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# Introduction to Discrete Event Systems

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# Introduction to Discrete Event Systems

Third Edition

 Springer

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TO CAROL AND MONICA (C.G.C.)

TO JULIEN AND CLAIRE (S.L.)

# Preface — Third Edition

The third edition of *Introduction to Discrete Event Systems* is available as both a print copy and an e-book with hyperlinking capability. We hope the readers will appreciate this new option.

The third edition is a “superset” of the second one, with new material added principally in Chaps. 1, 2, 3, 5, 10, and 11. These additions are based on our teaching of discrete event systems courses at Boston University and at the University of Michigan, and they reflect active research trends in discrete event systems since the publication of the second edition. The additions consist of the inclusion of new topics as well as more thorough coverage of existing topics. For the benefit of readers familiar with the second edition, the main changes are summarized as follows.

- Chapter 1: additional examples of discrete event systems and more discussion on modeling.
- Chapter 2: new sections on opacity properties, labeled transitions systems, and formal verification and temporal logic; enhanced treatment of verification of diagnosability and codiagnosability properties, state space refinement, and strict subautomata; additional end-of-chapter problems.
- Chapter 3: new sections on state-based and liveness specifications, marking in specifications, maximal controllable and observable sublanguages, and marking supervisors; expanded treatment of control under partial observation, including state partition automata, supremal normal and controllable sublanguage, infimal observable and controllable superlanguage, and safe supervisors; more detailed treatment of verification of coobservability in decentralized control and safe decentralized supervision; additional end-of-chapter problems.
- Chapter 5: new section on event diagnosis.
- Chapter 10: updated section on discrete event simulation languages.
- Chapter 11: updated sections on extensions of IPA and on concurrent estimation.

While end-of-chapter references have been updated to reflect the new material included, we emphasize once again that these sections serve primarily as starting points for additional readings. The literature in discrete event systems is now vast and diverse, reflecting the growth in this field in the last 30 years.

Once again, we sincerely thank our colleagues, students, and readers for their constructive feedback over the last 12 years. We have tried to account for their comments in this third edition, but obviously our coverage of the growing field of discrete event systems is still very much incomplete. Nevertheless, we hope this book will continue to serve as a comprehensive *introduction* to the important class of dynamical systems known as *discrete event systems*. Several additional resources as well as software tools are mentioned throughout the book, although we have avoided explicit listings of URLs, since these tend to change frequently; however, the desired resources should be easily located by web searches.

Finally, it is a pleasure to acknowledge the leadership of Melissa Fearon and Wayne Wheeler at Springer throughout the course of this project.

Boston, USA  
Ann Arbor, USA

*Christos G. Cassandras*  
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# Preface — Second Edition

The second edition of *Introduction to Discrete Event Systems* improves upon the original edition in many respects. Immediately noticeable are the new cover and slightly larger format of this textbook. In terms of content, several revisions have been made to improve the presentation and enlarge the coverage. The most significant revisions are found in Chaps. 2, 3, 5, 10, and 11. We briefly describe the main changes for the benefit of readers familiar with the first edition.

- Several parts of Chap. 2 have been reorganized and additional material added on equivalence of automata and analysis (specifically, diagnosis) of discrete event systems.
- In Chap. 3, the topic of decentralized control is now covered in significantly more detail. In addition, a polynomial-time complexity test for the property of observability has been included.
- A brief introduction to the control of Petri nets based on the technique of place invariants has been added at the end of Chap. 4.
- Chapter 5 has been significantly expanded with two new topics: timed automata with guards and hybrid automata. These two modeling formalisms are introduced in a unified manner that builds on the earlier treatment of timed automata with clock structures.
- Chapter 10 now contains an updated section on discrete event simulation languages and related commercially available software.
- In Chap. 11, new material has been added on perturbation analysis for hybrid automata. In particular, Infinitesimal Perturbation Analysis (IPA) is presented for a class of hybrid automata known as stochastic fluid models. These are used as abstractions of complicated discrete event systems and are particularly useful in analyzing networks with very high traffic volumes.

The new material added in this second edition reflects to a large extent current active research trends in discrete event systems. The style of presentation remains as in the first edition, formal in nature but with numerous examples to help the reader. Whenever appropriate, additional end-of-chapter problems have been included for the new material.

There is a large and continuously growing literature in the field of discrete event systems. For this reason, it becomes more and more difficult to present a comprehensive list of references for any of the major topics covered in this book without inadvertently omitting some important ones. The sections titled “Selected References” at the end of the chapters should

thus be viewed as such, namely, small samples of relevant references, clearly biased by the authors' experience and knowledge.

We maintain a web site at Boston University<sup>1</sup> that provides additional information on the book and invites feedback for the authors. In particular, this web site points to a number of on-line resources, including a multimedia introduction to Discrete Event Systems, the UMDES-LIB library of routines, and a number of interactive java applets. Throughout the book, we also refer readers to various relevant web sites, including the site of the IEEE Control Systems Society Technical Committee on Discrete Event Systems,<sup>2</sup> whose aim is to promote communication between researchers and practitioners interested in discrete event systems.

This second edition would not have been possible without the constructive feedback that we have received over the last eight years from colleagues, students, and readers. The list is too long to enumerate here. We sincerely thank all of them. All of our graduate students over the last several years also deserve our most sincere gratitude, as working with them has continuously deepened our knowledge of the fascinating area of discrete event systems.

Finally, very special thanks go to Melissa Fearon at Springer for her persistent encouragement (and patience!) throughout this often-delayed second edition project.

*Christos G. Cassandras*  
*Stéphane Lafortune*

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<sup>1</sup>In 2021, the URL is <https://christosgassandras.org/introduction-to-discrete-event-systems/>.

<sup>2</sup>In 2021, the URL for this site is <http://discrete-event-systems.ieeecss.org/>.



# Preface

*Though this be madness, yet there is method in't.*

William Shakespeare, Hamlet

Over the past few decades, the rapid evolution of computing, communication, and sensor technologies has brought about the proliferation of “new” dynamic systems, mostly technological and often highly complex. Examples are all around us: computer and communication networks; automated manufacturing systems; air traffic control systems; highly integrated command, control, communication, and information (C<sup>3</sup>I) systems; advanced monitoring and control systems in automobiles or large buildings; intelligent transportation systems; distributed software systems; and so forth. A significant portion of the “activity” in these systems, sometimes all of it, is governed by operational rules designed by humans; their dynamics are therefore characterized by asynchronous occurrences of discrete *events*, some controlled (like hitting a keyboard key, turning a piece of equipment “on”, or sending a message packet) and some not (like a spontaneous equipment failure or a packet loss), some observed by sensors and some not. These features lend themselves to the term *discrete event system* for this class of dynamic systems.

The mathematical arsenal centered around differential and difference equations that has been employed in systems and control engineering to model and study the time-driven processes governed by the laws of nature is inadequate or simply inappropriate for discrete event systems. The challenge is to develop new modeling frameworks, analysis techniques, design tools, testing methods, and systematic control and optimization procedures for this new generation of highly complex systems. In order to face this challenge we need a multidisciplinary approach. First, we need to build on the concepts and techniques of system and control theory (for performance optimization via feedback control), computer science (for modeling and verification of event-driven processes), and operations research (for analysis and simulation of stochastic models of discrete event systems). Second, we need to develop new modeling frameworks, analysis techniques, and control procedures that are suited for discrete event systems. Finally, we need to introduce new paradigms that combine mathematical techniques with processing of experimental data. The role of the computer itself as a tool for system design, analysis, and control is becoming critical in the development of these new techniques and paradigms.

The capabilities that discrete event systems have, or are intended to have, are extremely exciting. Their complexity, on the other hand, is overwhelming. Powerful methodologies are needed not only to enhance design procedures, but also to prevent failures, which can indeed be catastrophic at this level of complexity, and to deliver the full potential of these systems.

## About this Book

A substantial portion of this book is a revised version of *Discrete Event Systems: Modeling and Performance Analysis*, written by the first author and published in 1993 (Irwin and Aksen Associates), which received the 1999 Harold Chestnut Prize, awarded by the International Federation of Automatic Control for best control engineering textbook. The present book includes additional material providing in-depth coverage of language and automata theory and new material on the supervisory control of discrete event systems; overall, it is intended to be a comprehensive introduction to the field of discrete event systems, emphasizing breadth of coverage and accessibility of the material to a large audience of readers with possibly different backgrounds. Its key feature is the emphasis placed on a unified modeling framework for the different facets of the study of discrete event systems. This modeling framework is centered on automata (and to a lesser extent on Petri nets) and is gradually refined: untimed models for logical properties concerned with the ordering of events, timed models for properties that involve timing considerations, and stochastic timed models for properties that involve a probabilistic setting. The unified modeling framework transcends specific application areas and allows linking of the following topics in a coherent manner for the study of discrete event systems: language and automata theory, supervisory control, Petri net theory,  $(\max, +)$  algebra, Markov chains and queueing theory, discrete-event simulation, perturbation analysis, and concurrent estimation techniques. Until now, these topics had been treated in separate books or in the research literature only.

The book is written as a textbook for courses on discrete event systems at the senior undergraduate level or the first-year graduate level. It should be of interest to students in a variety of disciplines where the study of discrete event systems is relevant: control, communications, computer engineering, computer science, manufacturing engineering, operations research, and industrial engineering, to name a few. In this regard, examples throughout the book are drawn from many areas such as control engineering, networking, manufacturing, and software engineering.

We have attempted to make this book as self-contained as possible. It is assumed that the background of the reader includes set theory and elementary linear algebra and differential equations. A basic course in probability theory with some understanding of stochastic processes is essential for Chaps. 6–11; a comprehensive review is provided in Appendix I. If readers have had an undergraduate course in systems and control, then the first part of Chap. 1 should be a refresher of fundamental modeling concepts. Some parts of Chaps. 3, 9, and 11 are more advanced and appropriate for graduate courses.

A senior-level one-semester course taught at the University of Massachusetts at Amherst covered the material in Chap. 1, parts of Chaps. 2 and 4, and most of Chaps. 5–8, and 10. A more advanced graduate-level course taught at Boston University is based on Chaps. 6 and 8–11, assuming knowledge of elementary random processes. At the University of Michigan, a first-year graduate course for students in electrical engineering and computer science covers Chaps. 1–5; no prerequisite is required.

## Acknowledgements

As mentioned earlier, a large part of this book is a revised version of *Discrete Event Systems: Modeling and Performance Analysis*, written by the first author and published in 1993. Therefore, several of the acknowledgements included in this earlier book are still relevant here (see below). Regarding the present book as a joint effort by the two authors, special acknowledgements go to George Barrett, Rami Debouk, Feng Lin, and Karen Rudie, for carefully reviewing earlier versions of the new contributions in this book and making numerous suggestions for improvement. Special thanks also go to Jianyang Tai, whose help with the formatting of the book and with figure generation has been vital, and to Christos Panayiotou for his problem-solving contributions to some sticky formatting issues (he is also an instrumental contributor to some of the material of Chap. 11).

### First Author

A large part of the material included in this book was written while I was on sabbatical at the Division of Applied Sciences at Harvard University (September 1990–January 1991). Professor Y. C. Ho was instrumental in providing me with a truly comfortable environment to do some serious writing during that period. I am also grateful to the Lilly Foundation for providing me with a Fellowship for the academic year 1991–1992, during which another substantial portion of this book was written. In addition, I would like to acknowledge the support of the National Science Foundation, which, through a grant under its Combined Research and Curriculum Development Program, supported the creation of a course on discrete event systems at the Department of Electrical and Computer Engineering at the University of Massachusetts at Amherst, and, since 1997, at the Department of Manufacturing Engineering at Boston University as well. A similar acknowledgement goes toward several other funding organizations (the Air Force Office of Scientific Research and the Air Force Research Laboratory, the Office of Naval Research and the Naval Research Laboratory, DARPA, and United Technologies/OTIS) that have provided, over the past several years, support for my research work; some of this work has given rise to parts of Chaps. 5, 6, 9, and 11.

A number of colleagues, friends, reviewers, and former and current students have contributed in a variety of ways to the final form of this book. I am particularly grateful to Y.C. (Larry) Ho, because my continuing interaction with him (from Ph.D. thesis advisor to friend and colleague) has helped me realize that if there is such a thing as “*joie de vivre*”, then there surely is something like “*joie de recherche*”. And it is under the intoxicating influence of such a “*joie de recherche*” that books like this can come to be.

For many stimulating discussions which directly or indirectly have influenced me, many thanks to Wei-Bo Gong, Rajan Suri, Xiren Cao, R. (R.S) Sreenivas, Al Sisti, Jeff Wieselthier, Yorai Wardi, and Felisa Vázquez-Abad. For their constructive suggestions in reviewing my prior book (of which this is largely a revised version), I am grateful to Mark Andersland (University of Iowa), Ed Chong (Purdue University), Vijay K. Garg (University of Texas at Austin), P. R. Kumar (University of Illinois), Peter Ramadge (Princeton University), Mark Shayman (University of Maryland), and Pravin Varaiya (University of California at Berkeley). My co-author, Stéphane Lafortune, was originally also a reviewer; I am now delighted to have had the opportunity to collaborate with him on this follow-up project. His deep knowledge of supervisory control has contributed material in Chaps. 2–4 which is a major asset of this new, truly comprehensive, book on DES.

For developing and testing the algorithm in Appendix II, I would like to thank Jie Pan and Wengang Zhai, former graduate students at the Control Of Discrete Event Systems (CODES) Laboratory at the University of Massachusetts at Amherst. All of my former and current

graduate students at the University of Massachusetts and at Boston University have had a role to play in the body of knowledge this book encompasses. Special thanks must go to my very first Ph.D. student, Steve Strickland, who made important contributions to the material of Chap. 11, but all the rest are also gratefully acknowledged.

Finally, I was grateful to Carol during the writing of my last book, and am even more grateful now after completing this one.

*Christos G. Cassandras*  
Boston, 1999

*Second Author*

I wish to thank all the graduate students and colleagues with whom I have built my knowledge of discrete event systems. My contributions to this book have been shaped by my interactions with them. I thank Hyuck Yoo, Enke Chen, Sheng-Luen Chung, Nejib Ben Hadj-Alouane, Raja Sengupta, Meera Sampath, Yi-Liang Chen, Isaac Porche, George Barrett, and Rami Debouk; I also thank all the students who have taken “EECS 661” at Michigan over the last 11 years. I thank Feng Lin (Wayne State University), Karen Rudie (Queen’s University), Kasim Sinnamohideen (Johnson Controls), Demosthenis Teneketzis (University of Michigan), and John Thistle (École Polytechnique de Montréal); doing research with them has been, and continues to be, inspiring and enjoyable. It is a pleasure to acknowledge the constant support that I have received since 1986 from the Department of Electrical Engineering and Computer Science at the University of Michigan regarding my research and teaching activities in discrete event systems. The National Science Foundation and the Army Research Office are also acknowledged for supporting my research in this area.

I extend very special thanks to Peter Ramadge (Princeton University), who first introduced me to the topic of discrete event systems and who has been a constant source of encouragement, and to W. Murray Wonham (University of Toronto), whose vision and seminal contributions have influenced greatly the field of discrete event systems. I am grateful to W. M. Wonham for kindly sharing with me his “Course Notes on Discrete Event Systems” for several years; these notes have been an important source of inspiration in the writing of Chap. 3.

Finally, I wish to thank Christos for asking me to contribute to the new version of this book. It has been a great experience.

*Stéphane Lafortune*  
Ann Arbor, 1999

## Organization of Book

The basic road map and organizational philosophy of this book are as follows.

- *Chapter 1* introduces the defining characteristics of discrete event systems and places the different chapters of this book in perspective. This is preceded by an introduction to the fundamental concepts associated with the theory of systems and control engineering in Sect. 1.2. The motivation for starting with Sect. 1.2 is to help the reader appreciate the distinction between *continuous-variable (time-driven) dynamic systems* and *event-driven dynamic systems*. Readers with a background in linear systems and control are likely to be familiar with the material in Sect. 1.2 but not with that in Sect. 1.3 where discrete event systems (DES) are introduced. On the other hand, readers with a background in computing science and systems are likely to be familiar with the concept of event-driven systems but not with the notions of the state space model and feedback control covered in Sect. 1.2, which are important for the subsequent chapters. Hybrid systems, which combine time-driven and event-driven dynamics, are introduced in Sect. 1.3.5.

The next 10 chapters are organized according to the level of abstraction (as defined in Sect. 1.3.3) chosen for modeling, analysis, control, performance optimization, and simulation of DES: *untimed* or *logical* (Chaps. 2, 3, and 4), *timed* and *hybrid* (Chap. 5), and *stochastic timed* (Chaps. 6–11).

- *Chapter 2* introduces language models of DES and the representation of languages by automata. The automaton modeling formalism is discussed in detail, including composition operations on automata by product and parallel composition, observer automata, and diagnoser automata. While many parts of Sects. 2.1–2.4 include “standard material” in formal languages and automata theory, the presentation is adapted to the needs and objectives of this book. Other parts of these sections are specific to DES research. Section 2.5 builds upon the techniques of the earlier sections and solves analysis problems for fully-observed and partially-observed DES. In particular, a detailed treatment of event diagnosis is included, followed by an introduction to opacity properties.
- *Chapter 3* presents an introduction to supervisory control theory. The goal is to study how to control a DES in order to satisfy a given set of logical (or qualitative) performance objectives on states and event ordering. The control paradigm is language-based and accounts for limited controllability of events and limited observability of events. The duality between language concepts and algorithmic techniques based on (finite-state) automata is emphasized. Generally speaking, this chapter is more advanced and suited for a graduate course. The material is based on research papers and monographs. Chapter 2 is a necessary prerequisite for this chapter.
- *Chapter 4* presents the modeling formalism of Petri nets and discusses the analysis and control of untimed Petri net models. While many references are made to concepts introduced in Chap. 2 (and in Chap. 3 for Sect. 4.5), this chapter is to a large extent self-contained.
- In *Chap. 5*, the two classes of untimed models studied in Chaps. 2 (automata) and 4 (Petri nets) are refined to include “time” by means of the *clock structure* mechanism, resulting in *timed automata* and *timed Petri nets*. An introduction to the technique of the

$(\max, +)$  algebra for analyzing certain classes of timed DES, particularly timed marked graphs, is presented. The modeling formalism of *timed automata with guards* is discussed. The chapter concludes with a brief introduction to *hybrid automata* for modeling a large class of hybrid systems.

It is suggested that Chaps. 1–5 may form the content of a course on modeling, analysis, diagnosis, and control of DES, with no consideration of probabilistic models.

- Starting with *Chap. 6*, the focus is on a *probabilistic setting* for the study of *stochastic timed* DES models. The timed automaton model of Chap. 4 is refined by the use of a *stochastic* clock structure, leading to *stochastic timed automata* and their associated *generalized semi-Markov stochastic processes* (GSMP). The *Poisson process* is then presented in depth as a “building block” for the stochastic clock structure of timed DES models. Using this building block (which is based on simple, physically plausible assumptions), the class of Markov chain models emerges rather naturally from the general GSMP model.

By the end of Chap. 6, two general directions emerge regarding the analysis of stochastic timed DES models. The first direction is based on classical stochastic models for which *analytical techniques* have been developed based on probability theory (Chaps. 7–9). In particular, Chap. 7 (Markov chains) and Chap. 8 (queueing theory) cover a limited class of stochastic DES models that can be handled through fairly traditional analysis. This material will look quite familiar to many readers.

The second direction relies on *computer simulation* and on some new techniques based on the analysis of sample paths of DES (Chaps. 10 and 11). It should be pointed out that the reader can go directly from Chaps. 6 to 10, completely bypassing the first direction if so desired.

- *Chapter 7* is concerned with the analysis of *Markov chain models*, introduced at the end of Chap. 6. Both discrete-time and continuous-time Markov chains are considered. The chapter also includes a treatment of birth–death chains and uniformization of continuous-time chains.
- *Chapter 8* is an introduction to *queueing theory*. Queueing models are arguably the most well-known and studied class of stochastic DES models. The material in Chap. 8 includes the standard key results on simple Markovian queueing systems ( $M/M/1$ ,  $M/G/1$ , etc.), as well as results on some special classes of queueing networks.
- While Chaps. 7 and 8 cover the analysis part of Markov chains, *Chap. 9* covers the *control* part, based on the technique of *dynamic programming*. This chapter involves more advanced material and requires some additional mathematical maturity; it is, therefore, more suited to graduate students.
- *Chapter 10* brings the reader back to the “real world”, where complex systems do not always conform to the “convenient” assumptions made in Chaps. 7–9 in the analysis of stochastic timed DES models; hence, the need for *simulation*. The goal here is to help the reader become comfortable with building simulation models for DES, to introduce some

basic techniques for analyzing the output data of a simulation for purposes such as estimating the performance of a complex DES, and to appreciate the advantages and limitations of such techniques.

The stochastic-timed automaton framework developed in Chap. 6 allows the introduction of discrete-event simulation in Chap. 10 to be particularly smooth and natural. This is because of the concept of a “clock structure” driving a DES considered in Chaps. 5 (deterministic case) and 6 (stochastic case). When this clock structure is supplied through a computer random number generator, a “simulation” is simply a software implementation of a stochastic timed automaton.

- *Chapter 11* presents sensitivity analysis and concurrent estimation techniques for DES. It develops the theory behind perturbation analysis and the fundamental *sample path constructability problem*, based on which methodologies and concrete algorithms for “rapid learning” in the control and optimization of DES have been developed. This material is based exclusively on recent research. The emphasis is on presenting key ideas and basic results from which the reader can proceed to more detailed and advanced material.

It is suggested that Chap. 6 along with Chaps. 8–11 may form the content of a more advanced graduate course on stochastic modeling, analysis, control, performance optimization, and simulation of DES.

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