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El Hassan Zerrik · Oscar Castillo

Stabilization of Infinite Dimensional Systems

With Contribution by Lahcen Ezzaki
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Preface

Systems theory is a branch of applied mathematics, it is interdisciplinary and develops activities in fundamental research which are at the frontier of mathematics, automation and engineering sciences. It is everywhere, innumerable and daily, moreover is there something which is not system: it is present in medicine, in commerce, economy, in psychology, in biological sciences, in finance, in architecture (construction of towers, bridges, etc …), weather forecast, robotics, automobile, aeronautics, localization systems and so on. These are the few fields of application that are useful or even essential for our society. It is a question of studying the behavior of systems and acting on their evolution. Among the most important notions in system theory which attracted the most attention is stability.

The notion of stability is crucial. Indeed, the evolution of a dynamic system is described by a system of differential equations, partial differential equations... These systems generally have an infinite number of solutions, unless initial conditions are fixed. Depending on the choice of these initial values, there can be an infinity of operating regimes. To illustrate this idea, we cite the example of a clock. This one works with a well-determined amplitude of the pendulum although at the starting time, the pendulum can deviate more or less from its vertical position. If at the starting time of the clock one does not deviate sufficiently the pendulum, it will stop after a few oscillations. On the other hand, if the deviation is large enough, the amplitude of the oscillations of the pendulum will become fully determined after a short time and the clock will operate with this amplitude, for an infinite time. So the equations describing the clock functioning have two stationary solutions: an equilibrium position corresponding to the pendulum at rest and a periodic solution corresponding to the normal operation of the clock. So to understand the evolution of any dynamic system, it is important to know all the stable solutions of its equations, these are equilibrium states.

The existing literature on systems stability is quite important but disparate and the purpose of this book is to bring together in one document the essential results on the stability of infinite dimensional dynamical systems. In addition and as such systems evolve in time and space, explorations and research on their stability were mainly focused on the whole domain in which the system evolved. We have strongly felt that, in this sense, important considerations were missing: those which are consist in considering that the system of interest may be unstable on the whole domain but stable in a certain region of the whole domain. This is the case in many applications ranging from engineering sciences to living science. For this reason, we have dedicated this work to extension of classical results on stability to the regional case.

This book has its origins in the book I wrote with Professor El Jai [1]. An important consideration was that it should be accessible to mathematicians and to graduate engineering with a minimal background in functional analysis. Moreover, for the majority of the students this would be their only acquaintance with infinite dimensional system. It is organized by following increasing difficulty. The two first chapters deal with stability and stabilization of infinite dimensional linear systems described by partial differential equations.

The following chapters concern original and innovative aspects of stability and stabilization of certain classes of systems motivated by real applications, that is to say bilinear and semilinear systems. The stability of these systems has been considered from a global and regional point of view.

A particular aspect concerning the stability of the gradient has also been considered for various classes of systems.

This book is for students of master's degrees, engineering students and researchers interested in the stability of infinite dimensional dynamical systems, in various aspects.

Our thanks to Prof. Abdelhaq El Jai the president of the "Systems Theory" network who initiated research on applied mathematic in Morocco in 1978. He has greatly contributed to the development of research in systems theory in most Moroccan universities. To all the researchers of MACS Laboratory particularly M. Ouzahra and Y. Benslimane with whom I initiated the concept of regional stabilization. This new concept has led to developments of important results and to openings on other lines of research that can enrich approaches in systems theory, as well as all the colleagues who were part of their constructive remarks.

Finally we would like to thank the Hassan II Academy of Sciences and Techniques for its support, through the network *Systems Theory*, which made it possible to finalize this work.

Meknes, Morocco El Hassan Zerrik
August 2020

Reference

El Jai, A. and Zerrik, E. *Stabilité des systèmes dynamiques*, Presses Universitaires De Perpignan, 2014.

Contents

Notations

Ω	open of $\mathbb{R}^n$
$\partial\Omega$	boundary of Ω
$f(.,t)$	function : $x \in \Omega \mapsto f(x,t)$
$\frac{\partial f}{\partial x_i}$	partial derivative with respect to x_i
ω	subregion of Ω
χ_ω	characteristic function of $\omega \subset \Omega$
γ_0	trace application of order zero
∇y	gradient of y : $\nabla y = (\frac{\partial y}{\partial x_1}, ..., \frac{\partial y}{\partial x_n})$
Δy	Laplacien of y : $\Delta y = \sum_{i=1}^n \frac{\partial^2 y}{\partial x_i^2}$
$\Delta^2 y$	bilaplacien of y : $\Delta^2 y = \sum_{i=1}^n \frac{\partial^4 y}{\partial x_i^4}$
$D(A)$	domain of A linear operator
$N(A)$	kernel set of A
$Im(A)$	images set of A
A^{-1}	Inverse of A
A^*	adjoint operator of A
$\rho(A)$	resolvent set of A
$\sigma(A)$	pointwise spectrum of A
$r(A)$	spectral radius of A
$L^2(\Omega)$	space of square integrable functions on Ω
$H^1(\Omega)$	$\left\{y \in L^2(\Omega) \mid y' \in L^2(\Omega)\right\}$
$H_0^1(\Omega)$	$\left\{y \in H^1(\Omega) \mid y = 0 \text{ on } \partial\Omega\right\}$
$H^{\frac{1}{2}}(\partial\Omega) = Im(\gamma_0)$	Sobolev space of order $\frac{1}{2}$ on $\partial\Omega$
$H^s(\Omega)$	Sobolev space of order s
$\lvert\lvert\lvert . \rvert\rvert\rvert$	uniform norm on $\mathcal{L}(X)$
$<,>$	inner product defined on X
$\lvert\lvert . \rvert\rvert$	norm defined on X
$\lvert\lvert . \rvert\rvert_A$	graph norm
X_w	X endowed with its weak topology

$V \hookrightarrow X$	V is dense in X and the injection is continous
$y(t)$	solution of a system
$u(t), v(t)$	controls that act on the system
$\frac{dy(t)}{dt}$	derivative of the function y with respect to the variable t
$\frac{\partial y}{\partial x}$	partial derivative of the function y with respect to the variable x
$\frac{\partial^2 y}{\partial x^2}$	second partial derivative of the function y with respect to the variable x
X	the state space
X'	dual space of X
$\mathcal{L}(X,Y)$	bounded linear operators from X to Y
$\mathcal{L}(X)$	bounded linear operators from X to X
$\mathcal{R}(\lambda, A)$	resolvent of the operator A
$\rightarrow$	strong convergence
$\rightharpoonup$	weak convergence