John Cage's Number Pieces, a geometric interpretation of ”time brackets” notation

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John Cage's Number Pieces, a geometric interpretation of “time brackets” notation

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Abstract. Conceptual musical works that lead to a multitude of realizations are of special interest. One can’t talk about a performance without considering the rules that lead to the existence of that version. After dealing with similar works of open form by Iannis Xenakis, Pierre Boulez and Karlheinz Stockhausen, the interest in John Cage’s music is evident. His works are “so free” that one can play any part of the material; even a void set is welcomed. The freedom is maximal and still there are decisions to consider in order to make the piece playable. Our research was initially intended to develop a set of conceptual and software tools that generates a representation of the work as an assistance to performance. We deal here with the *Number Pieces* Cage composed in the last years of his life. Over time, we realized that the shape used to represent time brackets, brought important information for the interpretation and musical analysis. In the present text, we propose a general geometric study of these time brackets representations, while trying to make the link with their musical properties to improve the performance.

Keywords: Computer Aided Performance, Notation, Musical Graphic Representation

1 Introduction

The interpreter who approaches the music of John Cage composed after the middle of the 20th century is often disconcerted by a great freedom of execution, associated with a set of precise instructions. The result is that, each time, the musician is led to determine “a version,” and to decide on a choice among the free elements proposed by the piece. A fixed score is thus created, which can be used several times. The musician interprets “his version” while thinking that it conforms to the composer’s intentions. But in fact, most works of Cage composed after the 1950s should not be preconceived, prepared, “pre-generated” for several executions. Each interpretation should be unique and “undetermined.” It is in this sense that the use of the computer can help the performer: a program will allow the latter to discover without being able to anticipate what and when he plays. The performance of the work thus escapes the intention of the musician to organize the musical text.

2 John Cage's Number Pieces

The corpus of John Cage's late compositions (composed between 1987 and 1992) is known today as *Number Pieces*. Each work is named after the number of musicians involved; and the exponent indicates the order of the piece among the other compositions containing the same number of musicians [1].

Silence and Indeterminacy

In the course of his creative research as a composer, Cage has laid down essential structural elements. Thus, silence has been posited as an element of structure to be thought of in a new and positive way; not as an absence of sound, but as a diachronic element, a presence, an acoustic space. This innovative work concerning silence has itself evolved: at first it was conceived as giving the work its cohesion by alternating with sound, then Cage extended the reflection to a spatial conception: the silence is composed of all the ambient sounds which, together, form a musical structure. Finally, silence was understood as "unintentional," sound and silence being two modes of nature's being unintentional [2].

Moreover, in this desire to give existence to music by itself, Cage has resorted to various techniques of chance in the act of composition and principles of performance.

The principles of indetermination and unintentionality go in that direction. The principle of indetermination leads the musician to work independently from the others, thus introducing something unexpected in what the musical ensemble achieves. The performer, unaware of the production of his fellow musicians, concentrates on his own part and on the set of instructions. This requires great attention, even if the degree of freedom of the playing is high [3].

Time Brackets

In Cage's Number Pieces each individual part contains musical events with *time brackets*. Generally, an event consists of a score endowed with two pairs of numbers: time brackets (Fig. 1).

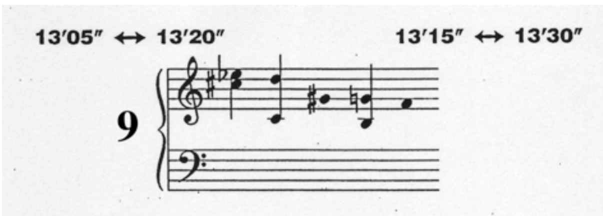

Fig. 1. John Cage's *Two*^{5}, piano, 9th event

This gives the interpreter lower and upper-time bounds to begin and end each event. The composition has a defined total duration and the events are placed inside a pair of the *time brackets*. Although there are only individual parts, a score for the group is implicitly present and leads to a form.

Earlier research

In previous work [8] we modeled these time brackets by parallelograms (see Figures 2 and 3) to build computer interfaces for interpretation assistance in the context of Cage’s *Two*^{5} (Fig. 2).

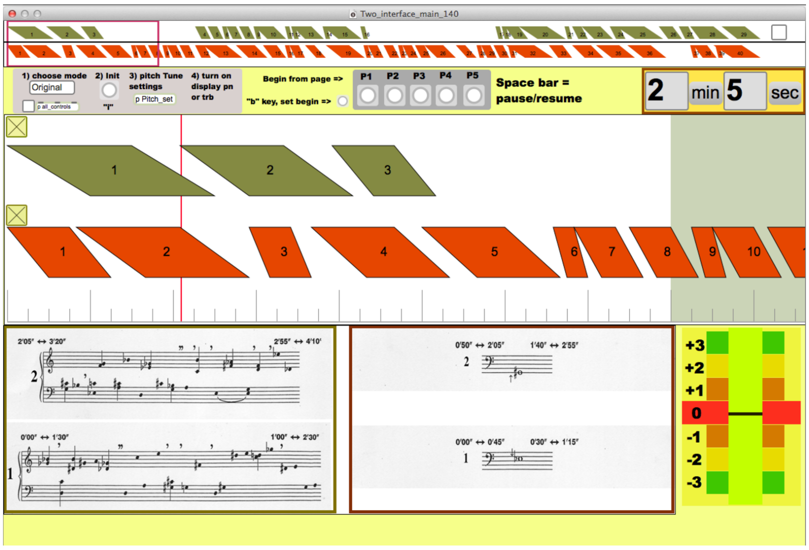

Fig. 2. Cage’s *Two*^{5} main computer interface

Over time ([9], [10], [11]), we realized that the shape used to represent time brackets, brought important information for the interpretation and musical analysis. The unusually long duration of this piece, 40 minutes, and the use of time brackets show that the temporal question, and its representation, is essential in the Number Pieces, in general, and in *Two*^{5} in particular.

The computer interface whose use has become obvious, has created for us a climate of confidence in our relationship to the piece. Random encounters of synchronicity as well as intervals bring unexpected situations…[12]

In the present text, we propose a general geometric study of these time brackets representations, while trying to make the link with their musical properties to improve the performance.

3 The Geometry of Time Bracket

The first step in the process was to model a graphic representation of each part as a succession of musical events in time. For this purpose, the temporal structure of the piece has been represented as quadruples on a timeline. $(s_l(k),\ s_u(k),\ e_l(k), e_u(k)\ )$.

In order to place an event k on the timeline, time brackets are defined as quadruples to indicate the time span allocated to it. Each quadruple consists of two pairs. More precisely, each pair gives the interpreter lower and upper time bounds to start $(s_l(k),\ s_u(k)\ )$ and to end $(e_l(k),\ e_u(k)\ )$. Theses closed time intervals give to the performer, a choice of the pair $(s(k),\ e(k)\ )$, where $(s_l(k)\ \le\ s(k) \le\ s_u(k)\ )$ and $(e_l(k)\ \le\ e(k) \le\ e_u(k)\ )$. One could choose the starting time $(s(k))$, while performing and, then accordingly, the end time $(e(k))$. This is the way one would employ when actually performing the work.

To obtain a graphic representation of each event in time we consider the quadruple:

$$(s_l(k),\ s_u(k),\ e_l(k), e_u(k)\ )$$

where $(s_l(k),\ s_u(k)\ )$ is the *Starting Time Zone* and $(e_l(k),\ e_u(k)\ )$ the *Ending Time Zone*. As the two intervals have, in our case, a designed superposition, we prefer to distinguish starting and ending zones by using two parallel lines (Fig. 3).

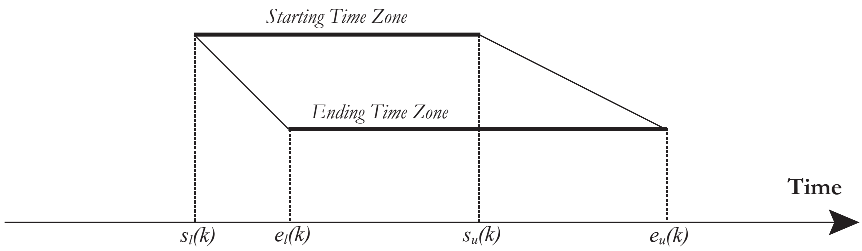

Fig. 3. Graphic representation for a generic time event

The graphic event obtained by connecting the four points has a quadrilateral shape. The height has no particular meaning. The *starting duration* $\delta_s(k)$ is defined as the difference: $(\ s_u(k) - s_l(k)\ )$, which is the time span the performer has to start the event. In the same way the *ending duration* $\delta_e(k)$ will be the time span given to end the event $(\ e_u(k) - e_l(k)\ )$. In the general case, these values are not the same, and the form we get is asymmetrical. When dealing with Cage's *Number Pieces*, one generally has: $\delta_s(k) =\ \delta_e(k)$, both durations are the same, and the figure to represent an event is a trapezoid (Fig. 4). This is the case in the majority of the corpus we are treating. Special cases will be mentioned later on.

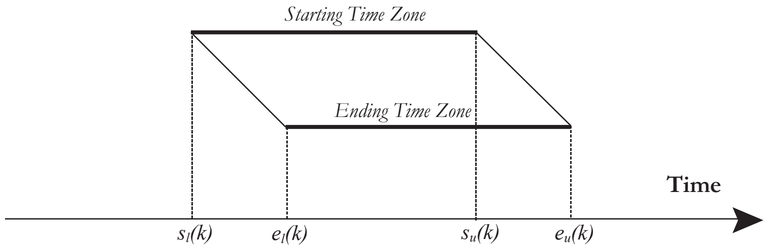

Fig. 4. Graphic representation for a time event in Cage's *Number Pieces*

There is mostly an overlapping of the two time zones, $(s_l(k),\ s_u(k))$ and $(e_l(k),\ e_u(k))$ but it can happen that those are disjoined. We can define a variable $\gamma(k)$ where: $s_l(k) + \gamma(k) = e_l(k)$. In Cage's *Number Pieces,* $\gamma(k)$ depends generally on the event duration. Thus, we don't have a big variety of forms. For example, in *Five*[3], we have only 4 different time brackets sorts, for a total number of 131 events for the five instruments and $\gamma(k) = \frac{2}{3}\delta\ (k)$ for all quadruples.

We make a distinction between a *generic musical event* and a *real* (or determined) *musical event*. A real musical event is the one whose starting points (s) and end points (e) are defined, that is, where there is a concretization of choice. One could represent this by a straight line from $s(k)$ to $e(k)$ (Fig. 5).

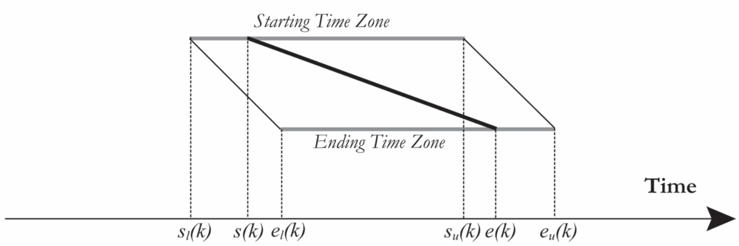

Fig. 5. A real music event represented by a straight line, joining the starting to ending time zones

There are certain properties of a generic event that can easily be deduced from the trapezoidal graphic representation:

1. The *starting* or *ending durations*: $\delta_s(k)$ or $\delta_e(k)$ are a kind of a nominal duration that Cage gives to an event.

2. The maximum duration, $e_u(k) - s_l(k) = \delta_{max}(k)$, is the maximum length (duration) an event can have.

3. The fact that, $s_u(k) > e_l(k)$ means that we can choose a starting point *s(k)* placed after the end, which leads to an empty musical event $\emptyset$ (an important idea of Cage: he often indicates that the artist can choose, all of, a part of, or nothing of the material placed at its disposal). In this case, *s(k)* > *e(k)*.

4. An alternative way to present a quadruple will be: $(s_l(k), \delta_s(k), \delta_e(k), \gamma(k))$ where $\gamma(k)$ is the value previously discussed. This representation can easily display the regularity in the time brackets construction (Fig. 6). It is easy to see that

$$\delta_{max}(k) = \frac{(\delta_s(k) + \delta_e(k))}{2} + \gamma(k).$$

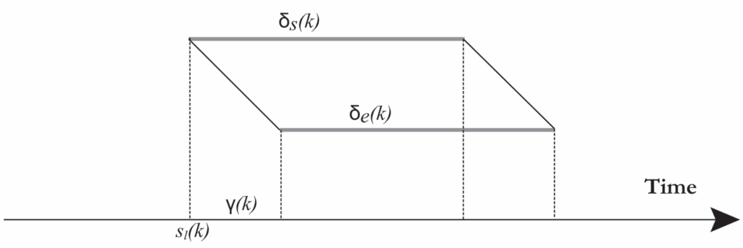

Fig. 6. An event represented as $(\boldsymbol{s_l(k)}, \boldsymbol{\delta_s(k)}, \boldsymbol{\delta_e(k)}, \boldsymbol{\gamma(k)})$

5. An implicit parameter that is important is the straight line's slope of the concrete event (Fig. 5). This value is inversely proportional to the concrete event duration. The slope is strongly related to performance: it shows how much time the performer has for a particular event k. In regard to a wind instrument part, often only composed by held notes, knowledge of this parameter allows the artist to better manage his air capacity, in order to respect the composer's indications. As far as the pianist is concerned, the slope gives some information that allows him to manage his interpretation with reference to the time indications. When the straight line of a concrete event is close to the vertical, the event will be short and concentrated.

The relationships of the generic events

Concerning the placement of two contiguous events *k* and *k+1* we can define a variable $\varepsilon(k)$, the gap between the elements *k* and *k+1* where:
$\varepsilon(k) = s_l(k+1) - e_u(k)$ (Fig. 7).

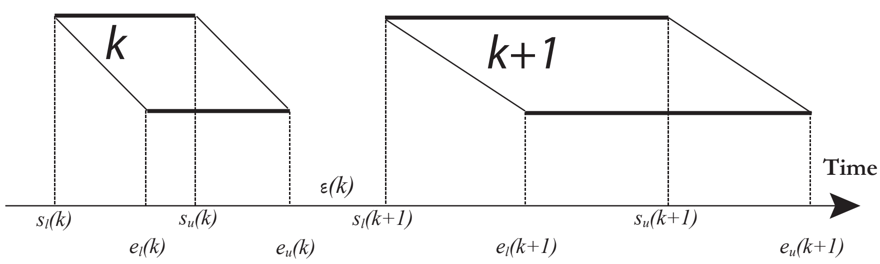

Fig. 7. $\boldsymbol{\varepsilon(k)}$**,** The gap between the elements *k and k+1*

We will observe five typical placements of two contiguous events.

1. $\varepsilon > 0$.

The two events are separated on the timeline. There is a minimum length of silence between the two events, which will probably be longer according to the choice of $e(k)$ and $s(k+1)$. In *Five*[3] for example, we have events 1 and 2 of violin 2 separated by more than 8 minutes, or 3 minutes between events 6 and 7 of violin 1. Here the piece could also be considered from the point of view of the relative density of the musical elements. One should mention the global statistical approach done elsewhere [4] [5].

2. $\varepsilon = 0$.

The two events are adjacent (Fig. 8).

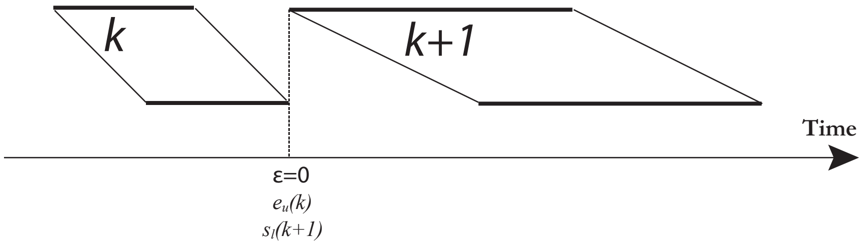

Fig. 8. $\varepsilon = 0$

Again, a gap may occur between the two events as the actual ending of event *k:* $e(k)$, and/or the actual starting of event *k+1,* $s(k+1)$ will differ from $e_u(k)$, and $s_l(k+1)$ correspondingly. For example, *Two*[5], trombone, events 21 and 22 (Fig. 9), events 27 and 28.

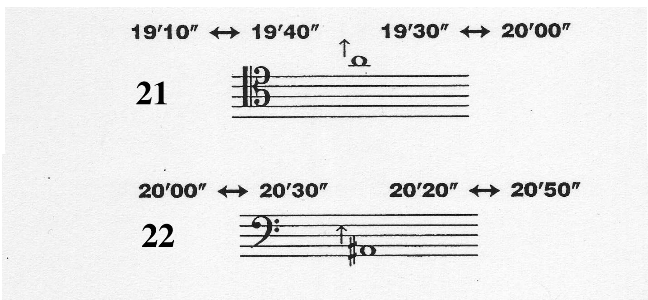

Fig. 9. *Two*[5], trombone, events 21 and 22

3. $\varepsilon < 0$.

In this case, the performer's opinion and attitude can determine the performance. There are many remarkable cases of interest in this situation; we could mention some cases that presently occur in Cage's *Number Pieces* (Fig. 10). For example, *Two*[5], trombone events 28 and 29, and piano events 6 and 7.

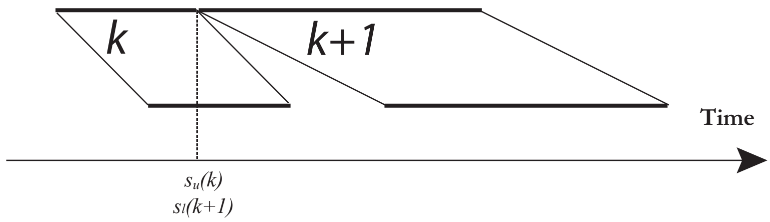

Fig. 10. $\varepsilon < 0, \boldsymbol{s_l(k+1) = s_u(k)}$

While performing event k, the player could start the event $k+1$ when not yet ending event k. We can encounter a superposition as shown in Fig. 11. For example, *Two*[5], trombone events 37 and 38; piano events 9 and 10, events 12 and 13.

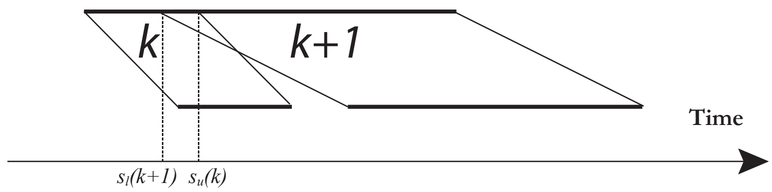

Fig. 11. $\varepsilon < 0, \boldsymbol{s_l(k+1) < s_u(k)}$

And even the same starting time for the two events: $s_l(k+1) = s_l(k)$ (Fig. 12). For example, *Two*[5], piano, events 14 and 15 (Fig. 13).

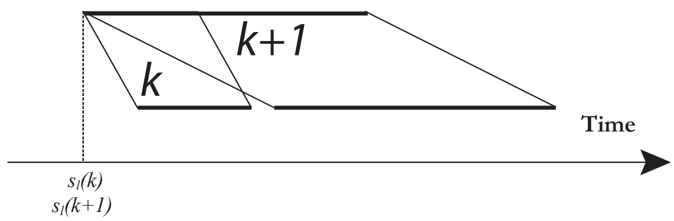

Fig. 12. $\varepsilon < 0, \boldsymbol{s_l(k+1) = s_l(k)}$

Fig. 13. *Two*[5], piano, events 14 and 15

As the events have an order given by Cage, one may assume that the sequence of events is to be respected. But the performer may consider mixing the two events and choosing the respective ending times, $e(k)$ and $e(k+1)$.

In some case one has the configuration shown in Fig. 14. For example, *Two*[5], trombone events 31 and 32, events 39 and 40.

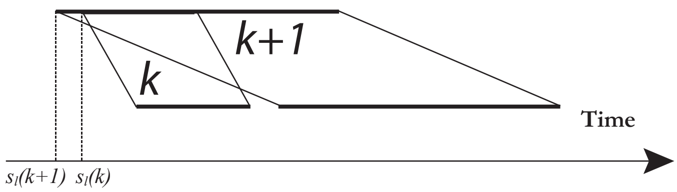

Fig. 14. $\varepsilon < 0$, $\boldsymbol{s_l(k+1) < s_l(k)}$

This may be a mistake, in calculation or in printing. Again, without change the order of events, one could start with the event k, and continue with the event $k+1$, mixing or separating. Starting with the event $k+1$ would mean that mixing has to happen, or the event k, should be skipped, that an idea dear to Cage: the event k wouldn't be performed.

The presentation of the time brackets as geometric figures and the variables we have defined lead to calculate some constants related to each of the instruments involved. The *average filling rate* ($\overline{Fr}$) gives an indication of how much a particular instrument is present during the piece. This value will be the ratio of the sum of all the events' duration by the overall length of the work (Δ), where the event duration, $\delta(k)$, is the arithmetic mean between $\delta_s(k)$ and $\delta_e(k)$ (**1**).

$$\overline{Fr} = \frac{\sum_1^n \delta(k)}{\Delta} \qquad \textbf{(1)}$$

In the analog way, if we set: $\varepsilon(0)$ be the gap before the first event, and $\varepsilon(n)$ the gap after the last event n, the *average silence rate* ($\overline{Sr}$) will be the ratio of the sum of all the gaps between the events by the overall length of the work (**2**).

$$\overline{Sr} = \frac{\sum_0^n \varepsilon(k)}{\Delta} \qquad (2)$$

These interesting values are based on the lengths of events, the gaps between them and their number, independent of the contents of the events.

If instead of using $\delta(k)$, the event duration, we consider $\delta_{max}(k)$, then:

$$\sum_{k=1}^{n} \delta_{max}(k) + \sum_{0}^{n} \varepsilon(k) \;= \Delta \qquad (3)$$

4 Musical Analysis Application

Table 1 shows the values for the 21 events of violin 1 in *Five*[3], and the constants we just defined. The time values, onsets and durations, are defined in seconds.

Table 1. Data for *Five*[3], first violin

	s_l (k)	δ_s (k)	δ_e (k)	γ (k)	ε (k)		
1	10	45	45	30	10	$\Delta=$	**2400**
2	150	45	45	30	65		
3	215	30	30	20	-10		
4	290	45	45	30	25	$\sum_{k=1}^{n} \delta(k) =$	**810**
5	405	45	45	30	40		
6	465	45	45	30	-15		
7	740	45	45	30	200	$\sum_{k=0}^{n} \varepsilon(k) =$	**1030**
8	1225	45	45	30	410		
9	1315	15	15	10	15		
10	1325	45	45	30	-15	$\overline{Fr} =$	**0,3375**
11	1475	15	15	10	75		
12	1570	30	30	20	70	$\overline{Sr} =$	**0,4292**
13	1625	45	45	30	5		
14	1685	45	45	30	-15		
15	1865	30	30	20	105		
16	1900	45	45	30	-15		
17	2060	45	45	30	85		
18	2165	45	45	30	30		
19	2235	15	15	10	-5		
20	2245	45	45	30	-15		
21	2305	45	45	30	-15		

The following Table 2, compares these constants for the five instruments. We can observe how these two constants ($\overline{Fr}$ *and* $\overline{Sr}$) are strongly related to the presence of the instruments. For example, trombone will be more present, more active than the string instruments. One can see that $\overline{Sr}$ may be negative. This occurs when many of the events are superposed (All cases with $\varepsilon < 0$).

Table 2. Comparison values in *Five³*

	#Events	$\overline{Fr}$	$\overline{Sr}$
Violin 1	21	0.34	0.43
Violin 2	12	0.16	0.74
Viola	26	0.34	0.44
Violoncello	25	0.23	0.5
Trombone	47	0.74	-0.24

These values are clearly reflected in the form of the piece seen in the upper part of Fig. 15. We had implemented several models, some offline in “OpenMusic”[1] computer aided composition software, and in a real-time “Max” software [8]. Fig. 15 presents a generic computer interface we are exploring, to perform most part of Cage’s *Number Pieces*.

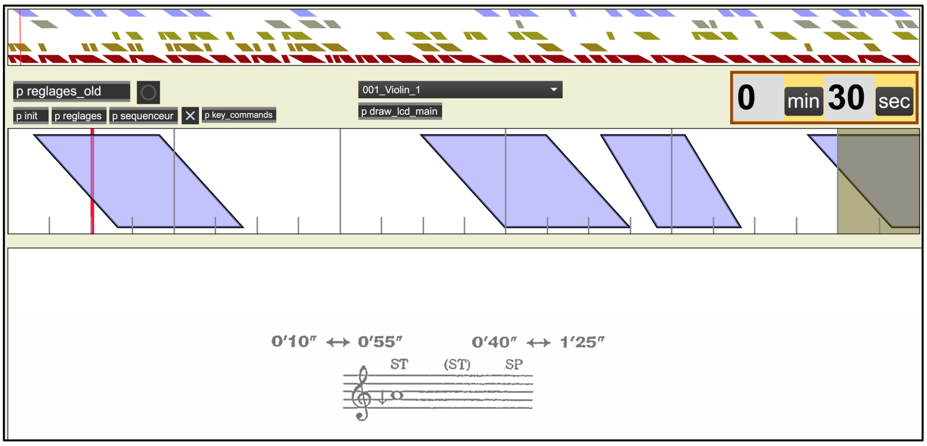

Fig. 15. Computer interface used for performing *Five³*

The medium part of this figure, displays one of the instruments chosen (here violin 1) and bottom part displays the musical score corresponding to the time (here 30 seconds after beginning). The global view displays a presentation of the entire duration of *Five³*, using the trapezoidal event representation. It allows the performer to have a global view of the piece at a glance. As Cage mention about the context-specific character of his time-bracket notation:

> *Then, we can foresee the nature of what will happen in the performance, but we can’t have the details of the experience until we do have it.* [6]

This global representation enables another perspective of the piece. The printed score orients a natural local view. More than being a graphic representation for each time bracket, it allows us to identify similarities between generic musical events. Fig. 16, a detail from Fig. 15, presents the first ten minutes of the global representation of *Five³*.

[1] “OpenMusic” is a software developed by Ircam by Gerard Assayag, Carlos Augusto Agon and Jean Bresson. See: http://recherche.ircam.fr/equipes/repmus/OpenMusic/.

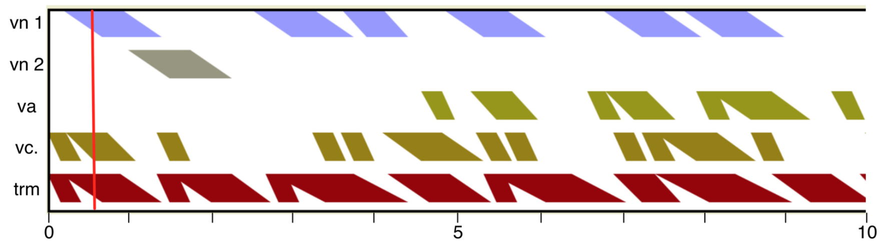

Fig. 16. The first ten minutes of the global representation in Cage's *Five*^{3}

In an analog way Table 3 presents $\overline{Fr}$ and $\overline{Sr}$ constants for *Two*^{5}, and Fig. 17 shows the global structure of the piece. One can clearly distinguish the difference in the presence of the two instruments.

Table 3. Comparison values in *Two*^{5}

	#Events	$\overline{Fr}$	$\overline{Sr}$
Piano	29	0.33	0.15
Trombone	40	0.46	-0.14

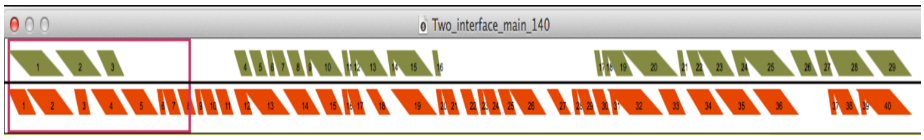

Fig. 17. Two^5 global structure

5 Conclusions

At the present time we work to offer the musicians a way to approach other pieces from the same family, constructing a generic interface. The task may be somewhat complicated. The works called *Number Pieces*, share the same principal described earlier, but often contain particularities and exceptions in the instructions for performance. The interface then has to be adapted to cover these.

The interface is a substitute to the printed score. It reveals the structure of the work and provides the performer with the tool to achieve the "meditative concentration" needed. The few instructions given by Cage are integrated in the interface.

Considering the graphic representation, we presented above, our main goal was to find geometric properties and strategies to enhance the performance of these pieces through computer interfaces. John Cage's works have been the target of our work for several years now. We have developed computer tools for the interface, and used it in practice. Both concerts and recordings have been the tests for the usefulness of the approach towards performance. The modeling process is transformed in a pragmatic analysis of the musical phenomena that leads us, step by step, to model some of Cage's concepts. Mentioning first the *Concert for Piano and Orchestra* (1957), an earlier work that has become important step of his output [7]. Followed by two of his number pieces for a small number of performers [8]. These works were also the object of a recording and performance sessions ([9], [10], [11]).

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