's message (line 66), and acknowledges it, infinitely often, so that p_i receives p_j 's acknowledgment (line 67), infinitely often, while making sure that j is included in r (line 52). Since a single round-trip is required for the latter to hold, the cost measure is 2.

Invariant (iii). This is implied by Remark 2.3 applied to the proof of Invariant (ii).

5.3 The convergence property

Theorem 5.2 shows that the system reaches a state that satisfies Definition 5.1.

Theorem 5.2 (Convergence) Let R be a fair execution of Algorithm 5 that starts in an arbitrary system state. Within $\mathcal{O}(\text{bufferUnitSize})$ asynchronous cycles, the system reaches a state, $c \in R$, after which a suffix R' of R starts, such that R' is complete with respect to the urbBroadcast invocations in R. Moreover, seq and buffer are consistent in any $c' \in R'$ (Definition 5.1).

Proof. The proof is implied by arguments (1) to (6).

Argument (1): The case in which $\mathsf{MSG}(m, \bullet)$ (or its correspondent $\mathsf{MSGack}(\bullet)$) appears in a communication channel at R's starting system state. Suppose that in R's starting system state, it holds that $\mathsf{MSG}(m, \bullet)$ appears in an incoming communication channel to p_k . Since R is fair, then by Remark 2.1 it holds that within $\mathcal{O}(1)$ asynchronous cycles, $\mathsf{MSG}(m, \bullet)$, or respectively, $\mathsf{MSGack}(\bullet)$ arrives at its destination, p_k . For the case of $\mathsf{MSG}(m, \bullet)$, this arrival results in the execution of line 66 and then line 51 if m's record was not already in $buffer_k$. For the case of $\mathsf{MSGack}(\bullet)$ and $(m, \bullet) \in buffer_k$, line 67 has a similar effect. Moreover, by the code of Algorithm 5, the case of $\mathsf{MSGack}(\bullet)$ and $(m, \bullet) \notin buffer_k$ does not change p_k 's state. Therefore, without loss of generality, the rest of the proof can simply focus on the case in which R's starting system state, it holds that $\exists_{p_k \in \mathcal{P}} : (m, \bullet) \in buffer_k$. Note that by similar arguments, we can also consider the case in which the communication channels include message MSGack with a message identifier (id, seq) that refers to m.

Argument (2): Definition 5.1's Invariant (i) holds for the case in which $\exists_{p_k \in \mathcal{P}} : (m, \bullet) \in buffer_k$ in *R's starting system state.* Within $\mathcal{O}(1)$ asynchronous cycles, p_k runs a complete iteration of its do forever loop (lines 53 to 64). Invariant (i) is implied by lines 54 to 58.

Argument (3): seq_i is greater than or equal to any p_i 's sequence values in the variables and fields related to seq in c'. Within $\mathcal{O}(1)$ asynchronous cycles, every message that was present in a communication channel in R's starting system state arrives at the receiver. Therefore, without loss of generality, the proof can focus on the values of seq_i at the non-failing nodes $p_i, p_k \in \mathcal{P}$. Other than in seq_i , every sequence value that is related to p_i can only be stored in records of the form $(\bullet, id = i, seq = s', \bullet)$ that are stored in $buffer_k$. Suppose that in R's starting system state, it holds that $s' > seq_i$. By lines 65 and 68, within $\mathcal{O}(1)$ asynchronous cycles, p_k gossips $s_k \geq s'$ to p_i and the latter updates seq_i upon reception. The argument proof is complete because only p_i (line 47) can introduce new seq values that are associated with p_i , and thus, the argument invariant holds for any system state in R'.

Argument (4): Definition 5.1's Invariant (ii) holds in c'. The case of seq values is covered by Argument (3). Within an asynchronous cycle, every message arrives at the receiver. Therefore, without loss of generality, the proof can focus on the values of sMj at the non-failing node $p_i \in \mathcal{P}$. Suppose that in R's starting system state, the predicate $txObsS_i[j] \leq rxObsS_j[i]$ does not hold for some non-failing $p_j \in \mathcal{P} : j \in trusted_i$. By lines 65 and 68, within $\mathcal{O}(1)$ asynchronous cycles, p_j gossips $rxObsS_j[i]$ to p_i and the latter updates $txObsS_i[j]$ upon reception as well as p_i gossips $txObsS_i[j]$ to p_j and the latter updates $rxObsS_j[i]$ upon reception. Thus, Definition 5.1's Invariant (ii) holds is any system state that follows.

The rest of the proof assumes, without loss of generality, that Definition 5.1's invariants (i) and (ii) hold throughout R. Generality is not lost due to arguments (1) to (4).

Argument (5): $\forall k \in trusted_i : |\{(\bullet, id = i, \bullet) \in buffer_k\}| \leq bufferUnitSize holds in c' (first part of Definition 5.1's Invariant (iii)). Let <math>p_i, p_k \in \mathcal{P}$ be two non-faulty nodes. For the case of p_k 's records in $buffer_i$, Definition 5.1's Invariant (i) says that $\forall (\bullet, id = k, seq = s_k, \bullet) \in buffer_i : \max\{s'_k : (\bullet, id = k, seq = s'_k, \bullet) \in buffer_i\} \leq (s_k + bufferUnitSize)$. In other words, the largest sequence number, $\max\{s'_k : (\bullet, id = k, seq = s'_k, \bullet) \in buffer_i\}, of a p_k$'s records in $buffer_i$ minus bufferUnitSize must be smaller than s_k of any p_k 's records in $buffer_i$.

Argument (6): $seq_i < \min \mathsf{TxObsS}_i() + \mathsf{bufferUnitSize}$ holds in c' (second part of Definition 5.1's Invariant (iii)). Let $c \in R$ and $x_c = (seq_i - \min \mathsf{TxObsS}())$. Assume, towards a contradiction,

that $x_c \geq \text{bufferUnitSize}$ for at least $\mathcal{O}(\text{bufferUnitSize})$ asynchronous cycles. Let $A_c = \bigcup_{k \in trusted_i} \{r \in buffer_k : \neg \text{obsolete}_k(r) \land r.id = i\}$ and $B_c = \bigcup_{k \in trusted_i} \{r \in buffer_i : r.id = i \land txObsS_i[k] < r.seq\}$ as well as $rec \in A_c$ and $rec' \in B_c$ be the records with the smallest sequence number (among all the records with id = i) that p_k , and respectively, p_i stores in c. We start the proof by showing that, within $\mathcal{O}(1)$ asynchronous cycles, the system reaches a state $c' \in R$ for which $rec \notin A_{c'}$ and $rec' \notin B_{c'}$ hold. We then show that $x_{c'} < \text{bufferUnitSize}$ holds.

Showing that $rec \notin A_{c'}$ because obsolete(rec) holds. Let $p_i \in \mathcal{P}$ be a non-faulty node. Suppose that $\exists p_k \in \mathcal{P} : (\bullet, id = i, delivered = d_k, recBy = r_k, \bullet) \in buffer_k \land k \in trusted_i \land d_k = \mathsf{False}$ holds for some value of r_k in c. For any $p_j \in \mathcal{P}$ for which $j \in trusted_k$ holds throughout R's first $\mathcal{O}(1)$ asynchronous cycles, we know that the system reaches, within $\mathcal{O}(1)$ asynchronous cycles, a state in which $j \in r_k$ is true (invariants (i) and (ii) of Lemma 5.1). Once $\forall j \in trusted_k : j \in r_k$ holds, p_k assigns True to d_k (line 61). Thus, $rec \notin A_{c'}$ holds within $\mathcal{O}(1)$ asynchronous cycles in R. Moreover, obsolete(rec) holds due to the choice of rec as the one with the smallest sequence number.

Showing that $rec' \notin B_{c'}$. As long as $rec' \in B_c$, node p_i sends $\mathsf{MSG}(rec'.msg, rec'.id, rec'.seq)$ to p_k infinitely often (line 64). Within an asynchronous cycle, p_k receives this MSG message. Lines 66 and 49 imply that either $(rec'.msg, rec'.id, rec'.seq, \bullet) \in A_{c'}$ or $rec'.seq \leq rxObsS_k[i]$. Within $\mathcal{O}(1)$ asynchronous cycles, $(rec'.msg, rec'.id, rec'.seq, \bullet) \notin A_{c'}$ and $\mathsf{obsolete}(rec)$ hold (due the $rec \notin A_{c'}$ case) as well as $rxObsS_k[rec'.id] \geq rec'.seq$ (line 57). Moreover, by Argument (4)'s proof, $rec' \notin B_{c'}$ since $txObsS_k[rec'.id] \geq rec'.seq$ holds.

Showing that $x_{c'} < \mathsf{bufferUnitSize}$. Due to the assumption at the start of this proof, throughout R's first $\mathcal{O}(\mathsf{bufferUnitSize})$ asynchronous cycles, p_i does not increment seq_i and call $\mathsf{update}()$ (line 47). Thus, on the one hand, no new p_i 's record is added to buffer_k throughout R's first $\mathcal{O}(\mathsf{bufferUnitSize})$ asynchronous cycles (due to the assumption that appears in the start of this case), while on the other hand, within $\mathcal{O}(1)$ asynchronous cycles, the system reaches a state in which either p_i stops including p_k in $trusted_i$ or it removes at least one record from A_c and B_c . The latter can repeat itself at most $\mathsf{bufferUnitSize}$ times due to Argument (5). This completes the proof of the argument and the proof of the theorem.

5.4 The closure property

Theorem 5.3 considers system executions that reach suffixes, R, that satisfy definitions 5.1 and 5.2. Theorem 5.3 then shows that R satisfies Definition 2.1, *i.e.*, $R \in LE_{\text{URB}}$ is a legal execution.

Theorem 5.3 (Closure) Let R be an execution of Algorithm 5 that is complete with respect to urbBroadcast invocations (or R is a suffix of an execution $\mathcal{R} = \mathcal{R}' \circ R$ for which R is complete with respect to the invocation of urbBroadcast messages in \mathcal{R} , cf. Definition 5.2) and seq's sequence values are consistent in $c \in R$ (Definition 5.1). Algorithm 5 demonstrates in R a construction of the URB communication abstraction. Moreover, each invocation of the operation urbBroadcast() incurs $\mathcal{O}(n^2)$ messages and has the cost measure of 2 (Section 2.3).

Proof. We note that the property of validity holds with respect to Algorithm 5 due to this theorem's assumption about R's completeness, which can be made due to Theorem 5.2. We observe that Algorithm 5 guarantees, within $\mathcal{O}(1)$ asynchronous cycles, the property of integrity since only non-delivered messages can be delivered (line 61) and once delivered they cannot be delivered again (line 62).

The proof demonstrates claims 5.4 and 5.5 before showing the termination and quiescent properties. Let R be an execution of Algorithm 5. We note that no processor removes the processor identity ℓ from the set recBy throughput R.

Claim 5.4 Let $p_i, p_j \in \mathcal{P}$ be two non-faulty nodes. The fact that $diffuse_i(m)$ (Definition 5.3) holds for every system state $c \in R$ implies that eventually, $diffuse_i(m)$ holds in $c' \in R$.

Proof of claim. The proof is implied by arguments (2) and (3), which consider Argument (1).

Argument (1): Only due to lines 47 and 66 can diffuse_i(m) hold during R. Only line 51 can add $(m, k, s, \bullet) : m \neq \bot$ to buffer_i. This can only happen due to an earlier invocation of operation urbBroadcast(m) (line 47) or MSG message arrival (line 66).

Argument (2): Suppose that R includes a system state c'' in which $(m, k, s, r, \bullet) \in buffer_i : m \neq \perp \land j \in r$ holds. Then, $diffuse_j(m)$ holds in R (possibly before c''). Processor p_i adds j to the recBy field value, r, only due to the reception of MSG(m) (or it's correspondent $MSGack(\bullet)$) from p_j (lines 66 and 67). By the theorem's assumption that m does not appear in R's starting system state, it follows that p_j receives MSG(m). Immediately after the first reception, it holds that $(m, k, s, \bullet) \in buffer_j : m \neq \bot$ in $c' \in R$ (due to the execution of line 66 and then line 51). Thus, the predicate $diffuse_j(m)$ holds in c'.

Argument (3): Suppose that R includes no system state in which $(m, k, s, r, \bullet) \in buffer_i : m \neq \perp \land j \in r$ holds. Then, the predicate $diffuse_j(m)$ holds eventually. By the assumption that p_j is non-faulty and the HB-liveness property (Section 2.1), eventually, p_i does not suspect p_j and thus, it follows that p_i sends infinitely often the message MSG(m) to p_j (line 65). Due to the fair communication assumption, p_j receives MSG(m) infinitely often from p_i . Immediately after the first time in which p_j receives MSG(m), it holds that $(m, k, s, \bullet) \in buffer_j : m \neq \perp$ in $c' \in R$ (due lines 66 and 51). Thus, the predicate $diffuse_j(m)$ holds in c'.

Claim 5.5 Suppose that for any non-faulty $p_i \in \mathcal{P}$, there is a system state $c_i \in R$, after which diffuse_i(m) holds. Eventually, in R, any non-faulty $p_i \in \mathcal{P}$ raises urbDeliver_i(m).

Proof of claim. Note that this claim's assumption that $diffuse_i(m)$ always holds after c_i implies that eventually (m, \bullet) is always included in the foreach loop of line 59. Moreover, p_j raises the event $urbDeliver_j(m)$ only when the if-statement condition in line 61 holds. By diffuse()'s definition, $\exists (msg = m, \bullet, delivered = \mathsf{False}, recBy = r, \bullet) \in buffer_i$ for some value of r in every system state in R. The rest of the proof shows that $trusted_i \subseteq r$ holds eventually.

Argument (1): $(m, \bullet, recBy = r_i, \bullet) \in buffer_i \wedge Correct \subseteq r_i holds$. Lemma 5.1 and the claim's assumption that $diffuse_i(m)$ holds in every system state after c_i imply that $j \in r_i$ holds eventfully. Using arguments that are symmetric to the ones above, also $i \in r_j : (m, \bullet, recBy = r_j, \bullet) \in buffer_j$ holds eventfully. Thus, $Correct \subseteq r_i$ holds eventually.

Argument (2): $(m, \bullet, recBy = r_i, \bullet) \in buffer_i \wedge trusted_i \subseteq r_i holds$. By the Θ -liveness property, $trusted_i \subseteq Correct$ eventually. From Argument (1), we have that the system eventually reaches a state after which $trusted_i \subseteq recBy_i$ always holds. \Box

Proof of the termination property. Let $p_i, p_j \in \mathcal{P}$ be two non-faulty nodes. The proof is implied by the following arguments (1) and (2) as well as Argument (6) of Lemma 5.2.

Argument (1): p_j raises urbDeliver_j(m) eventfully when p_i invokes urbBroadcast_i(m). Since $i \in Correct$, the invocation of urbBroadcast_i(m) makes sure that $(m, \bullet) \in buffer_i$ in a way that

implies $diffuse_i(m)$ (by lines 47 and 51 since by Theorem 5.2's Argument (3), line 52 is not executed in steps that invoke $urbBroadcast_i(m)$, which increment seq_i). Since $diffuse_i(m)$ holds, $diffuse_k(m)$ holds eventually for any non-failing $p_k \in \mathcal{P}$ (Claim 5.4). This implies that p_j invokes urbDeliver(m)(Claim 5.5).

Argument (2): Suppose that a (correct or faulty) processor p_k invokes $urbDeliver_k(m)$. Any nonfailing node invokes urbDeliver(m). Immediately before p_k invokes $urbDeliver_k(m)$, it holds that $(m, \bullet, recBy = r_k, \bullet) \in buffer_k \land trusted_i \subseteq r_k$ (line 61). Due to the Θ -accuracy property, $\exists j \in$ $trusted_k \cap Correct$. By this theorem's assumption about R, the only way in which $j \in trusted_k \subseteq r_k$ can hold, is if, before p_k invokes $urbDeliver_k(m)$, node p_j had received $MSG(m, \bullet)$ (or its corresponding acknowledgment, $MSGack(\bullet)$) that was sent by p_k (due to reasons that are similar to the ones that appear in the proof of Argument (2) of Claim 5.4). Upon p_j 's first reception of $MSG(m, \bullet)$ (or $MSGack(\bullet)$), processor p_j stores m, such that $diffuse_j(m)$ holds immediately after (by similar reasons that appear in the proof of Argument (1)). Since $diffuse_j(m)$ holds, then $diffuse_\ell(m)$ holds eventually for any non-failing processor $p_\ell \in \mathcal{P}$ (Claim 5.4) and p_ℓ invokes $urbDeliver_\ell(m)$ (Claim 5.5).

Proof of the quiescence property. In the context of self-stabilization [12], quiescence is demonstrated by showing that any call to urbBroadcast(m) can result only in a finite number of $MSG(m, \bullet)$ (or corresponding $MSGack(\bullet)$) messages. Note that the reception of a MSGack() does not result in the sending of a message and the sending of a MSGack() is always due to the reception of a $MSG(m, \bullet)$ message. Moreover, processors that fail eventually, can only send a finite number of messages. Thus, without loss of generality, the proof focuses on the sending of $MSG(m, \bullet)$ messages by processors that never fail. The non-faulty $p_i \in \mathcal{P}$ can send $MSG(m, \bullet)$ message to $p_k \in \mathcal{P}$ only when $rec = (msg = m, id = j, seq = s, delivered, recBy = r, prevHB = e) \in buffer_i : (k \notin r \lor (i = j \land s = txObsS_i[k] + 1)) \land (e < u[k])$ holds (lines 59 and 64), where $u = HB_i$. In other words, once the system reaches $c \in R$ for which $j \in r$ holds and rec is obsolete, p_i stops sending $MSG(m, \bullet)$ to p_j . If this occurs for any $j \in Correct$, the proof is done. Thus, arguments (1) to (3) assume that the system does not reach c.

Argument (1): The case of $j \notin Correct$. Within a finite time, $HB_i[j]$ does not increase (the HB-completeness property). Thus, the system reaches within a finite time a suffix for which in any state the predicate (e[j] < u[j]) (line 64) does not hold, where $u = HB_i$. Thus, p_i does not send $\mathsf{MSG}(m, \bullet)$ messages to p_j during that suffix.

Argument (2): The case of $j \in Correct$ and $\neg(i = j \land s = txObsS_i[k] + 1)$. Suppose, forwards a contradiction, that p_i never stop sending $\mathsf{MSG}(m, \bullet)$ messages to p_j or that p_j never stop sending (the corresponding) $\mathsf{MSGack}(\bullet)$ messages to p_i . By Invariant (ii) of Lemma 5.1, p_i adds j to $r_i : (msg = m, \bullet, recBy = r_i, \bullet) \in buffer_i$. By Algorithm 5's code, p_i never removes j from r_i , and thus, the predicate $j \in r_i$ remains true forever. This contradicts the assumption made in the start of this case, and thus, the quiescence property holds for Algorithm 5.

Argument (3): The case of $j \in Correct$ and $(i = j \land s = txObsS_i[k] + 1)$. The proof here is by showing that p_i eventually stores a higher value in $txObsS_i[k]$ and thus this argument is true due to Argument (2). Assume, towards a contradiction, that $txObsS_i[k] = s - 1$ always holds. Since the transmission conditions (line 63) always holds, the sender p_i sends $\mathsf{MSG}(m, j, s, \bullet)$ infinitely often to p_k . Due to the communication fairness assumption, p_k eventually receives $\mathsf{MSG}(m, j, s, \bullet)$ and makes sure that $buffer_k$ stores m's record (line 66) or that $s \leq rxObsS_k[i]$ (line 49). It turns out that even when only the former case holds, eventually the latter case holds. Specifically, by Invariant (ii) of Lemma 5.1 and the proof of the URB-termination property, $\exists rec = (msg = m, id = j, seq = s, delivered = d, recBy = r, \bullet) \in buffer_k : d = \mathsf{True} \land trusted_k \subseteq r_k$ and thus $\mathsf{obsolete}(rec)$ holds eventually. But then, it must be that $rxObsS_k[i] \ge s$, which in turn implies that $txObsS_i[k] \ge s$ (due to lines 65 and 68 and the communication fairness assumption). This is a contradiction with this case assumption that $txObsS_i[k] = s - 1$. Thus, the argument and Algorithm 5 is quiescent.

6 Extension: FIFO Message Delivery

We discuss an extension for Algorithm 5 for ensuring "First-in, First-out" (FIFO) message delivery. This extension is marked by Algorithm 5's boxed code lines. Our solution uses a well-known approach, which can be found in [15, 20]. For the sake of completeness, we bring the definition of the FIFO-URB abstraction before discussing the implementation details and proof. The abstraction includes the operation fifoBroadcast(m) and the event fifoDeliver(m). Definition 6.1 requires FIFO-URB-broadcast messages to be delivered by their sending orders (per individual sender).

Definition 6.1 (FIFO Uniform Reliable Broadcast [20]) Let R be a system execution. We say that the system demonstrates in R a construction of the FIFO-URB communication abstraction if URB-validity, URB-integrity, and URB-termination requirements are satisfied (Definition 2.1) as well as the following property.

• **FIFO** message delivery. Suppose that $p_i \in \mathcal{P}$ takes a step that includes a call to fifoBroadcast(m) and calling to fifoBroadcast(m') (possibly in another step). No $p_j \in \mathcal{P}$ raises the event fifoDeliver(m') before taking raising the event fifoDeliver(m) (possibly in another step).

Our solution (Algorithm 5 including the boxed code lines) considers a well-known approach for ensuring the FIFO message delivery, which can be found in [15, 20]. We associate each message arrival with a predicate, cf. the boxed part of the if-statement condition of line 60, which is based on Definition 6.1's requirement. The predicate uses the array next[1..n] (line 42), which holds at the *j*-th entry the sequence number of the next-to-be-delivered message from p_j that receiver p_i is allowed to FIFO-deliver. As long the predicate does not hold, the message is buffered by the receiving end. The receiver can then FIFO deliver a pending message as soon as Definition 6.1-based predicate holds. The proposed self-stabilizing solution advances the one in [15, 20] with respect to recovery after the occurrence of transient faults, which requires also the use of bounded buffer size.

Lines 45, 56, 65 and 68 help to deal with the case in which node p_i is a receiver that holds at $next_i[j]$ a sequence number that is higher than the sender's sequence number, seq_j . This situation can only occur due to a transient fault and the concern here is that p_i might omit up to $next_i[j] - seq_j$ messages broadcast by p_j . Algorithm 5 overcomes such concerns by gossiping to the sender's next-to-be-delivered sequence number, cf. the boxed part of line 45's code, which is called in line 65. Upon the arrival of such gossip messages from p_j to p_j , node p_j can assure that seq_j is not smaller then $next_i[j]$. Moreover, line 56 helps p_i to make sure that $next_i[j]$ does not refer to an obsolete message, *i.e.*, $next_i[j] \leq rxObsS_i[j]$.

Theorem 6.1 Algorithm 5 is a self-stabilizing construction of a FIFO uniform reliable broadcast communication abstraction in any system in which URB can be built. Moreover, the operation of FIFO uniform reliable broadcast has URB's cost measures.

Proof. Claims 6.2 and 6.3 demonstrate the proof.

Claim 6.2 (Convergence) Let R be a fair execution of Algorithm 5 that starts in an arbitrary system state. Within $\mathcal{O}(1)$ asynchronous cycles, the system reaches a state, $c \in R$, after which a suffix R' of R starts, such that R' is complete with respect to the fifoBroadcast invocations in R. Moreover, seq and buffer are consistent in any $c' \in R'$ (Definition 5.1).

Proof of claim. The proof is along the same lines as the one of Theorem 5.2 with a minor revision of Argument (3). There is a need to consider not only the largest sequence number stored in *buffer* for a p_k 's message record, but also the value of next[k]. The boxed part of line 45's code implies the latter case.

Claim 6.3 (Closure) Let R be an execution of Algorithm 5 that is not necessarily fair but it is complete with respect to fifoBroadcast invocations (or R is a suffix of an execution $\mathcal{R} = \mathcal{R}' \circ R$ for which R is complete with respect to the invocation of fifoBroadcast messages in \mathcal{R} , cf. Definition 5.2). Moreover, seq and buffer are consistent in any $c \in R$ (Definition 5.1). Algorithm 5 demonstrates in R a construction of the FIFO uniform reliable broadcast.

Proof of claim. The proof is implied using the sequence numbers in seq and next[] as well as the URB communication abstraction and its properties, which Theorem 5.3 shows.

7 Bounded Self-stabilizing Uniform Reliable Broadcast

In this section, we explain how to transform our unbounded self-stabilizing URB algorithm to a bounded one. We note the existence of several such techniques, *e.g.*, Awerbuch *et al.* [5], Dolev *et al.* [13, Section 10] and Georgiou *et al.* [14]. The ideas presented in these papers are along the same lines. They present a transformation that takes a self-stabilizing algorithm for message passing systems that uses unbounded operation indices and transforms it into an algorithm that uses bounded indices. The transformation uses a predefined maximum index value, say, MAXINT = $2^{64} - 1$, and it has two phases. (Phase A) As soon as p_i discovers an index that is at least MAXINT, it disables new invocations of operations. (Phase B) Once all non-failing processors have finished processing their operations, the transformation uses an agreement-based global restart for initializing all system variables. After the end of the global restart, all operations are enabled. For further details, please see [5, 13, 14].

8 Conclusions

We showed how non-self-stabilizing algorithms [1, 15, 20] for (quiescent) uniform reliable broadcast can be transformed into one that can recover after the occurrence of arbitrary transient faults. This requires non-trivial considerations that are imperative for self-stabilizing systems, such as the explicit use of bounded buffers. To that end, we developed a flow-control scheme that allows our URB solution to serve as a basis for explicitly bounding the buffer size at the application layer. The need to have this new scheme shows that currently there no conclusive evidence for the existence of a meta-self-stabilizing scheme that can transfer any (or large family of) non-self-stabilizing algorithm from the textbooks into a self-stabilizing one. We simply need to study one problem at a time (and its non-self-stabilizing state-of-the-art) until we have an algorithmic toolkit that is sufficiently generic.

References

- Marcos Kawazoe Aguilera, Wei Chen, and Sam Toueg. Heartbeat: A timeout-free failure detector for quiescent reliable communication. In Distributed Algorithms, 11th International Workshop, WDAG '97, Saarbrücken, Germany, September 24-26, 1997, Proceedings, volume 1320 of Lecture Notes in Computer Science, pages 126–140. Springer, 1997. doi:10.1007/BFb0030680.
- [2] Marcos Kawazoe Aguilera, Wei Chen, and Sam Toueg. On quiescent reliable communication. SIAM J. Comput., 29(6):2040–2073, 2000. doi:10.1137/S0097539798341296.
- [3] Marcos Kawazoe Aguilera, Sam Toueg, and Borislav Deianov. Revising the weakest failure detector for uniform reliable broadcast. In Prasad Jayanti, editor, *Distributed Computing*, 13th International Symposium, Bratislava, Slovak Republic, September 27-29, 1999, Proceedings, volume 1693 of Lecture Notes in Computer Science, pages 19–33. Springer, 1999. doi:10.1007/ 3-540-48169-9_2.
- [4] Karine Altisen, Stéphane Devismes, Swan Dubois, and Franck Petit. Introduction to Distributed Self-Stabilizing Algorithms. Synthesis Lectures on Distributed Computing Theory. Morgan & Claypool Publishers, 2019. doi:10.2200/S00908ED1V01Y201903DCT015.
- [5] Baruch Awerbuch, Boaz Patt-Shamir, and George Varghese. Bounding the unbounded. In Proceedings IEEE INFOCOM '94, The Conference on Computer Communications, Thirteenth Annual Joint Conference of the IEEE Computer and Communications Societies, Networking for Global Communications, Toronto, Ontario, Canada, June 12-16, 1994, pages 776–783. IEEE Computer Society, 1994. doi:10.1109/INFCOM.1994.337661.
- [6] Kenneth P. Birman. A review of experiences with reliable multicast. Softw., Pract. Exper., 29(9):741-774, 1999.
- [7] Peva Blanchard, Shlomi Dolev, Joffroy Beauquier, and Sylvie Delaët. Practically self-stabilizing Paxos replicated state-machine. In NETYS, volume 8593 of LNCS, pages 99–121. Springer, 2014.
- [8] Tushar Deepak Chandra and Sam Toueg. Unreliable failure detectors for reliable distributed systems. J. ACM, 43(2):225–267, 1996. doi:10.1145/226643.226647.
- [9] Sylvie Delaët, Stéphane Devismes, Mikhail Nesterenko, and Sébastien Tixeuil. Snap-stabilization in message-passing systems. J. Parallel Distrib. Comput., 70(12):1220-1230, 2010. doi: 10.1016/j.jpdc.2010.04.002.
- [10] Edsger W. Dijkstra. Self-stabilizing systems in spite of distributed control. Commun. ACM, 17(11):643-644, 1974. doi:10.1145/361179.361202.
- [11] Shlomi Dolev. Self-Stabilization. MIT Press, 2000.
- [12] Shlomi Dolev, Mohamed G. Gouda, and Marco Schneider. Memory requirements for silent stabilization. Acta Inf., 36(6):447–462, 1999. doi:10.1007/s002360050180.
- [13] Shlomi Dolev, Thomas Petig, and Elad Michael Schiller. Self-stabilizing and private distributed shared atomic memory in seldomly fair message passing networks. CoRR, abs/1806.03498, 2018. URL: http://arxiv.org/abs/1806.03498, arXiv:1806.03498.

- [14] Chryssis Georgiou, Oskar Lundström, and Elad Michael Schiller. Self-stabilizing snapshot objects for asynchronous failure-prone networked systems. In Peter Robinson and Faith Ellen, editors, Proceedings of the 2019 ACM Symposium on Principles of Distributed Computing, PODC 2019, Toronto, ON, Canada, July 29 - August 2, 2019., pages 209–211. ACM, 2019. Also appeared in the proceedings of the 7th International Conference on Networked Systems NETYS as well as a technical report in CoRR abs/1906.06420. doi:10.1145/3293611.3331584.
- [15] Vassos Hadzilacos and Sam Toueg. A modular approach to fault-tolerant broadcasts and related problems. Technical report, Cornell University, Ithaca, NY, USA, 1994.
- [16] Damien Imbs, Achour Mostéfaoui, Matthieu Perrin, and Michel Raynal. Set-constrained delivery broadcast: Definition, abstraction power, and computability limits. In 19th Distributed Computing and Networking, ICDCN, pages 7:1–7:10. ACM, 2018. doi:10.1145/3154273. 3154296.
- [17] Leslie Lamport. The implementation of reliable distributed multiprocess systems. Computer Networks, 2:95–114, 1978. doi:10.1016/0376-5075(78)90045-4.
- [18] Leslie Lamport. Time, clocks, and the ordering of events in a distributed system. Commun. ACM, 21(7):558-565, 1978. doi:10.1145/359545.359563.
- [19] Michel Raynal. A case study of agreement problems in distributed systems: Non-blocking atomic commitment. In 2nd High-Assurance Systems Engineering Workshop (HASE '97), August 11-12, 1997, Washington, DC, USA, Proceedings, pages 209–214. IEEE Computer Society, 1997. doi:10.1109/HASE.1997.648067.
- [20] Michel Raynal. Fault-Tolerant Message-Passing Distributed Systems An Algorithmic Approach. Springer, 2018. doi:10.1007/978-3-319-94141-7.
- [21] Fred B. Schneider. Implementing fault-tolerant services using the state machine approach: A tutorial. ACM Comput. Surv., 22(4):299–319, 1990. doi:10.1145/98163.98167.